

2012

VCE Further Mathematics Trial Examination 1

Suggested Solutions

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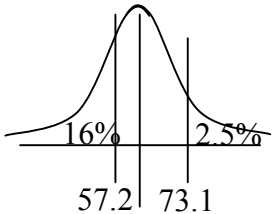
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Core: Data analysis

Question 1 C The data can be put in a frequency table and then a histogram can be drawn to show the frequency distribution of a numerical variable, length. Bar graphs are used for categorical data. Scatter plots are used for bivariate data. Time graphs are used where the values of the dependent variable are plotted in time order. Back-to-Back stem and leaf requires two groups.	Question 2 A Number of female part-time workers = 205 Total number interviewed = $734 + 391 + 170 + 205 = 1500$ $\text{Percentage} = \frac{205}{1500} \times 100 = 13.7\%$
Question 3 C There are 22 numbers. The median is half way between the 11th and the 12th number. i.e. half way between 17.6 and 18.2 $\frac{17.6 + 18.2}{2} = 17.9$	Question 4 D  $62.5 - 5.3 = 57.2$, so 57.2 is one standard deviation from the mean. 68% lie within $\pm$ one SD of the mean so 32% lie outside, half on each side. This means that 16% are less than 57.2 $62.5 + 2 \times 5.3 = 73.1$, so 73.1 is two standard deviations from the mean. 95% lie within $\pm$ two SD of the mean so 5% lie outside, half on each side. This means that 2.5% are greater than 73.1 $16 + 2.5 = 18.5\%$ lie outside the given values, so $100 - 18.5 = 81.5\%$ lie within the given boundaries.
Question 5 B The first two points lie below the regression line so not C or E. The first point is when $x = 0$, so not A. The first point is 0.5 below the regression line, so not D.	Question 6 E Using $\log x$ will move the x values in the plot horizontally so that the large x values are compressed.
Question 7 A The data needs to be divided into 3 sections with 3 points in each of the lower and upper groups. The median x value in the lower group is 2 and the median y value is 21. This gives the point (2, 21). The median x value in the upper group is 9 and the median y value is 35. This gives the point (9, 35). $\text{gradient} = \frac{35 - 21}{9 - 2} = 2.0$	

Core: Data analysis**Question 8 D**

Average for the four seasons

$$= \frac{2073 + 2414 + 2339 + 1967}{4} = 2198.25$$

$$\text{Spring seasonal index} = \frac{1967}{2198.25} = 0.8948$$

$$= 89.48\%$$

This means that the number must be increased to 100% by $100 - 89.48 = 10.52\%$

Question 9 A

The number of employees shows a general trend down and the graph is becoming less variable with time.

Question 10 B

This is quarter 34.

When $x = 34$

$$\text{Number of employees} = 23 - 0.3 \times 34 = 12.8$$

Question 11 C

Year	Season	Sales
1	Sum	30
	A	20
	W	8
	Sp	15
2	Sum	35
	A	25
	W	12
	Sp	20

$$\frac{15 + 35 + 25 + 12}{4} = 21.75$$

$$\frac{35 + 25 + 12 + 20}{4} = 23$$

$$\frac{23 + 21.75}{2} = 22.38$$

Question 12 E

$$y = a + bx$$

$$b = r \frac{s_y}{s_x}$$

$$a = \bar{y} - b\bar{x}$$

$$b = 0.71 \times \frac{25.7}{29.3} = 0.62$$

$$a = 61 - 0.62 \times 59 = 24.4$$

Question 13 E

The coefficient of determination is

$$r^2 = 0.89^2 = 0.7921 \text{ so A is true.}$$

Other factors such as where the product is placed in the supermarket, could affect sales, so B is true.

There is a high positive correlation coefficient of 0.89 so C and D are true.

About 79% of the variation in the number of sales of the product can be explained by the variation in the price, so E is not true.

Module 1 Number patterns

Question 1 D The pattern of matches is 3, 5, 7 This is an arithmetic sequence where each term is 2 more than the term before it. Hence, the fourth term will have $7 + 2 = 9$ matches.	Question 2 E This is a geometric sequence with a common ratio of $6 \div 3 = 2$, and the first term is 3. $t_n = ar^{n-1}$ $t_7 = 3 \times 2^6 = 192$
Question 3 A Each term is multiplied by -0.5 Hence, it is a geometric sequence with a common ratio of -0.5 $S_\infty = \frac{a}{1-r}$ $S_\infty = \frac{20}{1 - \left(-\frac{1}{2}\right)} = \frac{20}{\frac{3}{2}} = \frac{40}{3} = 13\frac{1}{3}$	Question 4 C If $A_{n+2} = A_{n+1} - 3A_n$ where $A_1 = -6$ and $A_2 = 4$, $A_3 = A_2 - 3 \times A_1 = -4 + 18 = 14$ $A_4 = A_3 - 3 \times A_2 = 14 + 12 = 26$ $A_5 = A_4 - 3 \times A_3 = 26 - 42 = -16$
Question 5 B This is an arithmetic sequence with $a = 193$ and $d = -5$ $t_n = a + (n-1) \times d$ $68 = 193 + (n-1) \times -5$ $68 = 193 - 5n + 5$ $5n = 130$ $n = 26$	Question 6 D $t_3 = ar^2 = 54$ $t_6 = ar^5 = 16$ $\frac{ar^5}{ar^2} = \frac{16}{54} = \frac{8}{27}$ $r^3 = \frac{8}{27}$ $r = \frac{2}{3}$ $t_3 = a \left(\frac{2}{3}\right)^2 = 54$ $a = 54 \times \frac{9}{4} = \frac{243}{2} = 121.5$

Module 1 Number patterns**Question 7 D**

This is a geometric sequence with $a = 2400$, and $r = 0.96$

$$t_n = ar^{n-1}$$

$$t_5 = 2400 \times (0.96)^4 = 2038$$

Question 8 A

Geometric sequences have a common ratio.

$$\frac{4n-4}{n-1} = \frac{12n+16}{4n-4}$$

Solving this equation with the CAS calculator gives $n = 8$

Hence, the first three terms are

$$8 - 1, 4 \times 8 - 4, \text{ and } 12 \times 8 + 16$$

This gives 7, 28, 112, which is a geometric sequence with a common ratio of 4.

Hence, the 4th term is $112 \times 4 = 448$

The sum of these four terms is

$$7 + 28 + 112 + 448 = 595$$

Question 9 A

Each row will have one less can than the row below it. This gives an arithmetic sequence with a common difference of -1, a first term of 34 and the sum of the terms is 244.

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$244 = \frac{n}{2}[2 \times 34 + (n-1) \times -1]$$

Solving this equation on the CAS calculator gives $n = 8$ or $n = 61$

But $n < 34$ because the number of cans in each row decreases by 1 for each row, so there must be 8 rows.

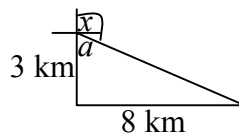
The number of cans in the 8th row is $t_8 = a + 7d = 34 + 7 \times -1 = 27$

Module 2 Geometry and trigonometry**Question 1 B**

$$\frac{23}{\sin 68^\circ} = \frac{18}{\sin \angle ACB}$$

$$\angle ACB = \sin^{-1} \left(\frac{18 \times \sin 68^\circ}{23} \right) = 46.52^\circ$$

This is closest to 47°

Question 2 C

$$\tan a = \frac{8}{3}$$

$$a = \tan^{-1} \left(\frac{8}{3} \right) = 69.44^\circ$$

$$x = 180 - 69.44 = 110.56$$

this is closest to 111°

Question 3 A

$$20 : 100 = 1 : 5$$

$$\text{Volume ratio} = 1^3 : 5^3 = 1 : 125$$

$$1 : 125$$

$$\frac{1}{125} : 1$$

$$\frac{1}{125} \times 1800 : 1800$$

$$\frac{1}{125} \times 1800 = 14.4$$

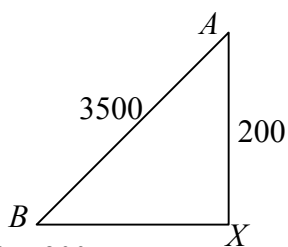
Volume of model is closest to 14 cm^3

Question 4 D

2 cm represents 1000 m

1 cm represents 500 m

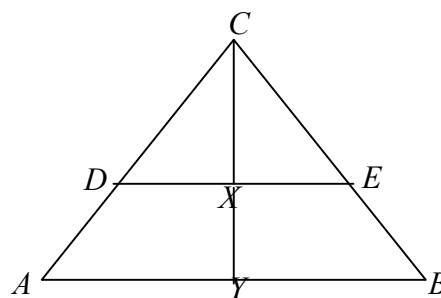
7 cm represents $7 \times 500 = 3500 \text{ m}$

Question 5 A

$$AX = 250 - 50 = 200$$

$$BX = \sqrt{3500^2 - 200^2} = 3494.28$$

$$\text{Gradient} = \frac{\text{rise}}{\text{run}} = \frac{200}{3494.28} = 0.06$$

Question 6 D

Length of $CY = 12 \text{ cm}$ (Pythagorean triad, 5:12:13)

Triangles ABC and DEC are similar, AAA.

Ratio of sides is $12:8 = 3:2$

Ratio of areas is $3^2:2^2 = 9:4 = 2.25:1$

The big triangle is 2.25 times the area of the small triangle.

Module 2 Geometry and trigonometry**Question 7 E**

In triangle CDO , $\tan 40^\circ = \frac{CO}{12}$

$$CO = 12 \tan 40^\circ = 10.069$$

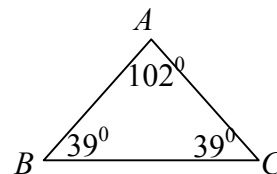
In triangle CBO , $\tan 60^\circ = \frac{10.069}{OB}$

$$OB = \frac{10.069}{\tan 60^\circ} = 5.813$$

In triangle OBA , $\sin 20^\circ = \frac{5.813}{AB}$

$$AB = \frac{5.813}{\sin 20^\circ} = 16.99$$

This is closest to 17 cm

Question 8 C

For an isosceles triangle $\angle ABC = \angle ACB$

$$\angle ABC + \angle ACB = 180 - 102 = 78$$

$$\angle ABC = 78 \div 2 = 39^\circ$$

Let the equal sides be x

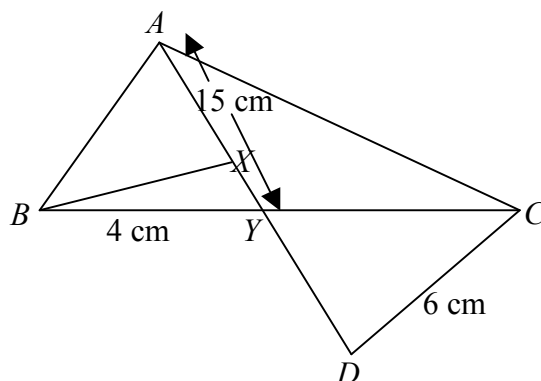
$$\therefore BC = 320 - 2x$$

$$\frac{320 - 2x}{\sin 102^\circ} = \frac{x}{\sin 39^\circ}$$

Solve for x

$$x = 90.03$$

This is closest to 90 cm

Question 9 D

$$XY = \frac{1}{5} \times 15 = 3$$

$$BX = \sqrt{4^2 - 3^2} = 2.6458$$

$\Delta s BXY$ and CDY are similar AAA.

$$\therefore \frac{YC}{4} = \frac{6}{2.6458}$$

$$YC = 9.07$$

This is closest to 9 cm

Module 3 Graphs and relations

Question 1 B For the first section of the graph $m = \frac{4-8}{2-0} = -2$ $c = 8$ $y = 8 - 2x \quad 0 \leq x \leq 2$ So not A or C For the second section of the graph $y = 4 \quad 2 \leq x \leq 10$ So not E For the third section of the graph $m = \frac{4-1}{10-13} = -1$ $y = -x + c$ When $x = 10, y = 4$ $4 = -10 + c$ $c = 14$ $y = 14 - x \quad 10 < x \leq 13$ So not D	Question 2 A The equation of the line is of the form $y = mx + c$ $m = \frac{0 - -3}{2 - 0} = \frac{3}{2}$ $c = -3$ $y = \frac{3}{2}x - 3$ $y \geq \frac{3}{2}x - 3$ $2y \geq 3x - 6$ $2y - 3x \geq -6$ $3x - 2y \leq 6$
Question 3 E When $x = 3$ For $AB, 9 + y = 15 \Rightarrow y = 6$ The point(3,6) lies on the line AB so not C For $BC, 3 - 3y = 12 \Rightarrow y = -3$ The point(3,-3) lies on the line BC so not A or B Gradient of $AB, y = -3x + 15$ is -3 Gradient of $BC, -3y = x + 12$ $\Rightarrow y = -\frac{1}{3}x - 4$ is $-\frac{1}{3}$ Gradients are not equal so not parallel. Gradients multiply to give -1, so the lines are perpendicular	Question 4 C If $y = \frac{36}{x}$ when $x = 3, y = 12$, so not A If $x = \frac{36}{y^2}$ when $y = 4, x = 2.25$, so not B If $y = 4x + 24$ when $x = 3, y = 36$, so not D If $y = \frac{4x}{3}$ when $x = 3, y = 4$, but when $x = 1, y = \frac{4}{3}$ so not E If $y = \frac{36}{x^2}$ when $x = 3, y = 4$, when $x = 1, y = 36$, when $x = 36, y = \frac{1}{36}$, so C

Module 3 Graphs and relations

Question 5 D $2y = -4x + 12$ $y = -2x + 6$ Gradient of line parallel to this line is -2 so not A, C or E Using CAS calculator, solve the two equations simultaneously This gives $x = 2$ and $y = -1$ If $y = -2x + c$ and $x = 2$ when $y = -1$ $-1 = -4 + c$ $c = 3$ $y = -2x + 3$	Question 6 C Simultaneous equations have no solutions when the lines are parallel. The only two sets of simultaneous equations that have the same gradient are C and D. The gradients of C are $-\frac{3}{2}$, and the gradients of D are $-\frac{2}{3}$. However, the two equations in D are the same line and so have an infinite number of solutions. Only C has parallel lines and hence no solutions.
Question 7 D The gradient does not show American dollars or Australian dollars so not A or B. $D = 0.85A + c$ $34 = 0.85 \times 40 + c$ $c = 0$, so not E $D = 0.85A + 0 = 0.85A$ So to get the equivalent in American dollars you multiply the Australian dollars by the gradient.	Question 8 B For $x - y$ to be a maximum, want x to be as large as possible and y to be as small as possible. This would occur at R. For $x - y$ to be a minimum, want x to be as small as possible and y to be as large as possible. This would occur at P.
Question 9 D The hybrid equation is $y = \begin{cases} 0.8x + 8800 & 0 \leq x \leq 5000 \\ x + 12000 & 5001 \leq x \leq 10000 \\ 2.1x + 14400 & 10001 \leq x \leq 15000 \end{cases}$ Income = $3x$ For the first stage $3x = 0.8x + 8800 \therefore x = 4000$. This means that if the number of mugs made and sold is between 4000 and 5000, including 5000 then a profit will be made. Second stage: $3x = x + 12000 \therefore x = 6000$. This means that if the number of mugs made and sold is between 6000 and 10000, including 10000 then a profit will be made. Third stage: $3x = 2.1x + 14400 \therefore x = 16000$. This is outside the maximum number of 15000, so no profit will be made in this section. For a profit need to sell between 4000 up to and including 5000 or between 6000 up to and including 10000.	

Module 4 Business-related mathematics

Question 1 C 600 tickets cost $600 \times \\$135 = \\81000 $110\% = 81000$ $1\% = 81000 \div 110 = 736.3636$ $10\% = 736.3636 \times 10 = \\7363.636 This is closest to \$7364	Question 2 B $I = \frac{PRT}{100}$ $P = \frac{100 I}{RT} = \frac{100 \times 486}{6 \times 3} = \\2700
Question 3 D Amount borrowed = $400 - 120 = 280$ Amount repaid = \$20 a month for 24 months = \$480 Interest paid = $480 - 280 = 200$ $I = \frac{PRT}{100}$ $R = \frac{100 I}{PT} = \frac{100 \times 200}{280 \times 2} = 35.7\%$ This is closest to 36%	Question 4 E Number of months = 60 $R = 7.2 \div 12 = 0.6\%$ per month Total value of investment = $P \left(1 + \frac{r}{100} \right)^n$ $= 4500 \left(1 + \frac{0.6}{100} \right)^{60}$ $= 4500 (1.006)^{60}$
Question 5 D Depreciation = $25000 - 14500 = \\$10500$ Number of 5 cents in \$10500 = Number of \$0.05 in \$10500 $= 10500 \div 0.05 = 210000$ km.	Question 6 C March minimum balance on 2nd March = $465 - 200 = 265$ Monthly interest = $5.8 \div 12 = 0.483333$ Interest for March = $0.483333 \div 100 \times 265 = 1.28$ April minimum balance on 3rd April = 754.50 Interest for March = $0.483333 \div 100 \times 754.50 = 3.65$ Total interest for the two months = $1.28 + 3.65 = \\$4.93$
Question 7 E $A = 300000 \left(1 + \frac{3}{100} \right)^{10} = 403174.91$ This is closest to \$403175	

Module 4 Business-related mathematics**Question 8 A**

Use TVM solver

$$N = 5 \times 12 = 60$$

$$I = 0.54 \times 12 = 6.48$$

$$PV = 20000$$

$$PMT = -365.8$$

$$FV = ?$$

$$P/Y = 12$$

$$C/Y = 12$$

This gives $FV = 1789.65$

This is closest to \$1790

Question 9 C

Number of months to repay loan in full with no change in interest rate

$$N = ?$$

$$I = 0.54 \times 12 = 6.48$$

$$PV = 20000$$

$$PMT = -365.8$$

$$FV = 0$$

$$P/Y = 12$$

$$C/Y = 12$$

This gives $N = 64.972$

Amount owing after 2 years

$$N = 24$$

$$I = 0.54 \times 12 = 6.48$$

$$PV = 20000$$

$$PMT = -365.8$$

$$FV =$$

$$P/Y = 12$$

$$C/Y = 12$$

This gives $FV = 13412.92$

Amount to now repay

$$N = 64.97 - 24 = 40.97$$

$$I = 0.65 \times 12 = 7.8$$

$$PV = 13412.91873$$

$$PMT =$$

$$FV = 0$$

$$P/Y = 12$$

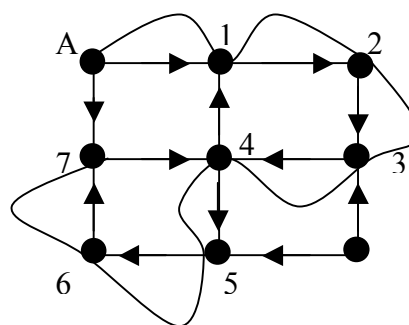
$$C/Y = 12$$

This gives $PMT = \$373.97$

This is closest to \$374

Module 5 Networks and decision mathematics**Question 1 E**

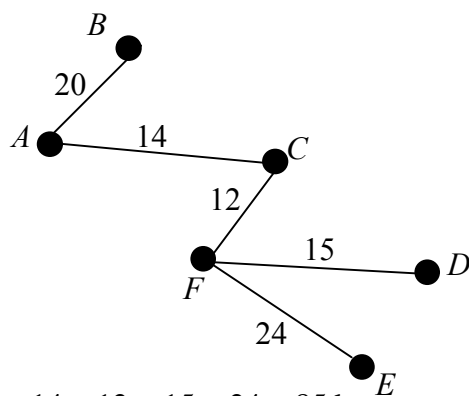
A Hamiltonian circuit is a circuit, which passes through every vertex only once, starting and finishing on the same vertex.

Question 2 D**Question 3 A**

For an Eulerian circuit to exist, all the vertices must be of even degree. C and G are of odd degree, so if a line is added between these two points, they will both be of even degree.

Question 4 C

This information could be represented as a tree.

Question 5 B

$$20 + 14 + 12 + 15 + 24 = 85 \text{ km.}$$

Question 6 D

$$v + f = e + 2$$

$$4 + 6 = e + 2$$

Original number of edges = 8

$$v + f = e + 2$$

$$2 + 2 = e + 2$$

Final number of edges = 2

$$\text{Number of edges removed} = 8 - 2 = 6$$

Question 7 B

A critical path is the longest path.
A-E-I-H-M

Question 8 E

The minimum time is the time along the critical path $8 + 9 + 3 + 8 + 3 = 31$ days.

Question 9 E

Earliest starting time for F is 10 days

Latest starting time for F is $31 - (3 + 8 + 4) = 16$

Float time = $16 - 10 = 6$ days.

Module 6 Matrices

Question 1 A $\begin{bmatrix} 3 & -1 \end{bmatrix} \begin{bmatrix} -2 \\ 3 \end{bmatrix} = [3 \times -2 + -1 \times 3]$ $= [-6 - 3] = [-9]$	Question 2 C Matrices with a determinant of zero do not have an inverse. Hence, not A. To add matrices, they must be of the same order. Hence, not B. To multiply matrices, the number of columns in the first matrix must equal the number of rows in the second matrix. Hence, not D. A 4 by 1 matrix is a column matrix. Hence, not E. Two matrices that multiply to give the identity matrix are commutative.
Question 3 B $\begin{bmatrix} -3 & 0 \\ 7 & 2 \end{bmatrix} - 2 \times \begin{bmatrix} 4 & -3 \\ -1 & 5 \end{bmatrix}$ $= \begin{bmatrix} -3 & 0 \\ 7 & 2 \end{bmatrix} - \begin{bmatrix} 8 & -6 \\ -2 & 10 \end{bmatrix}$ $= \begin{bmatrix} -3-8 & 0+6 \\ 7+2 & 2-10 \end{bmatrix} = \begin{bmatrix} -11 & 6 \\ 9 & -8 \end{bmatrix}$	Question 4 E Transition matrices are square matrices and the sum of each column is 1.
Question 5 C The ones represent “is the particular relative of” And the zeros represent “is not the particular relative of”. In the given matrix, there are only 4 ones. From the original matrix the column is the relative of the row. Here, Emily is the relative of Jack, Daisy is the relative of Laura, Laura is the relative of Daisy and Daisy is the relative of Laura, Gabriella is the relative of Paul The relationship must be “is the sister of”	Question 6 A $\begin{bmatrix} 1 & -3 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} -14 & -11 \\ 5 & 16 \end{bmatrix}$ Inverse of $\begin{bmatrix} 1 & -3 \\ -1 & 2 \end{bmatrix}$ is $\begin{bmatrix} -2 & -3 \\ -1 & -1 \end{bmatrix}$ $\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} -2 & -3 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} -14 & -11 \\ 5 & 16 \end{bmatrix}$ $\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 13 & -26 \\ 9 & -5 \end{bmatrix}$ $b - c = -26 - 9 = -35$

Module 6 Matrices**Question 7 D**

D^{-1} has the same order as D which is $n \times n$.

C does not have n columns

and A does not have n rows

Hence, A and B are not defined.

A and B do not have the same order

and so cannot be added.

Hence, not E

AB has the order $m \times m$

and so cannot be added to D but can be added to C^{-1} which is of the same order.

Question 8 E

$$\text{Let } A = \begin{bmatrix} 2 & 3 & -1 \\ 1 & -2 & 3 \\ 3 & 2 & -4 \end{bmatrix}$$

$$A \times \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} -4 \\ 19 \\ -11 \end{bmatrix}$$

$$A^{-1} \times A \times \begin{bmatrix} a \\ b \\ c \end{bmatrix} = A^{-1} \times \begin{bmatrix} -4 \\ 19 \\ -11 \end{bmatrix}$$

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = A^{-1} \times \begin{bmatrix} -4 \\ 19 \\ -11 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ 4 \end{bmatrix}$$

$$a = 3, b = -2, c = 4$$

$$a + b + c = 3 - 2 + 4 = 5$$

Question 9 D

Answer E cannot be done. In answers A and C the running hours are being multiplied by the swimming times. When the multiplication is completed, the answer will be a 3×1 matrix, so the answer will be in the third row first column.

Hence, D is the correct answer.

End of Suggested Solutions 2012 Further Mathematics VCE Trial Examination 1

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