

Year 2012

VCE

Mathematical Methods

CAS

Solutions

Trial Examination 2



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SECTION 1

ANSWERS

1	A	B	C	D	E
2	A	B	C	D	E
3	A	B	C	D	E
4	A	B	C	D	E
5	A	B	C	D	E
6	A	B	C	D	E
7	A	B	C	D	E
8	A	B	C	D	E
9	A	B	C	D	E
10	A	B	C	D	E
11	A	B	C	D	E
12	A	B	C	D	E
13	A	B	C	D	E
14	A	B	C	D	E
15	A	B	C	D	E
16	A	B	C	D	E
17	A	B	C	D	E
18	A	B	C	D	E
19	A	B	C	D	E
20	A	B	C	D	E
21	A	B	C	D	E
22	A	B	C	D	E

SECTION 1

Question 1

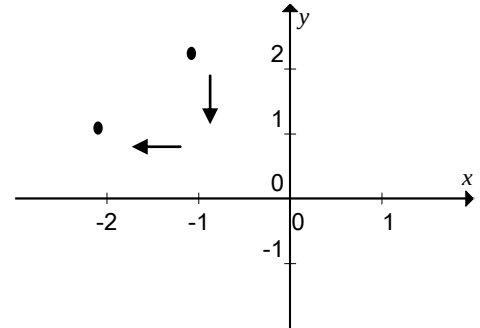
Answer B

The point is translated one unit down parallel to the y -axis and one unit to the left parallel to the x -axis.

$$g(x) = f(x+1) - 1$$

$$f(-1) = 2 \text{ and } g(-2) = 1$$

$$g(-2) = f(-1) - 1 = 2 - 1 = 1$$



Question 2

Answer A

$$\Delta = \begin{vmatrix} -2 & p-1 \\ p & -6 \end{vmatrix} = 12 - p(p-1) = 12 - p^2 + p$$

$$\Delta = -(p^2 - p - 12) = -(p-4)(p+3)$$

There is no unique solution when $p = 4$ or $p = -3$.

When $p = -3$ the equations become $-2x - 4y = 4$ and $-3x - 6y = 6$, and when $p = 4$ the equations become $-2x + 3y = 4$ and $4x - 6y = -8$, in both cases, the equations are just multiples of each other, and hence there is an infinite number of solutions when $p = 4$ or $p = -3$.

Question 3

Answer D

$$y_1 = x^2 + 5x, \quad y_2 = kx - 2 \quad y_1 = y_2$$

$$x^2 + 5x = kx - 2$$

$$x^2 + (5-k)x + 2 = 0$$

$$\Delta = (5-k)^2 - 8 = (5-k+2\sqrt{2})(5-k-2\sqrt{2})$$

For two distinct solutions, $\Delta > 0 \Rightarrow k > 5 + 2\sqrt{2}$ or $k < 5 - 2\sqrt{2}$

Question 4

Answer E

A. B. C. and D. are all true, $f(x) = x^3$ $f(a+b) = (a+b)^3 \neq f(a) + f(b) = a^3 + b^3$

Question 5

Answer C

A. B. D. and E. are all true. When $n = \frac{1}{2}$, the graph of $y = 2\sqrt{x-5} + 4$, has an endpoint at $x = 5$, it is not a vertical asymptote.

Question 6**Answer B**

$$h(x) = f(g(x)) \Rightarrow h(2) = f(g(2)) = f(3) = 4$$

$$h'(x) = g'(x) f'(g(x))$$

$$h'(2) = g'(2) f'(g(2)) = g'(2) f'(3) = 5 \times 7 = 35$$

Question 7**Answer C**

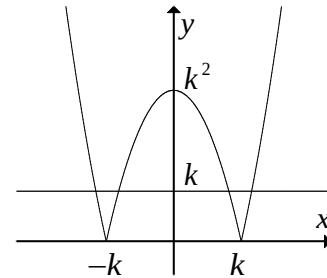
$$f(x) = \frac{g(x)}{\sqrt{x}} \quad \text{using the quotient rule}$$

$$f'(x) = \frac{g'(x)\sqrt{x} - \frac{g(x)}{2\sqrt{x}}}{x}$$

$$f'(4) = \frac{g'(4)\sqrt{4} - \frac{g(4)}{2\sqrt{4}}}{4} = \frac{6 \times 2 - \frac{8}{4}}{4} = \frac{10}{4} = \frac{5}{2}$$

Question 8**Answer E**

The graphs of $y = |x^2 - k^2|$ and $y = k$ have 4 points of intersection, when $k > 1$.

**Question 9****Answer C**

Although all the graphs, join up and are continuous at $x = 2$, however **C.** has a vertical asymptote at $x = \frac{3}{2}$ and is therefore, not continuous, over its maximal domain.

Question 10**Answer D**

$$a^{2x} = \frac{1}{b} \Leftrightarrow 2x = \log_a\left(\frac{1}{b}\right)$$

$$x = \frac{1}{2} \log_a\left(\frac{1}{b}\right) = -\frac{1}{2} \log_a(b) = -\frac{1}{2 \log_b(a)} \quad \text{by change of base rule, A. and B. are true.}$$

$$x = \frac{\log_{10}\left(\frac{1}{b}\right)}{2 \log_{10}(a)} = \frac{-\log_{10}(b)}{\log_{10}(a^2)} \quad \text{C. is true.}$$

$$x = \frac{1}{2} \log_a\left(\frac{1}{b}\right) = \log_a\left(\frac{1}{\sqrt{b}}\right) \quad \text{E. is true.}$$

D. is false.

Question 11**Answer D**

$$f: y = \frac{1}{x^2} + a \quad \text{swap } x \text{ and } y$$

$$f^{-1}: x = \frac{1}{y^2} + a \Rightarrow \frac{1}{y^2} = x - a \Rightarrow y^2 = \frac{1}{x - a} \Rightarrow y = \frac{\pm 1}{\sqrt{x - a}}$$

and $\text{ran } f = (a, \infty) = \text{dom } f^{-1}$ and $\text{ran } f^{-1} = (-\infty, 0) = \text{dom } f$, we must take the minus.

$$f^{-1}: (a, \infty) \rightarrow \mathbb{R}, f^{-1}(x) = \frac{-1}{\sqrt{x - a}}$$

Question 12**Answer E**

$$y = a - \frac{ax}{x - a} = a - \frac{a(x - a) - ax}{x - a} = -\frac{a^2}{x - a}$$

this graph has asymptotes at $x = a$ and $y = 0$

Question 13**Answer A**

Let the volume be $V \text{ cm}^3$, surface area $S \text{ cm}^2$, side length $L \text{ cm}$, at time t minutes.

$$V = L^3 \quad \frac{dV}{dL} = 3L^2 \quad \frac{dV}{dt} = -24 = \frac{dV}{dL} \frac{dL}{dt} = 3L^2 \frac{dL}{dt} \quad \text{when } L = 2 \Rightarrow \frac{dL}{dt} = -2$$

$$S = 6L^2 \quad \frac{dS}{dL} = 12L \quad \frac{dS}{dt} = \frac{dS}{dL} \frac{dL}{dt} = 12L \times -2 \quad \text{when } L = 2 \quad \frac{dS}{dt} = -48$$

Question 14**Answer C**

$$f(x) = e^{-2x} \cos\left(\frac{\pi x}{4}\right), \text{ now } f(2) = e^{-4} \cos\left(\frac{\pi}{2}\right) = 0 \quad f(0) = e^0 \cos(0) = 1$$

average rate of change over $0 \leq x \leq 2$

$$\frac{f(2) - f(0)}{2 - 0} = \frac{0 - 1}{2} = -\frac{1}{2}$$

Question 15**Answer B**

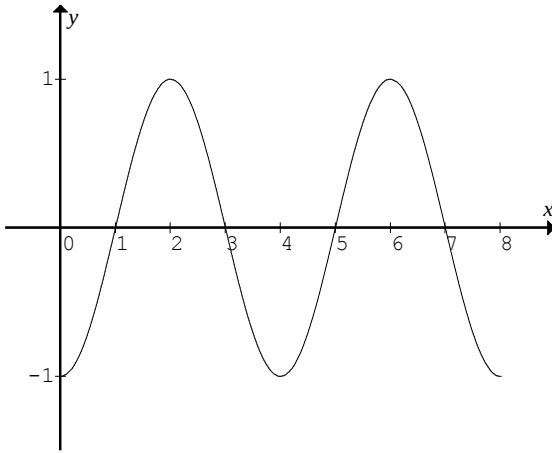
$$y = e^{-\frac{x}{2}} \sin\left(\frac{x}{2}\right) \quad \text{since } e^{-\frac{x}{2}} \neq 0 \text{ it crosses the } x\text{-axis at } \sin\left(\frac{x}{2}\right) = 0 \Rightarrow x = 2n\pi$$

$$\text{for turning points } \frac{dy}{dx} = \frac{1}{2} e^{-\frac{x}{2}} \left(\cos\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right) \right) = 0$$

$$\Rightarrow \tan\left(\frac{x}{2}\right) = 1 \Rightarrow x = (4n - 3) \frac{\pi}{2}$$

Question 16**Answer E**

$$f : (0, 8) \rightarrow \mathbb{R}, \quad f(x) = 2 \left(\sin \left(\frac{\pi(x-4)}{4} \right) \right)^2 - 1$$



All of **A. B. C.** and **D.** are correct, **E.** is false, the range is $[-1, 1]$

Question 17**Answer A**

$$y = \tan\left(\frac{x}{2}\right) \text{ at } x = \frac{\pi}{3} \quad \tan\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{3}$$

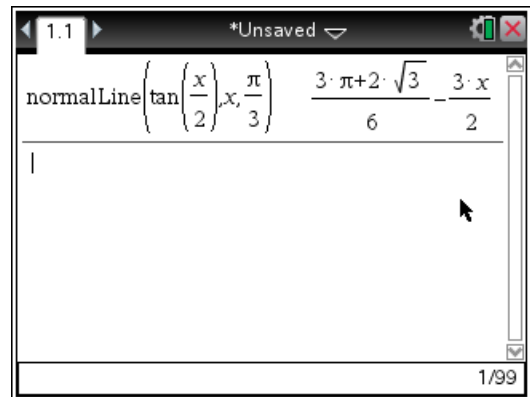
$$P\left(\frac{\pi}{3}, \frac{\sqrt{3}}{3}\right) \quad \frac{dy}{dx} = \frac{1}{2\cos^2\left(\frac{x}{2}\right)} \text{ at } x = \frac{\pi}{3}$$

$$\frac{dy}{dx} = m_T = \frac{1}{2\cos^2\left(\frac{\pi}{6}\right)} = \frac{2}{3} \quad m_N = -\frac{3}{2}$$

the equation of the normal

$$y - \frac{\sqrt{3}}{3} = -\frac{3}{2} \left(x - \frac{\pi}{3} \right)$$

$$y = -\frac{3x}{2} + \frac{\pi}{2} + \frac{\sqrt{3}}{3}$$

**Question 18****Answer D**

$$X \stackrel{d}{=} Bi(n=20, p=0.45), \quad E(X) = np = 20 \times 0.45 = 9$$

$$\Pr(X > 9) = \Pr(X \geq 10) = 0.409$$

Question 19**Answer B**

$$X \stackrel{d}{=} N(\mu = 240, \sigma^2 = 35^2)$$

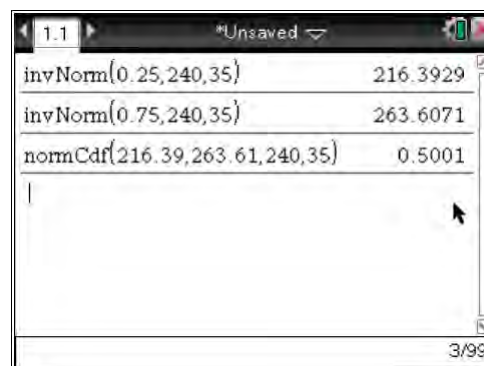
$$\Pr(a < X < b) = 0.50$$

$$a = ? \quad b = ?$$

$$\Pr(X < a) = 0.25$$

$$\Pr(X < b) = 0.75$$

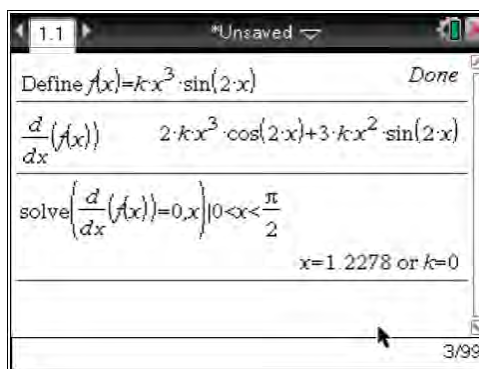
$$a = 216.39 \quad b = 263.61$$

**Question 20****Answer C**

The mode is the x value of the highest point on the graph, and occurs when $f'(x) = 0$

$$f'(x) = kx^2(3\sin(2x) + 2x\cos(2x)) = 0$$

solving for x , with $0 < x < \frac{\pi}{2}$ gives, $x = 1.23$

**Question 21****Answer A**

$$\sum \Pr(X = x) = c + 4c + 3c + 3c + 4c + 5c = 20c = 1 \Rightarrow c = \frac{1}{20}$$

X	49	50	51	52	53	54
$\Pr(X = x)$	0.05	0.20	0.15	0.15	0.20	0.25

The mean is $E(X) = \sum x \Pr(X = x)$

$$E(X) = 49 \times 0.05 + 50 \times 0.20 + 51 \times 0.15 + 52 \times 0.15 + 53 \times 0.20 + 54 \times 0.25 = 52.$$

Since $\sum_{49}^{52} \Pr(X = x) = 0.55 > 0.50$ the median is 52, the highest probability, 0.25 occurs

when $x = 54$, this is the mode.

Question 22**Answer E**

$\mu_2 > \mu_1$ and $\sigma_1 > \sigma_2$, since the mean of X_2 is to the right of X_1 , but the graph of X_1 is wider (more spread) than the graph of X_2 .

END OF SECTION 1 SUGGESTED ANSWERS

SECTION 2**Question 1**

a.i. $A(0,2) \quad f(0)=2 \Rightarrow \sqrt{p}=2 \Rightarrow p=4$
 $B(2,0) \quad f(2)=0 \Rightarrow \sqrt{4+2q}=0 \Rightarrow q=-2$ M1

ii. $A = \int_0^2 \sqrt{4-2x} \, dx = \frac{8}{3} = 2\frac{2}{3} \text{ metres}^2$ A1

iii. $s(x) = \sqrt{x^2 + y^2}$ with $y = f(x) = \sqrt{4-2x}$
 $s(x) = \sqrt{x^2 + (\sqrt{4-2x})^2}$ A1
 $s(x) = \sqrt{x^2 + 4 - 2x}$

iv. $\frac{ds}{dx} = \frac{2x-2}{2\sqrt{x^2+4-2x}} = 0$ for a maximum or minimum A1
 $\Rightarrow x=1$ A1
and $s_{\min} = s(1) = \sqrt{3}$ metres A1

b.i. $B(2,0) \quad f(2)=0 \Rightarrow 4 - e^{2k} - e^{-2k} = 0$ let $u = e^{2k}$
 $4 - u - \frac{1}{u} = 0 \Rightarrow u + \frac{1}{u} - 4 = 0$, multiply by u
 $u^2 - 4u + 1 = 0$ M1
 $u^2 - 4u + 4 = 3$
 $(u-2)^2 = 3$ M1
 $u-2 = \pm\sqrt{3}$
 $u = e^{2k} = 2 \pm \sqrt{3}$ but $k > 0$ take the positive M1
 $u = e^{2k} = 2 + \sqrt{3}$
 $k = \frac{1}{2} \log_e(2 + \sqrt{3})$

ii. $A = 4 - \int_0^2 (4 - e^{kx} - e^{-kx}) \, dx$ with $k = \frac{1}{2} \log_e(2 + \sqrt{3})$
 $A = \frac{4\sqrt{3}}{\log_e(2 + \sqrt{3})} - 4 = \frac{2\sqrt{3}}{k} - 4 \text{ metres}^2$ A1
 $a=2 \quad b=3 \quad \text{and} \quad c=-4$

iii. $s(x) = \sqrt{x^2 + (f(x))^2}$

with $f(x) = 4 - e^{kx} - e^{-kx}$ and $k = \frac{1}{2} \log_e(2 + \sqrt{3})$ A1

$\frac{ds}{dx} = 0$ for a maximum or minimum, $\Rightarrow x = 1.467$ A1

$s_{\min} = s(1.467) = 1.771$ metres A1

Define $f(x) = 4 - e^{kx} - e^{-kx}$	Done
solve($f(2)=0, k$) $ k>0$	$k = \frac{\ln(\sqrt{3}+2)}{2}$
$k := \frac{\ln(\sqrt{3}+2)}{2}$	$\frac{\ln(\sqrt{3}+2)}{2}$
$4 - \int_0^2 f(x) dx$	$\frac{2 \cdot (\sqrt{3}+2) - \frac{2}{\sqrt{4 \cdot \sqrt{3}+7}} - 4}{\ln(\sqrt{3}+2)}$
$\text{comDenom}\left(\frac{2 \cdot (\sqrt{3}+2) - \frac{2}{\sqrt{4 \cdot \sqrt{3}+7}} - 4}{\ln(\sqrt{3}+2)}\right)$	$\frac{4 \cdot \sqrt{3} - 4 \cdot \ln(\sqrt{3}+2)}{\ln(\sqrt{3}+2)}$
Define $s(x) = \sqrt{x^2 + (f(x))^2}$	Done
solve($\frac{d}{dx}(s(x))=0, x$) $ 0 < x < 2$	$x = 1.0000E-38$ or $x = 1.4673$
$xm := 1.4673342655874$	1.4673
$s(xm)$	1.7709
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Question 2

a.i. H uses a hand razor, E uses an electric razor.

$$H \rightarrow E = 0.15, E \rightarrow H = 0.25, H \rightarrow H = 0.85, E \rightarrow E = 0.75$$

$$\Pr(E \text{ 3 times}) = EHEE + EEHE + EEEH$$

$$= 0.25 \times 0.15 \times 0.75 + 0.75 \times 0.25 \times 0.15 + 0.75^2 \times 0.25$$

$$= 0.197$$

A1

ii. $\Pr(E \text{ on Thursday}) = EEEE + EEHE + EHEE + EHHE$

$$= 0.75^3 + 0.75 \times 0.25 \times 0.15 + 0.25 \times 0.15 \times 0.75 + 0.25 \times 0.85 \times 0.15$$

$$= 0.510$$

A1

$$\begin{array}{cc} & H & E \\ \text{or alternatively} & \begin{bmatrix} 0.85 & 0.25 \\ 0.15 & 0.75 \end{bmatrix} \end{array}$$

$$\begin{bmatrix} 0.85 & 0.25 \\ 0.15 & 0.75 \end{bmatrix}^3 \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.49 \\ 0.51 \end{bmatrix}$$

iii. $\frac{0.15}{0.15 + 0.25} = 0.375$

or alternatively $\begin{bmatrix} 0.85 & 0.25 \\ 0.15 & 0.75 \end{bmatrix}^n \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.625 \\ 0.375 \end{bmatrix}$ for $n \geq 19$

in the long run, the probability of using an electric razor is 0.375

A1

b.i. $H \stackrel{d}{=} Bi(n = 20, p = 0.4)$

$$\Pr(8 \leq H \leq 12) = 0.5631$$

A1

ii. $H \stackrel{d}{=} Bi(n = 20, p = ?)$

$$\Pr(9 \leq H \leq 11) = 0.25$$

$$\binom{20}{9} p^9 (1-p)^{11} + \binom{20}{10} p^{10} (1-p)^{10} + \binom{20}{11} p^{11} (1-p)^9 = 0.25 \text{ by CAS}$$

$$-16796 p^9 (p-1)^9 (9p^2 - 9p + 10) = 0.25$$

M1

solving with $0 < p < 1$ gives $p = 0.3625$ or 0.6375

but since $E(X) < 10 \Rightarrow p < 0.5$ so $p = 0.3625$

A1

c. X is the time in seconds spent cleaning, $X \stackrel{d}{=} N(\mu = ?, \sigma^2 = ?)$

$$(1) \quad \Pr(X < 130) = 0.16$$

$$(2) \quad \Pr(X > 264) = 0.18$$

$$(1) \Rightarrow \frac{130 - \mu}{\sigma} = -0.9945 \quad \text{M1}$$

$$(2) \Rightarrow \frac{264 - \mu}{\sigma} = 0.9154$$

$$(1) \quad 130 - \mu = -0.9945\sigma \quad \text{M1}$$

$$(2) \quad 264 - \mu = 0.915\sigma$$

now subtract equations (2) – (1) solving gives

$$\sigma = 70 \text{ substituting gives } \mu = 200 \text{ seconds} \quad \text{A1}$$

d.i. Since the total area under the curve is equal to one.

$$\int_0^{\infty} k t e^{-\frac{t^2}{8}} dt = 4k = 1 \Rightarrow k = \frac{1}{4} \quad \text{A1}$$

$$\text{ii.} \quad \Pr(0 < T < 3) = \frac{1}{4} \int_0^3 t e^{-\frac{t^2}{8}} dt = 1 - e^{-\frac{9}{8}} \quad \text{M1}$$

$$\Pr(0 < T < 3) = 0.6753 \quad \text{A1}$$

$$\text{iii.} \quad E(T) = \frac{1}{4} \int_0^{\infty} t^2 e^{-\frac{t^2}{8}} dt = 2.5066$$

$$\text{mean time } E(T) = 2.51 \text{ minutes} \quad \text{A1}$$

$$E(T^2) = \frac{1}{4} \int_0^{\infty} t^3 e^{-\frac{t^2}{8}} dt = 8$$

$$sd(T) = \sqrt{8 - 2.5066^2} = 1.31 \text{ minutes} \quad \text{A1}$$

$$\text{iv.} \quad \text{median } m, \text{ satisfies } \frac{1}{4} \int_0^m t e^{-\frac{t^2}{8}} dt = \frac{1}{2} \text{ solving, with } m > 0, \text{ gives}$$

$$m = 2.355 \text{ minutes} \quad \text{A1}$$

$\begin{bmatrix} 0.85 & 0.25 \\ 0.15 & 0.75 \end{bmatrix}^3 \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix}$	$\begin{bmatrix} 0.4900 \\ 0.5100 \end{bmatrix}$
$\frac{0.15}{0.15+0.25}$	0.3750
$\begin{bmatrix} 0.85 & 0.25 \\ 0.15 & 0.75 \end{bmatrix}^{19} \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix}$	$\begin{bmatrix} 0.6250 \\ 0.3750 \end{bmatrix}$
$\text{binomCdf}(20, 0.4, 8, 12)$	0.5631
$nCr(20, 9) \cdot p^9 \cdot (1-p)^{11} + nCr(20, 10) \cdot p^{10} \cdot (1-p)^{10} + nCr(20, 11) \cdot p^{11} \cdot (1-p)^9$	$-16796 \cdot p^9 \cdot (p-1)^9 \cdot (9 \cdot p^2 - 9 \cdot p + 10)$
$\text{solve}(-16796 \cdot p^9 \cdot (p-1)^9 \cdot (9 \cdot p^2 - 9 \cdot p + 10) = 0.25, p) 0 < p < 1$	$p = 0.3625 \text{ or } p = 0.6375$
$\frac{130-m}{s} = \text{invNorm}(0.16, 0, 1)$	$\frac{130-m}{s} = -0.9945$
$\frac{264-m}{s} = \text{invNorm}(0.82, 0, 1)$	$\frac{264-m}{s} = 0.9154$
$\text{solve}\left(\frac{130-m}{s} = -0.99445789074249 \text{ and } \frac{264-m}{s} = 0.91536508202965, \{m, s\}\right)$	$s = 70.1636 \text{ and } m = 199.7747$

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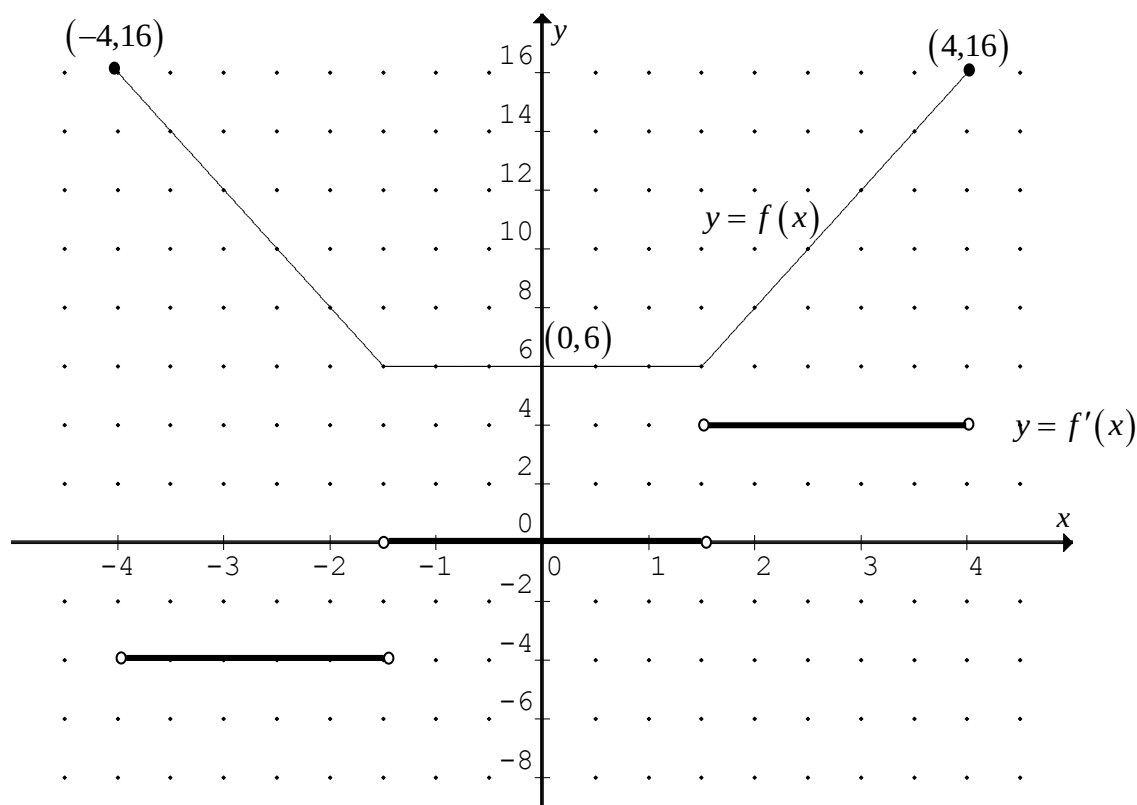
$\frac{-t^2}{8}$	Done
Define $f(t) = k \cdot t \cdot e^{\frac{-t^2}{8}}$	
$\text{solve}\left(\int_0^\infty f(t) dt = 1, k\right)$	$k = \frac{1}{4}$
$\frac{-t^2}{8}$	Done
Define $f(t) = k \cdot t \cdot e^{\frac{-t^2}{8}} k = \frac{1}{4}$	
$\int_0^3 f(t) dt$	$\left(\frac{9}{e^{\frac{9}{8}} - 1}\right) \cdot \frac{-9}{e^{\frac{9}{8}}}$
$\left(\frac{9}{e^{\frac{9}{8}} - 1}\right) \cdot \frac{-9}{e^{\frac{9}{8}}}$	0.6753
$\int_0^\infty (t \cdot f(t)) dt$	2.5066
$\int_0^\infty (t^2 \cdot f(t)) dt$	8
$\sqrt{8 - (2.5066282746378)^2}$	1.3103
$\text{solve}\left(\int_0^m f(t) dt = 0.5, m\right) m > 0$	$m = 2.3548$

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Question 3

a.i. $f(x) = |2x+3| + |2x-3|$

$f(-4) = |5| + |-11| = 16$, $f(4) = |11| + |-5| = 16$, $f(0) = 6$



Correct graph of $y = f(x)$, shape and continuous at $x = \pm \frac{3}{2}$, A1

y-intercept, $(0, 6)$, endpoints $(-4, 16), (4, 16)$. A1

ii. Correct shape of $y = f'(x)$, the graph of $y = f'(x)$ is not defined at A1

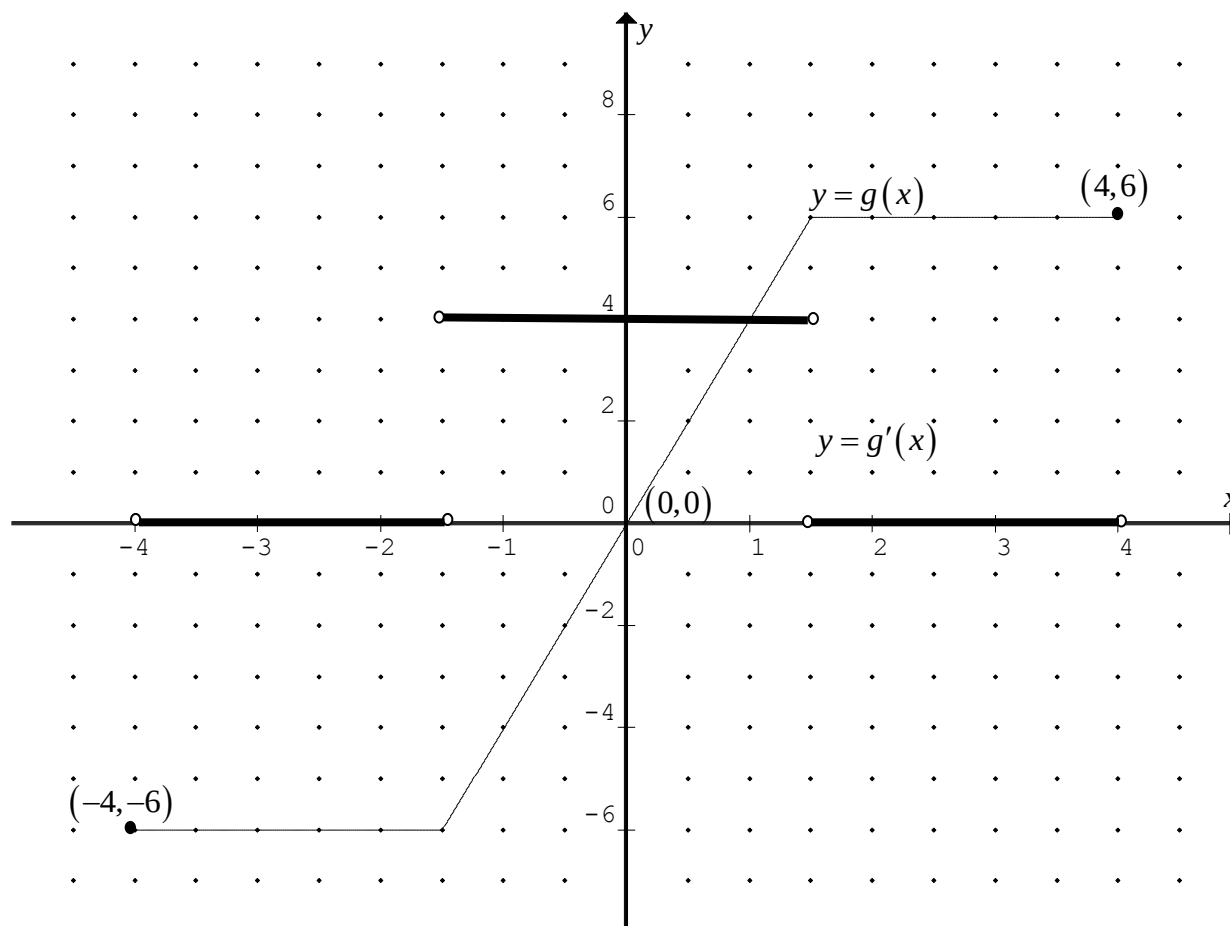
$x = \pm 4$ and $x = \pm \frac{3}{2}$ and must have open circles at $x = \pm 4$ and $x = \pm \frac{3}{2}$ A1

b. $f(x) = |ax+b| + |ax-b|$

$$f'(x) = \begin{cases} 2a & \text{for } x \in \left(\frac{b}{a}, \infty\right) \\ 0 & \text{for } x \in \left(-\frac{b}{a}, \frac{b}{a}\right) \\ -2a & \text{for } x \in \left(-\infty, -\frac{b}{a}\right) \end{cases} \quad \text{must have open brackets} \quad \text{A2}$$

c.i. $g(x) = |2x+3| - |2x-3|$

$g(-4) = |-5| - |-11| = -6$, $g(4) = |11| - |5| = 6$, $g(0) = 0$



Correct graph of $y = g(x)$, shape and continuous at $x = \pm \frac{3}{2}$,

A1

passes through the origin $(0,0)$ and endpoints $(-4,-6)$, $(4,6)$.

A1

ii. Correct shape of $y = g'(x)$, the graph of $y = g'(x)$ is not defined at

A1

$x = \pm 4$ and $x = \pm \frac{3}{2}$ and must have open circles at $x = \pm 4$ and $x = \pm \frac{3}{2}$

A1

d. $g(x) = |ax+b| - |ax-b|$

$$g'(x) = \begin{cases} 0 & \text{for } x \in \left(\frac{b}{a}, \infty\right) \\ 2a & \text{for } x \in \left(-\frac{b}{a}, \frac{b}{a}\right) \\ 0 & \text{for } x \in \left(-\infty, -\frac{b}{a}\right) \end{cases} \quad \text{must have open brackets}$$

A2

Question 4

a. $A_L = y_A + y_B + y_C + y_D = 1 + \frac{3}{2} + 2 + \frac{3}{2}$
 $A_L = 6 \text{ m}^2$ A1

b.i. period $T = \frac{2\pi}{n} = 4 \Rightarrow n = \frac{\pi}{2}$ A1

amplitude of inverted cosine wave $q = -\frac{1}{2}$ A1

translated upwards $p = \frac{3}{2}$ A1

ii. $\bar{g} = \frac{1}{4} \int_0^4 \left(\frac{3}{2} - \frac{1}{2} \cos\left(\frac{\pi x}{2}\right) \right) dx$, accept $\bar{g} = \frac{1}{4} \int_0^4 (p + q \cos(nx)) dx$ A1

iii. $\bar{g} = \frac{3}{2}$ metres A1

c.i. $f_2(x) = a_2 x^2 + b_2 x + c_2$
 $f_2(1) = \frac{3}{2} \Rightarrow a_2 + b_2 + c_2 = \frac{3}{2}$ (1)
 $f_2(2) = 2 \Rightarrow 4a_2 + 2b_2 + c_2 = 2$ (2) A1
 $f_2(3) = \frac{3}{2} \Rightarrow 9a_2 + 3b_2 + c_2 = \frac{3}{2}$ (3)

$f_2'(x) = 2a_2 x + b_2$
 $f_2'(2) = 0 \Rightarrow 4a_2 + b_2 = 0$ (4) A1

ii. (2) - (1) $\Rightarrow 3a_2 + b_2 = \frac{1}{2}$ (5)

(4) - (5) $\Rightarrow a_2 = -\frac{1}{2}$ substituting gives $b_2 = 2$ and $c_2 = 0$ A1

checks in all equations, so $f_2(x) = -\frac{1}{2}x^2 + 2x$

- iii. $f_1(x) = a_1x^2 + b_1x + c_1$
 $f_1(0) = 1 \Rightarrow c_1 = 1$ (1)
 $f_1(1) = \frac{3}{2} \Rightarrow a_1 + b_1 + c_1 = \frac{3}{2} \Rightarrow a_1 + b_1 = \frac{1}{2}$ (2) A1
 $f_2'(x) = -x + 2$ $f_2'(1) = -1 + 2 = 1 = f_1'(1)$ since the join is smooth
 $f_1'(x) = 2a_1x + b_1$ $f_1'(1) = 2a_1 + b_1 = 1$ (3) A1
- iv. $(3) - (2) \Rightarrow a_1 = \frac{1}{2}$ substituting gives $b_1 = 0$ since $c_1 = 1$ A1
 $f_1(x) = \frac{1}{2}x^2 + 1$
- v.
 • reflect in the y-axis A1
 • translate 4 units to the right parallel to the x-axis. A1
- vi. $f_3(x) = f_1(4-x)$ $a = -1$ $b = 4$ A1
 $f_3(x) = \frac{1}{2}(4-x)^2 + 1 = \frac{1}{2}(16 - 8x + x^2) + 1$
 $f_3(x) = \frac{1}{2}x^2 - 4x + 9$ $a_3 = \frac{1}{2}$, $b_3 = -4$, $c_3 = 9$ A1
- vii. $\bar{f} = \frac{1}{4} \left[\int_0^1 \left(\frac{1}{2}x^2 + 1 \right) dx + \int_1^3 \left(-\frac{1}{2}x^2 + 2x \right) dx + \int_3^4 \left(\frac{1}{2}x^2 - 4x + 9 \right) dx \right]$ A1
- viii. $\bar{f} = \frac{3}{2}$ metres A1

END OF SECTION 2 SUGGESTED ANSWERS