

**The Mathematical Association of Victoria**  
**Trial Examination 2017**  
**SPECIALIST MATHEMATICS**  
**Written Examination 2**

**STUDENT NAME** \_\_\_\_\_

**Reading time: 15 minutes**

**Writing time: 2 hours**

**QUESTION & ANSWER BOOK**

**Structure of Book**

| <i>Section</i> | <i>Number of Questions</i> | <i>Number of questions<br/>to be answered</i> | <i>Number of marks</i> |
|----------------|----------------------------|-----------------------------------------------|------------------------|
| A              | 20                         | 20                                            | 20                     |
| B              | 6                          | 6                                             | 60                     |
|                |                            |                                               | Total 80               |

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, full functionality may be used.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

**Materials supplied**

- Question and answer book of 23 pages.
- Formula sheet
- Answer sheet for multiple-choice questions.

**Instructions**

- Write your **name** in the space provided above on this page.
- Write your **name** on the multiple-choice answer sheet
- Unless otherwise indicated, the diagrams are **not** drawn to scale.
- All written responses must be in English.

**Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.**

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**SECTION A****Instructions for Section A**

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.

Choose the response that is **correct** for the question.

A correct answer scores 1, an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

No marks will be given if more than one answer is completed for any question.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude  $g \text{ ms}^{-2}$ , where  $g = 9.8$

**Question 1**

If  $z = \sqrt{3} - i$ ,  $w = 4(1 + i)$  and  $\text{Arg}\left(\frac{z^k}{w}\right) = \frac{11\pi}{12}$  then a possible value of  $k$  is

- A. 2
- B. 3
- C. 6
- D. 8
- E. 10

**Question 2**

If  $2 - i$  is a root of the equation  $z^3 + bz^2 + 17z - 15 = 0$  where  $b \in R$ , then  $b$  is

- A. -1
- B. 7
- C.  $-\frac{19}{2}$
- D. 1
- E. -7

**Question 3**

If  $(a + i)(1 - ai) = 6 + (b + 2)i$  where  $a, b \in R$  then

- A.  $ab = 30$
- B.  $b + a = 7$
- C.  $\frac{b}{a} = 4$
- D.  $a + b = -7$
- E.  $\frac{b}{a} = -4$
- F.

**SECTION A – continued**

**TURN OVER**

**Question 4**

If  $\operatorname{cosec}(x) = \frac{4\sqrt{7}}{7}$ ,  $\frac{\pi}{2} < x < \pi$  then  $\sec(x) + \tan(x)$  is

- A.  $-\frac{4}{3} + \frac{3\sqrt{7}}{16}$
- B.  $-\left(\frac{12 - 3\sqrt{7}}{12}\right)$
- C.  $-\frac{1}{3}(4 + \sqrt{7})$
- D.  $-\frac{4}{3}(1 - \sqrt{7})$
- E.  $-\frac{1}{3}(4 - \sqrt{7})$

**Question 5**

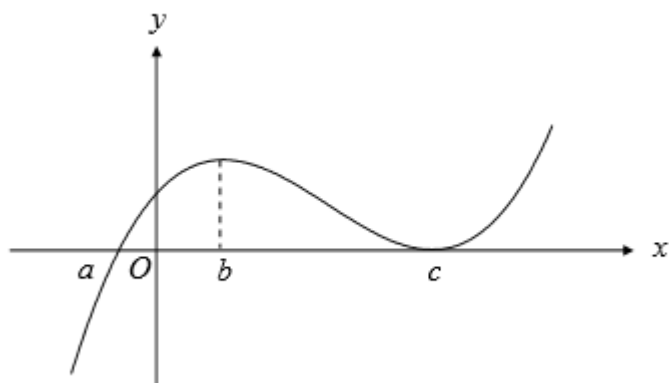
If  $3\cos(2\theta) + 5\sin(\theta) - 2 = 0$  then the following are solutions

- A.  $\sin(\theta) = 1$  and  $\sin(\theta) = -\frac{1}{6}$
- B.  $\cos(\theta) = 1$  and  $\cos(\theta) = -\frac{1}{6}$
- C.  $\sin(\theta) = -1$  and  $\sin(\theta) = -\frac{1}{6}$
- D.  $\sin(\theta) = -1$  and  $\cos(\theta) = \frac{1}{6}$
- E.  $\sin(\theta) = 1$  and  $\sin(\theta) = \frac{1}{6}$

**SECTION A – continue**

**Question 6**

The graph of  $y = f(x)$  is shown below.



If  $F(x)$  is an antiderivative of  $f(x)$ , then the graph of  $y = F(x)$  will have a

- A. local maximum at  $x = b$
- B. local maximum at  $x = a$
- C. point of inflexion at  $x = a$
- D. zero gradient at  $x = b$
- E. point of inflexion at  $x = c$

**Question 7**

If  $f'(t) = \ln(2t + 1)$  and  $f(0) = 2$  then the value of  $f(1)$  can be found by evaluating

- A.  $\int_0^1 (\ln(2t + 1) - 2) dt$
- B.  $\int_0^2 (\ln(2t + 1)) dt - 1$
- C.  $\int_0^1 (2 + \ln(2t + 1)) dt$
- D.  $\int_0^1 (2 - \ln(2t + 1)) dt$
- E.  $\int_0^1 \ln(2t + 1) dt + 2$

**SECTION A - continued**

**TURN OVER**

**Question 8**

The gradient of the curve  $y^2 + 2yx^4 = 33$  when  $y = 1$  in the first quadrant is equal to

- A.  $-\frac{31}{34}$
- B.  $-\frac{32}{17}$
- C.  $\frac{31}{34}$
- D.  $-32$
- E.  $\frac{31}{17}$

**Question 9**

$\int_{\pi/6}^{2\pi/3} (\tan^3(x) + \tan(x))dx$  is equivalent to

- A.  $\int_{-\sqrt{3}}^{1/\sqrt{3}} u du$
- B.  $\int_{1/\sqrt{3}}^{-\sqrt{3}} (u^3 + u) du$
- C.  $\int_{\pi/6}^{\pi/3} (u^3 + u) du$
- D.  $\int_{1/\sqrt{3}}^{-\sqrt{3}} u^3 du$
- E.  $-\int_{-\sqrt{3}}^{1/\sqrt{3}} u du$

**SECTION A – continued**

**Question 10**

The graph of the function  $y = \sqrt{\frac{16x^4 - 1}{x^4}}$ , between the lines  $y = 2$  and  $y = 3$  is rotated about the  $y$ -axis to form a solid of revolution. The volume formed is given by

- A.  $\cos^{-1}\left(\frac{3}{4}\right) - \frac{\pi}{3}$
- B.  $\pi \sin^{-1}\left(\frac{3}{4}\right) - \frac{\pi^2}{6}$
- C.  $\frac{\ln 7}{8} - \frac{\ln 3}{8}$
- D.  $\pi \sin^{-1}\left(\frac{3}{4}\right) + \frac{\pi^2}{6}$
- E.  $\frac{\pi}{4} \left( \tan^{-1}\left(\frac{3}{4}\right) - \tan^{-1}\left(\frac{1}{2}\right) \right)$

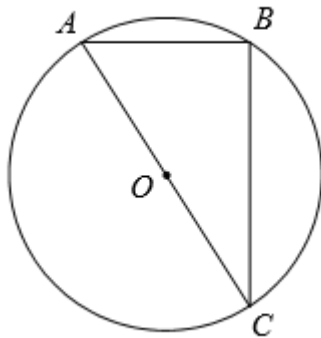
**Question 11**

$\mathbf{a} = 8\mathbf{i} - \mathbf{j} + 13\mathbf{k}$ ,  $\mathbf{b} = m\mathbf{i} - \mathbf{j} + 3\mathbf{k}$  and  $\mathbf{c} = \mathbf{i} + \mathbf{j} + 2\mathbf{k}$  are linearly dependent when

- A.  $m = \frac{1}{3}$
- B.  $m = -\frac{2}{3}$
- C.  $m = -2$
- D.  $m = -\frac{1}{3}$
- E.  $m = 2$

**SECTION A – continued**

**TURN OVER**

**Question 12**

If the points A, B and C lie on the circumference of the circle whose centre is O, which of the following statements is **not** true?

- A.  $\overrightarrow{AB} = \overrightarrow{AC} - \overrightarrow{BC}$
- B.  $\overrightarrow{AB} \cdot \overrightarrow{AC} = |\overrightarrow{BA}| |\overrightarrow{AC}| \cos(A)$
- C.  $(\overrightarrow{CA} + \overrightarrow{AB}) \cdot \overrightarrow{CB} = |\overrightarrow{CB}|^2$
- D.  $|\overrightarrow{AB}| + |\overrightarrow{BC}| = |\overrightarrow{AC}|$
- E.  $\overrightarrow{BA} \cdot \overrightarrow{BC} = 0$

**Question 13**

The acceleration  $a \text{ ms}^{-2}$  of a body moving in a straight line when it is  $x$  m from the origin is given by

$a = \frac{1}{\sqrt{9-x^2}}$ . If the velocity,  $v$ , of the body is  $2 \text{ ms}^{-1}$  when it passes through the origin, then the function for  $x$  is given by

- A.  $x = 3 \sin\left(\frac{v^2 - 4}{2}\right)$
- B.  $x = 2 \sin\left(\frac{2 - v^2}{3}\right)$
- C.  $x = 3 \cos\left(\frac{v^2 - 2}{2}\right)$
- D.  $x = 3 \sin(v^2 - 2)$
- E.  $x = 2 \sin^{-1}\left(\frac{v}{3}\right)$

**SECTION A** - continued

**Question 14**

The position vector of a particle at time  $t$ ,  $t \geq 0$ , is given by  $\underline{r} = 2\sqrt{t}\underline{i} + (5-t)\underline{j}$ . The particle is closest to the origin when

- A.  $t = 0$
- B.  $t = 1$
- C.  $t = \frac{3}{2}$
- D.  $t = 3$
- E.  $t = 5$

**Question 15**

A smooth inclined plane makes an angle of  $\theta^\circ$  to the horizontal. An object of mass  $m$  kg is held at rest on this plane by a force  $F$  newtons inclined at  $\alpha^\circ$  upwards to the plane. If the normal reaction of the mass on the plane is  $R$  newtons, which of the following statements is true?

- A.  $R = mg \cos \theta + F \sin \alpha$
- B.  $F \sin \alpha - mg \cos \theta = 0$
- C.  $\tan \alpha = \frac{mg \cos \theta - R}{mg \sin \theta}$
- D.  $F = \frac{m \sin \theta}{\cos \alpha}$
- E.  $\tan \alpha = \frac{F \cos \theta}{R - F \sin \theta}$

**Question 16**

A body of mass 10 kg is acted upon by three forces  $\underline{F}_1$ ,  $\underline{F}_2$  and  $\underline{F}_3$  all measured in Newtons. If, at time  $t$  seconds,  $\underline{F}_1 = 2\underline{i} + (t-3)\underline{j}$ ,  $\underline{F}_2 = -t^2\underline{i} - \underline{j}$  and  $\underline{F}_3 = 5t\underline{i} + 2\underline{j}$  then the magnitude of the acceleration of the body in  $\text{ms}^{-2}$  when  $t = 3$  is

- A.  $\frac{\sqrt{65}}{10}$
- B.  $\frac{73}{10}$
- C. 65
- D. 9
- E.  $\frac{\sqrt{65}}{5}$

**SECTION A – continued**  
**TURN OVER**

**Question 17**

A confidence interval estimate for a population mean can be used to test a null hypothesis about the population mean only if

- A. The distribution is symmetrical
- B. A one-tailed test is used
- C. A two-tailed test is used
- D. All of the above statements are true
- E. None of the above statements are true

**Question 18**

$X$  is a random variable with  $E(X) = 14.2$  and  $\text{Var}(X) = 2.3$ .  $Y$  is another random variable with  $E(Y) = 5.5$  and  $\text{Var}(Y) = 0.8$ . If  $X$  and  $Y$  are independent variables and  $W = 2X - Y + 3$  then

- A.  $E(W) = 22.9$  and  $\text{SD}(W) = \sqrt{13}$
- B.  $E(W) = 22.9$  and  $\text{SD}(W) = \sqrt{10}$
- C.  $E(W) = 33.9$  and  $\text{SD}(W) = 10$
- D.  $E(W) = 25.9$  and  $\text{SD}(W) = \sqrt{10}$
- E.  $E(W) = 25.9$  and  $\text{SD}(W) = \sqrt{13}$

**Question 19**

A Type II error is made when the

- A. alternative hypothesis is accepted when it is true
- B. null hypothesis is accepted when it is false
- C. null hypothesis is true and we accept it
- D. null hypothesis is rejected when it is true
- E. alternative hypothesis is accepted when it is false

**Question 20**

The distribution of serum vitamin E in a certain population is approximately normal with mean  $800\mu\text{g/dL}$  and standard deviation  $200\mu\text{g/dL}$ , ( $\mu\text{g/dL}$  stands for micrograms per decilitre). In a random sample of 20 people, the mean vitamin E level would lie within which range approximately 95% of the time

- A. Between  $780\mu\text{g/dL}$  and  $820\mu\text{g/dL}$
- B. Between  $408\mu\text{g/dL}$  and  $1192\mu\text{g/dL}$
- C. Between  $712\mu\text{g/dL}$  and  $888\mu\text{g/dL}$
- D. Between  $760\mu\text{g/dL}$  and  $839\mu\text{g/dL}$
- E. Between  $625\mu\text{g/dL}$  and  $975\mu\text{g/dL}$

**END OF SECTION A**

**SECTION B****Instructions for Section B**

Answer **all** questions in the spaces provided.

Unless otherwise specified, an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude  $g \text{ ms}^{-2}$ , where  $g = 9.8$

**Question 1** (11 marks)

A particle travels in a closed circuit. At time  $t$  secs it has a position vector, measured in metres, given by

$$\underline{r}(t) = 2\cos(t)\underline{i} + (2 + 3\sin(2t))\underline{j}, t \geq 0$$

- a. Show that the Cartesian equation of the path of the particle can be written as

$$9(x^2 - 2)^2 + 4(y - 2)^2 = 36$$

1 mark

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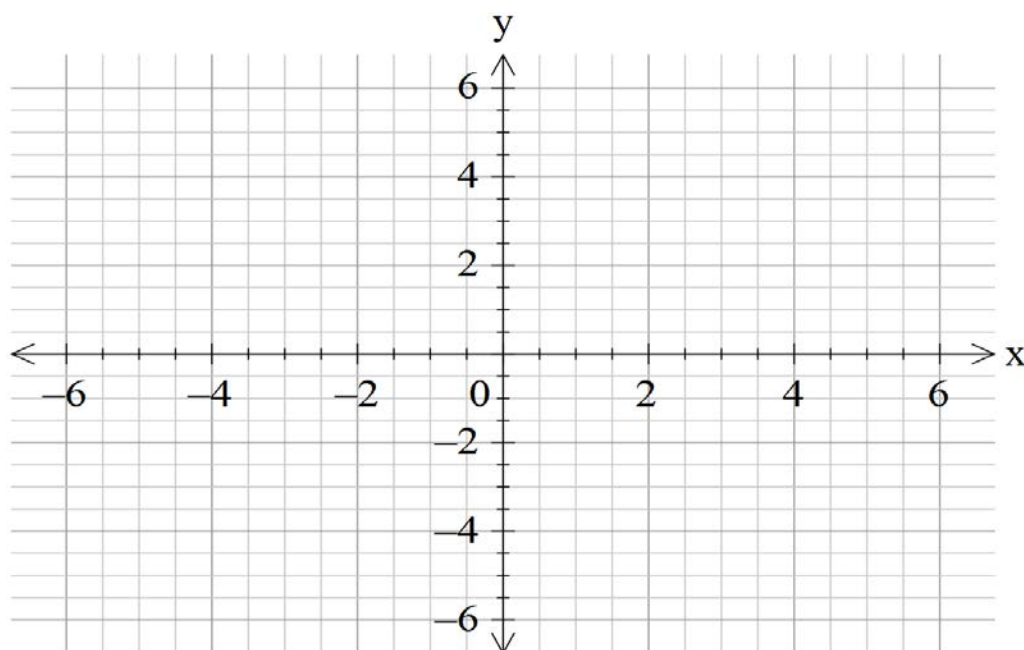
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- b. i Sketch the circuit of the particle on the set of axes below. There is no need to find the  $x$ -intercepts.

1 mark



**SECTION B – Question 1 – continued**  
**TURN OVER**

**ii.** Mark the starting point and the directions of motion of the particle on the diagram above. 1 mark

**c.** How long does it take the particle to complete one full circuit? 1 mark

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**d.** Find an expression for the speed of the particle at time  $t$  secs. 1 mark

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**e.** For  $0 < t \leq 2\pi$ , what are the maximum and minimum speeds of the particle? 2 marks

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**SECTION B – Question 1 – continued**

**f.**

- i.** Write down a definite integral for the length that the particle has travelled in the first 2 seconds.

1 mark

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- ii.** Find this length. Give your answer to two decimal places.

1mark

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- g.** Find the first time at which the particle will be travelling in a direction parallel to the line  $y = 3x$ . Give your answer correct to two decimal places.

2 marks

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**SECTION B** — continued  
**TURN OVER**

**Question 2** (9 marks)

Points A, B, C and D have position vectors  $\underline{a} = \underline{i} + m\underline{j}$ ,  $\underline{b} = -2\underline{i} + 2\underline{j}$ ,  $\underline{c} = n\underline{i} + 2\underline{j}$  and  $\underline{d} = 6\underline{i} + 6\underline{j}$  relative to the origin O, where  $m$  and  $n$  are real numbers.

- a.** Find  $\overrightarrow{AB}$  and  $\overrightarrow{AD}$  in terms of  $m$ . 1 mark

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- b.** If ABCD is a rhombus, find the values of  $m$  and  $n$ . 2 marks

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- c.** Find  $\overrightarrow{AE}$  the vector resolute of  $\overrightarrow{AB}$  parallel to  $\overrightarrow{AD}$ . 2 marks

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- d.** Find the area of triangle ABE. 2 marks

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**SECTION B – Question 2 – continued**

- e. If F divides  $\overline{CD}$  in the ratio 2:1, find the angle  $\theta^\circ$  between vector  $\overrightarrow{BD}$  and  $\overrightarrow{BF}$ . Give your answer correct to two decimal places. 2 marks

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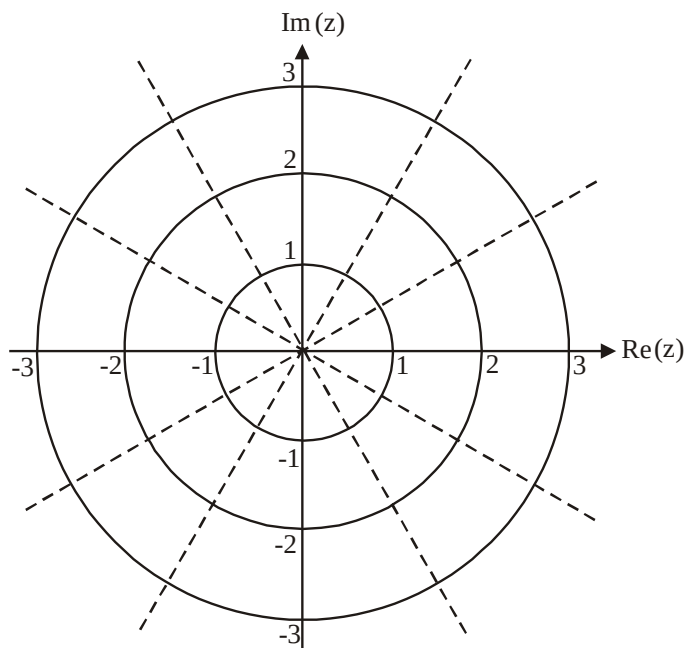
**SECTION B — continued**  
**TURN OVER**

**Question 3** (11 marks)

$w, \bar{w}$  and  $v$  are the solutions of the equation  $\{z : z^3 = k, z \in \mathbb{C}\}$  where  $w = 3\text{cis}\left(\frac{\pi}{3}\right)$

- a. Plot  $w, \bar{w}$  and  $v$  on the Argand diagram below.

1 mark



- b. Find the value of  $k$ .

1 mark

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- c. Show that  $w \in \{z : \sqrt{3} \text{Im}(z) - \text{Re}(z) = 3\}$ .

2 marks

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- d. On the Argand diagram given in part a.

- i. Sketch  $\{z : \sqrt{3} \text{Im}(z) - \text{Re}(z) = 3\}$

1 mark

**SECTION B – Question 3 – continued**

- ii. Sketch the ray given by  $\{z : \text{Arg}(z) = \text{Arg}(\bar{w}^{\frac{1}{2}})\}$

2 marks

- e. If  $u = 1 + i$

- i. Express  $\frac{4w}{u}$  in Cartesian form

1 mark

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- ii. Express  $\frac{4w}{u}$  in Polar form

1 mark

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- iii. Hence, express  $\tan\left(\frac{\pi}{12}\right)$  in the form  $a - \sqrt{b}$  where  $a$  and  $b$  are positive integers.

2 marks

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**SECTION B – continued**  
**TURN OVER**

**Question 4** (9 marks)

A tank initially contains 100 litres of water with 5kg of salt dissolved uniformly into it. A salt solution with a concentration of 0.1kg/litre flows into the tank at a rate of 2litres/min. The mixture is kept uniform by stirring and flows out at the rate of 2litres/min. If  $x$  kilograms is the amount of salt in the tank after  $t$  minutes

- a. Show that the differential equation for  $x$  in terms of  $t$  is

$$\frac{dx}{dt} = \frac{10 - x}{50}$$

1 mark

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- b. Solve this differential equation to give  $x$  as a function of  $t$ .

2 marks

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- c. Calculate the amount of salt in the tank after 2 minutes. Give your answer correct to 2 decimal places.

1 mark

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- d. Sketch a graph of  $x$  against  $t$

1 mark



**e.** The outflow is now adjusted to 3litres/min instead of 2litres/min

**i.** Set up a new differential equation for  $x$  in terms of  $t$ . Do not attempt to solve it.

1 mark

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**ii.** Use Euler's method with increments of 1 minute to predict the amount of salt in the tank after 2 minutes. Give your answer correct to 3 decimal places.

2 marks

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**iii.** Verify that the following is a solution for this differential equation

$$x = \frac{1}{10}(100 - t) - \frac{5}{1000000}(100 - t)^3$$

2 marks

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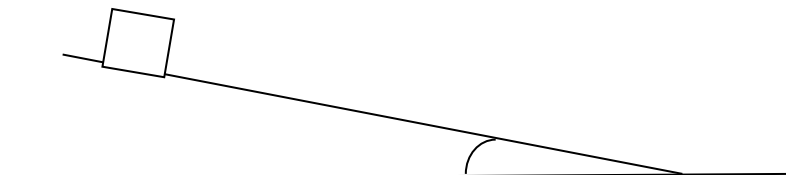
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**TURN OVER**

**Question 5** (11 marks)

A baggage ramp in an airport is 6 metres in length and is inclined at 25 degrees to the horizontal. A suitcase of mass 15kg is initially at rest at the top of the ramp. The suitcase slides down the ramp under the force of gravity with a constant frictional force of 40N acting upon it.

- a. Mark in all of the forces acting on the suitcase. 1 mark



- b. Find the acceleration of the suitcase down the ramp. Give your answer correct to 2 decimal places. 2 marks

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- c. Find the time taken for the suitcase to reach the bottom of the ramp. Give your answer correct to 2 decimal places. 2 marks

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- d. An identical suitcase, also initially at rest, is pushed down the ramp with a force of  $30 - 50t$  for the first 0.6 seconds. Show that the acceleration of this suitcase at time  $t$  where  $0 < t \leq 0.6$  is approximately

$$3.47 - \frac{10}{3}t$$

1 mark

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- e. Using this approximate acceleration, find the speed of this second suitcase when  $t = 0.6$ . Give your answer correct to 2 decimal places.

2 marks

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- f. How long does it take this suitcase to reach the bottom of the ramp? Give your answer correct to 2 decimal places.

3 marks

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**TURN OVER**

**Question 6** (9 marks)

The monthly turnover of a retail shop is normally distributed with an average of \$40500 per month and a standard deviation of \$4500

- a. What is the probability that in a three monthly period of time the mean turnover per month would be more than \$42000? Give your answer correct to three decimal places.

2 marks

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- b. What is the probability that the shop will turnover more than half a million dollars in a year? Give your answer correct to three decimal places.

2 marks

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- c. What is the probability of them turning over more than half a million dollars in a year, given that they are guaranteed to turn over at least \$480000? Give your answer correct to three decimal places.

2 marks

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**SECTION B – Question 6 – continued**

- d. Following an advertising campaign the next three month period produced an average monthly turnover of \$46500. At the 5% level, test whether the advertising campaign has lead to a significant increase in the monthly turnover.

3 marks

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**END OF QUESTION AND ANSWER BOOK**