

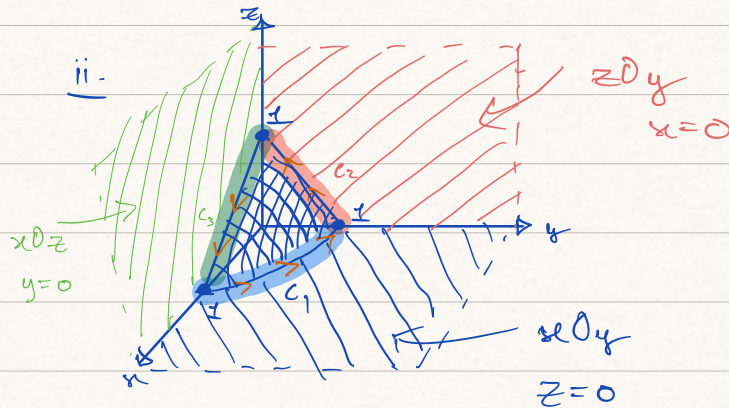
ASSIGNMENT 2

NAME : VISHNU PRASAD SIVAKUMAR , ID: 32491964

APPLIED CLASS DAY & TIME: FRIDAY 8AM - 10AM

APPLIED CLASS TUTOR: STEPHEN

1) i. Refer to matlab publication below.



$$\text{Circulation} = \oint_C y e^{xy} dx + x e^{xy} dy + xyz dz = \int_{c_1} + \int_{c_2} + \int_{c_3}$$

For $x dz$: $z=0, dz=0 \therefore (z^2-1)^2 = x^2 + y^2$

$$x^2 + y^2 = 1 \Rightarrow y = \sqrt{1-x^2}$$

$$\int_{c_1} = \int_{x=1}^{x=0} \sqrt{1-x^2} e^{x\sqrt{1-x^2}} + x e^{x\sqrt{1-x^2}} \cdot -\frac{x}{\sqrt{1-x^2}} dx \quad \frac{dy}{dx} = -\frac{x}{\sqrt{1-x^2}}$$

For $z dy$: $x=0, dx=0 \therefore \int_{c_2} = 0$

For $x dz$: $y=0, dy=0 \therefore \int_{c_3} = 0$

$$2) \quad U = e^{a \cdot r} = e^{(2,2,1) \cdot (x,y,z)} = e^{2x+2y+z}$$

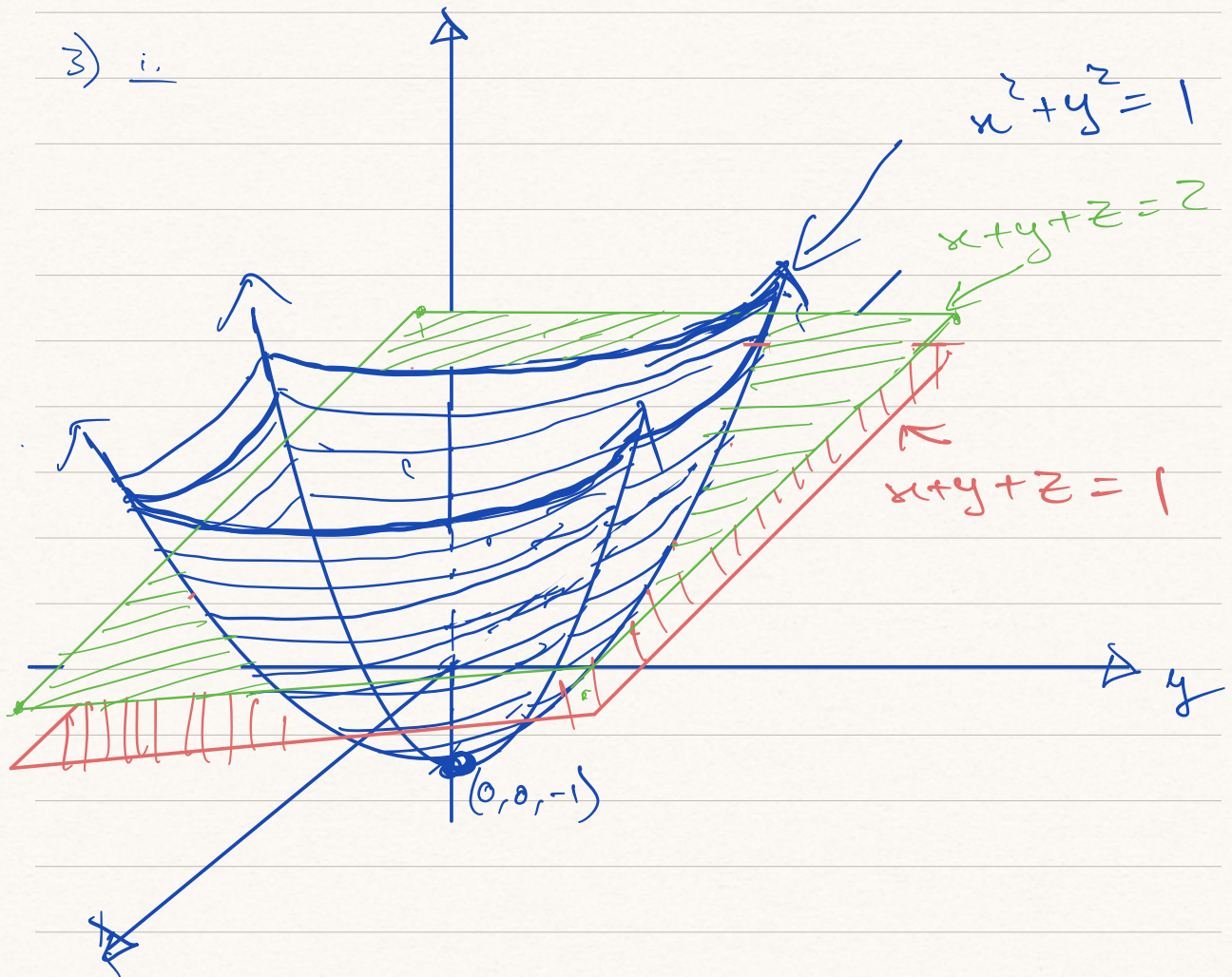
Therefore, level surface $U = e^{2x+2y+z}$

Hence, this is a family of rectangles

where $a \leq x \leq b$, $c \leq y \leq d$ $a \neq c$ & $b \neq d$

if $a = c$ and $b = d$, then this is a family of squares.

3) i.



3) ii. $x^2 + y^2 = 1$

By using cylindrical coordinates:

$$x = r \cos \theta, \quad y = r \sin \theta, \quad z = z$$

$$r^2 = 1 \quad \therefore r = 1 \quad (r \geq 0) \quad \therefore (x, y, z) = (\cos \theta, \sin \theta, z)$$

Hence r is constant

$$\therefore \text{Surface of cylinder} = C = \cos \theta \, \hat{i} + \sin \theta \, \hat{j} + z \, \hat{k}$$

Conditions: $0 \leq \theta \leq 2\pi$, $1 - \cos \theta \leq z \leq 2 - \cos \theta \Rightarrow 1 - \cos \theta - \sin \theta \leq z \leq 2 - \cos \theta \sin \theta$

$$\text{Normal of } C = \vec{N} = \frac{\partial C}{\partial \theta} \times \frac{\partial C}{\partial z}$$

$$\frac{\partial C}{\partial \theta} = -\sin \theta \, \hat{i} + \cos \theta \, \hat{j}, \quad \frac{\partial C}{\partial z} = \hat{k}$$

$$N = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{vmatrix} = \cos \theta \, \hat{i} + \sin \theta \, \hat{j} \quad \left(\text{which is positive outwards direction} \right)$$

$$\hat{n} = \frac{N}{|N|} = \frac{\cos \theta \, \hat{i} + \sin \theta \, \hat{j}}{\sqrt{\cos^2 \theta + \sin^2 \theta}}$$

$$\therefore \hat{n} = \cos \theta \, \hat{i} + \sin \theta \, \hat{j}$$

$$3) \text{ iii } \text{Flux} = \iint_S \mathbf{F} \cdot \hat{\mathbf{n}} \, ds = \int_{\theta=0}^{\theta=2\pi} \int_{z=1-x-y}^{z=2-x-y} (x, y, z) \cdot (\cos\theta, \sin\theta, 0) \, dz \, d\theta$$

$$\text{Flux} = \int_0^{2\pi} \int_{1-\cos\theta-\sin\theta}^{2-\cos\theta-\sin\theta} \{\cos\theta, \sin\theta, z\} \cdot \{\cos\theta, \sin\theta, 0\} \, dz \, d\theta$$

$$= \int_0^{2\pi} \int_{1-\cos\theta-\sin\theta}^{2-\cos\theta-\sin\theta} (\cancel{\cos^2\theta} + \cancel{\sin^2\theta}) \, dz \, d\theta = \int_0^{2\pi} \int_{1-\cos\theta-\sin\theta}^{2-\cos\theta-\sin\theta} 1 \, dz \, d\theta = \int_0^{2\pi} 1 \, d\theta = 2\pi$$

4) a Refer to matlab publication below.

b Refer to matlab publication below.

Approx cumulative length = 60.229 units.