

Formula Sheet

Coordinate Geometry

angle to the x-axis: $m = \tan \theta$

angle between two lines: $\tan \theta = \pm \frac{m_2 - m_1}{1 + m_1 m_2}$

distance between two points:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

midpoint: $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$

Vectors

$$\vec{r} = x\vec{i} + y\vec{j} \quad |\vec{r}| = r \quad |\vec{r}| = \sqrt{x^2 + y^2}$$

$$\vec{r}_1 \cdot \vec{r}_2 = |\vec{r}_1| |\vec{r}_2| \cos \theta = x_1 x_2 + y_1 y_2$$

scalar resolute: $\vec{a} \cdot \hat{b}$

vector resolute: $(\vec{a} \cdot \hat{b}) \hat{b}$

Complex Numbers

$$z = x + yi = r(\cos \theta + i \sin \theta) = r \operatorname{cis} \theta$$

where $r = \sqrt{x^2 + y^2}$ and $\theta = \tan^{-1} \frac{y}{x}$

$$-\pi < \operatorname{Arg}(z) \leq \pi$$

$$z_1 z_2 = r_1 r_2 \operatorname{cis}(\theta_1 + \theta_2) \quad \text{and} \quad \frac{z_1}{z_2} = \frac{r_1}{r_2} \operatorname{cis}(\theta_1 - \theta_2)$$

$$z^n = r^n \operatorname{cis}(n\theta) \quad (\text{de Moivre's Theorem})$$

Trigonometry

$$\text{sine rule: } \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\text{cosine rule: } c^2 = a^2 + b^2 - 2ab \cos C$$

$$\text{area of a triangle: } A = \frac{1}{2}bh = \frac{1}{2}ab \sin C$$

$$\text{Heron's formula: } A = \sqrt{s(s-a)(s-b)(s-c)}$$

$$\text{where } s = \frac{1}{2}(a+b+c)$$

Reciprocal trigonometric functions:

$$\sec \theta = \frac{1}{\cos \theta} \quad \operatorname{cosec} \theta = \frac{1}{\sin \theta} \quad \cot \theta = \frac{1}{\tan \theta}$$

Fundamental trigonometric identities:

$$\cos^2 x + \sin^2 x = 1$$

$$1 + \tan^2 x = \sec^2 x$$

$$\cot^2 x + 1 = \operatorname{cosec}^2 x$$

Compound angle (Addition) formulae:

$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$$

$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$

$$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$$

Double angle formulae:

$$\sin 2x = 2 \sin x \cos x$$

$$\cos 2x = 2 \cos^2 x - 1$$

$$= 1 - 2 \sin^2 x$$

$$= \cos^2 x - \sin^2 x$$

$$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$$

$$a \sin \theta \pm b \cos \theta = R \sin(\theta \pm \alpha)$$

$$a \cos \theta \pm b \sin \theta = R \cos(\theta \mp \alpha)$$

where: $R = \sqrt{a^2 + b^2}$ and $\tan \alpha = \frac{b}{a}$

Circle Mensuration

Arc length: $r\theta$

Chord length: $2r \sin\left(\frac{\theta}{2}\right)$

Sector area: $\frac{1}{2}r^2\theta$

Segment area: $\frac{1}{2}r^2(\theta - \sin \theta)$

Matrices

Determinant of the 2×2 matrix:

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$\det A = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

Inverse of a 2×2 matrix:

$$A^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \text{ where}$$

$$\det(A) \neq 0$$

Translation Matrix: $T_{(a,b)} = \begin{bmatrix} a \\ b \end{bmatrix}$

Rotation Matrix

(anticlockwise about (0,0)):

$$R_\theta = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$$

Matrix for Reflection in the X axis:

$$M_x = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

Matrix for Reflection in the Y axis:

$$M_y = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

Matrix for Reflection in the line $y = x$:

$$M_{y=x} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

Matrix for dilation from the X axis by a factor

of n : $D_y = \begin{bmatrix} 1 & 0 \\ 0 & n \end{bmatrix}$

Matrix for dilation from the Y axis by a factor

of m : $D_x = \begin{bmatrix} m & 0 \\ 0 & 1 \end{bmatrix}$

Matrix for dilation about the origin by a

factor of k : $D_k = \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$

Kinematics

$$v = u + at$$

$$v^2 = u^2 + 2as$$

$$s = ut + \frac{1}{2}at^2$$

$$s = \frac{1}{2}(u + v)t$$

Statistics & Probability

$$\mu_{\bar{x}} = \mu$$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

Hypergeometric:

$$Pr(X = x) = \frac{{}^D C_x \times {}^{N-D} C_{n-x}}{{}^N C_n}$$

Binomial:

$$Pr(X = x) = {}^n C_x \times p^x \times (1-p)^{n-x}$$

Sequences and Series

Arithmetic:

$$t_n = a + (n-1)d$$

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

Geometric:

$$t_n = ar^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$S_\infty = \frac{a}{1-r}$$