

Unit 2 Exam – Tech Access – Extra Revision

Question 20

If $A = \begin{bmatrix} a & 0 & 0 \\ 0 & 0 & a \\ 0 & a & 0 \end{bmatrix}$, then A^2 is:

A. $\begin{bmatrix} a^2 & 0 & 0 \\ 0 & 0 & a^2 \\ 0 & a^2 & 0 \end{bmatrix}$

B. $a^2 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

C. $\begin{bmatrix} a & 0 & 0 \\ 0 & 0 & a \\ 0 & a & 0 \end{bmatrix}$

D. aI , where I is the 3×3 identity matrix.

☒ E. $a^2 I$, where I is the 3×3 identity matrix.

using CAS

$$A^2 = \begin{bmatrix} a^2 & 0 & 0 \\ 0 & a^2 & 0 \\ 0 & 0 & a^2 \end{bmatrix}$$

$$= a^2 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= a^2 I_3$$

Question 13

The volume of a certain cylinder is numerically equal to its total surface area.

Which of the following equations would show the relationship between the height of the cylinder (H) and the radius of its base (R)?

☒ A. $H = \frac{2R}{R-2}$

B. $H = \frac{R+2}{R-2}$

C. $H = \frac{R-2}{R+2}$

D. $H = \frac{R+2}{2R}$

E. $H = \frac{R-2}{2R}$

$$V = \pi r^2 H$$

$$TSA = 2\pi r H + 2\pi r^2$$

$$\cancel{\pi} r^2 H = 2\cancel{\pi} r H + 2\cancel{\pi} r^2$$

$$H(r^2 - 2r) = 2r^2$$

$$rH(r-2) = 2r^2$$

$$H(r-2) = 2r \quad (r \neq 0)$$

$$H = \frac{2r}{r-2}$$

Question 19

The equation of the line that makes an angle of approximately $53^\circ 8'$ with the x -axis and passes through the point $(3, 5)$ is:

- A. $3y = 4x + 4$
 B. $3y = 4x - 4$
 C. $3y = -4x + 4$
 D. $3y = 4x + 3$
 E. $3y = 4x - 5$

$$m = \tan \theta$$

$$= \tan 53^\circ 8'$$

$$\approx 1.333$$

$$\approx 4/3$$

$$y - 5 = 4/3(x - 3)$$

$$3y - 15 = 4x - 12$$

$$3y = 4x + 3$$

Question 9

Consider the matrix:

$$A = \begin{bmatrix} \cos \theta - z & -\sin \theta \\ \sin \theta & \cos \theta - z \end{bmatrix}$$

- a) Find $\det(A)$ when $z = 0$. Hence show that A has an inverse when $z = 0$.

(Note: $\sin^2(\theta) + \cos^2(\theta) = 1$)

$$\begin{aligned} \det(A) &= \cos \theta \times \cos \theta + \sin \theta \times \sin \theta \\ &= \cos^2 \theta + \sin^2 \theta \\ &= 1 \end{aligned}$$

SINCE $\det(A) \neq 0$

- b) Now $z \neq 0$. Find an expression for $\det(A)$ as a quadratic polynomial in terms of z with coefficients as functions of θ .

A HAS AN INVERSE

$$\begin{aligned} \det(A) &= (\cos \theta - z)^2 + \sin^2 \theta = \cos^2 \theta - 2z \cos \theta + z^2 + \sin^2 \theta \\ &= z^2 - 2 \cos \theta z + 1 \end{aligned}$$

- c) Hence, considering A to be a matrix with complex entries, show that for $\det(A) = 0$, there are two possible complex numbers: $z = \cos \theta \pm i \sin \theta$. (Hint: use the quadratic formula).

$$\begin{aligned} z &= \frac{2 \cos \theta \pm \sqrt{4 \cos^2 \theta - 4}}{2} = \cos \theta \pm \sqrt{\cos^2 \theta - 1} = \cos \theta \pm \sqrt{-\sin^2 \theta} \\ &= \cos \theta \pm i \sin \theta \end{aligned}$$

- d) Hence, determine all the possible values of θ for which A is a singular matrix with real entries.

$$\text{REAL} \Rightarrow \sin \theta = 0 \Rightarrow \theta = n\pi, n \in \mathbb{Z} \Rightarrow \cos \theta = \pm 1$$

$$\text{SINGULAR} \Rightarrow \det(A) = 0 \Rightarrow z^2 \mp 2z + 1 = 0 \quad (z \mp 1) = 0$$

- e) If $z = \frac{1}{\sqrt{2}}(1 + i)$, then determine one possible angle, θ , for which $\det(A) = 0$. (Hint: Use c)). $\theta = n\pi$

$$\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i = \cos \theta \pm i \sin \theta$$

$$\cos \theta = \frac{1}{\sqrt{2}}, \sin \theta = \pm \frac{1}{\sqrt{2}}$$

$$\theta = \pm \pi/4$$

(2+2+2+2+2=10 marks)

Question 10

Solve the matrix equation below.

$$\begin{bmatrix} 1 & 2 & 5 & 7 \\ 3 & 2 & 3 & -10 \\ 4 & 5 & 15 & 8 \\ -5 & 8 & 11 & -2 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$$

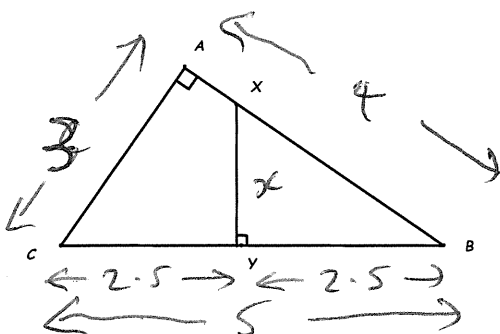
$$\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 1 & 2 & 5 & 7 \\ 3 & 2 & 3 & -10 \\ 4 & 5 & 15 & 8 \\ -5 & 8 & 11 & -2 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$$

$$\text{USING CAS} = \begin{bmatrix} 38/245 \\ 31/49 \\ -8/245 \\ -9/245 \end{bmatrix}$$

(2 marks)

Question 6

ABC is a right-angled triangle with $AB = 4$ and $AC = 3$. If the triangle is folded along the line XY , vertex C coincides with vertex B . Find the length of XY .



$$\triangle BXY \sim \triangle BCA$$

$$\frac{2.5}{4} = \frac{x}{3}$$

$$x = 1.875$$

(3 marks)

Question 7

A window that consists of a semicircle on top of a rectangle has the dimensions as shown.

- a If the perimeter of the window is 100 cm show that $y = \frac{100 - x(\pi + 2)}{2}$.

$$P = \frac{1}{2}(2\pi x) + 2y + 2x = 100$$

$$2y = 100 - \pi x - 2x$$

$$y = \frac{100 - x(\pi + 2)}{2}$$

AS REQ'D

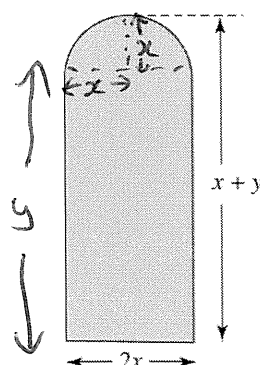
- b Show that the shaded area A can be expressed as: $A = x(100 - x(\pi + 2)) + \frac{1}{2}\pi x^2$.

$$A = 2xy + \frac{1}{2}\pi x^2$$

$$= 2x \left(\frac{100 - x(\pi + 2)}{2} \right) + \frac{1}{2}\pi x^2$$

$$= x(100 - x(\pi + 2)) + \frac{1}{2}\pi x^2$$

AS REQ'D



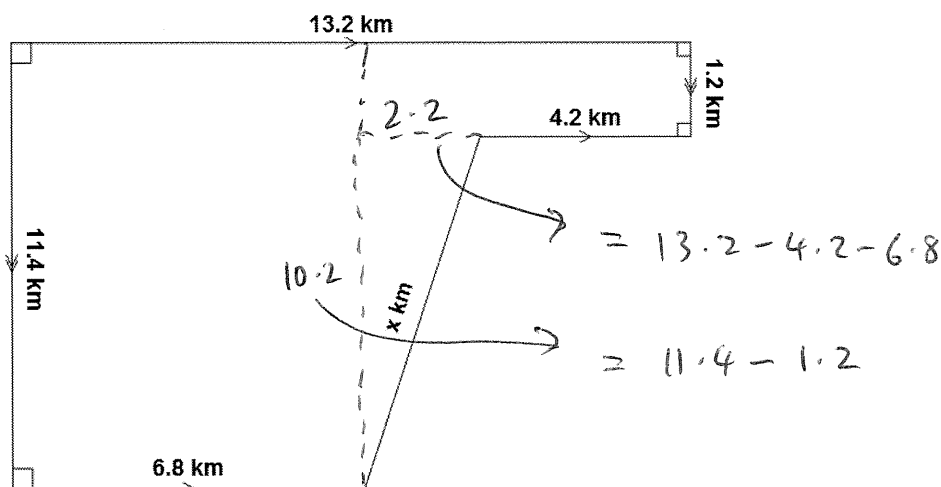
- c If $y = 10$, find the shaded area of the shape, expressed in exact form.

$$10 = \frac{100 - x(\pi + 2)}{2} \quad \rightarrow \quad A = \frac{180}{\pi + 2} \left(100 - \frac{180}{\pi + 2} (\pi + 2) \right) + \frac{1}{2} \pi \left(\frac{180}{\pi + 2} \right)^2$$

$$x = \frac{180}{\pi + 2} \quad = \frac{1600(3\pi + 2)}{(\pi + 2)^2} \quad \text{(USING CAS)} \quad (2+2+2=6 \text{ marks})$$

Question 8

Below is a proposed map of a new airport that is to be built in Melbourne:



- a Calculate the length of the edge labelled x km correct to 1 decimal place.

$$x = \sqrt{10.2^2 + 2.2^2} = 10.4$$

- b Calculate the land area in square kilometres (km^2) of the airport. Give your answer correct to 2 decimal places.

$$A = (11.4 \times 6.8) + \frac{1}{2} \times 2.2 \times 10.2 + 1.2 \times (13.2 - 6.8)$$

$$= 96.42 \text{ km}^2$$

A scale model of the site has been constructed. The scale model uses a 1:5000 scale.

- c Calculate the length of the side that is 13.2 km on the scale model in **metres**.

$$\frac{13200}{5000} = 2.64 \text{ m}$$

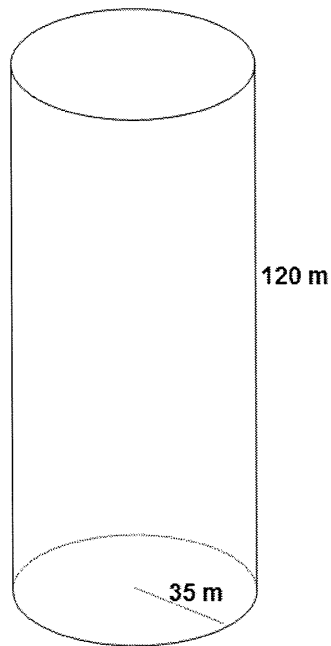
- d Calculate the area of the scale model, in m^2 , correct to 2 decimal places.

$$96.42 \text{ km}^2 \times 1000^2 = 9.642 \times 10^7 \text{ m}^2$$

$$\text{AREA FACTOR} = 5000^2$$

$$A = \frac{9.642 \times 10^7}{5000^2} \approx 3.86 \text{ m}^2 \quad (2+3+2+2=9 \text{ marks})$$

The control tower at this new airport is cylindrical in design with a radius of 35m and a height of 120m as shown below:



- e Calculate the volume of the control tower at this Airport in m^3 , correct to the nearest whole m^3 .

$$\begin{aligned} V &= \pi r^2 h \\ &= \pi \times 35^2 \times 120 \\ &\approx 461814 \text{ m}^3 \end{aligned}$$

- f Calculate the volume of the control tower in the scale model in **cubic centimetres, cm^3** , correct to 2 decimal places.

$$V = 461814 \text{ m}^3 \times 100^3 = 4.61814 \times 10^{11} \text{ cm}^3$$

$$V = \frac{4.61814 \times 10^{11}}{5000^3} \approx 3.69 \text{ cm}^3$$

(2+2=4marks)