

Specialist Unit 1&2

Headstart Program

Over the next 5 days you will be introduced to Unit 1 of Specialist Mathematics VCE course

You will be making a start on the topic of “Advanced ALGEBRA”

Throughout the week you will be expected to take course notes that your teacher will provide and to complete the following:

Exercises

The headstart booklet contains 5 textbook exercises and 1 worksheet.

Ex 1H Literal Equations

Ex 1I Literal Equations in CAS

Ex 5A Equating coefficients

Ex 5D Partial Fractions

Ex 5E Simultaneous Equations

One worksheet: Extra questions on Partial Fractions



Textbook required:

Cambridge Senior Mathematics: Australian Curriculum VCE
Specialist Mathematics Unit 1 and 2

Calculator: Casio Classpad fx-CP400

You are expected to have this calculator for Specialist Unit 1 and 2.

Calculator is expected to be in class for each lesson during headstart



Holiday Homework: Complete the exercises in this booklet

It is assumed that all the set work during this Headstart program are completed within the first week back in 2018.

The Headstart program work to be shown and submitted to your teacher at the start of term 1, 2018.

Exercise 1H

Example 20

1 Solve each of the following for x :

a $ax + n = m$

b $ax + b = bx$

c $\frac{ax}{b} + c = 0$

d $px = qx + 5$

e $mx + n = nx - m$

f $\frac{1}{x+a} = \frac{b}{x}$

g $\frac{b}{x-a} = \frac{2b}{x+a}$

h $\frac{x}{m} + n = \frac{x}{n} + m$

i $-b(ax + b) = a(bx - a)$

j $p^2(1-x) - 2pqx = q^2(1+x)$

k $\frac{x}{a} - 1 = \frac{x}{b} + 2$

l $\frac{x}{a-b} + \frac{2x}{a+b} = \frac{1}{a^2-b^2}$

m $\frac{p-qx}{t} + p = \frac{qx-t}{p}$

n $\frac{1}{x+a} + \frac{1}{x+2a} = \frac{2}{x+3a}$

2 For the simultaneous equations $ax + by = p$ and $bx - ay = q$, show that $x = \frac{ap + bq}{a^2 + b^2}$ and $y = \frac{bp - aq}{a^2 + b^2}$.

3 For the simultaneous equations $\frac{x}{a} + \frac{y}{b} = 1$ and $\frac{x}{b} + \frac{y}{a} = 1$, show that $x = y = \frac{ab}{a+b}$.

Example 21

4 Solve each of the following pairs of simultaneous equations for x and y :

a $ax + y = c$

b $ax - by = a^2$

$x + by = d$

$bx - ay = b^2$

c $ax + by = t$

d $ax + by = a^2 + 2ab - b^2$

$ax - by = s$

$bx + ay = a^2 + b^2$

e $(a+b)x + cy = bc$

f $3(x-a) - 2(y+a) = 5 - 4a$

$(b+c)y + ax = -ab$

$2(x+a) + 3(y-a) = 4a - 1$

5 Write s in terms of a only in the following pairs of equations:

a $s = ah$

b $s = ah$

c $as = a + h$

$h = 2a + 1$

$h = a(2 + h)$

$h + ah = 1$

d $as = s + h$

e $s = h^2 + ah$

f $as = a + 2h$

$ah = a + h$

$h = 3a^2$

$h = a - s$

g $s = 2 + ah + h^2$

h $3s - ah = a^2$

$h = a - \frac{1}{a}$

$as + 2h = 3a$

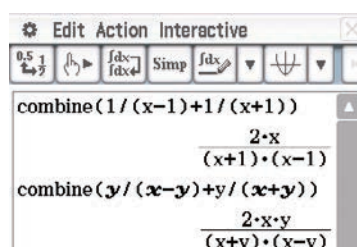


Combining fractions

This command returns the answer as a single fraction with the denominator in factored form.

Examples:

- Enter and highlight $1/(x-1) + 1/(x+1)$. Then select **Interactive** > **Transformation** > **combine**.
- Enter and highlight $y/(x-y) + y/(x+y)$. Then select **Interactive** > **Transformation** > **combine**.



Exercise 11

This exercise provides practice in some of the skills associated with a CAS calculator. Other exercises in this chapter can be attempted with CAS, but it is recommended that you also use this chapter to develop your 'by hand' skills.

- 1 Solve each of the following equations for x :

a $\frac{a(a-x)}{b} - \frac{b(b+x)}{a} = x$

b $2(x-3) + (x-2)(x-4) = x(x+1) - 33$

c $\frac{x+a}{x+b} = 1 - \frac{x}{x-b}$

d $\frac{x+a}{x-c} + \frac{x+c}{x-a} = 2$

- 2 Factorise each of the following:

a $x^2y^2 - x^2 - y^2 + 1$

b $x^3 - 2 - x + 2x^2$

c $a^4 - 8a^2b - 48b^2$

d $a^2 + 2bc - (c^2 + 2ab)$

- 3 Solve each of the following pairs of simultaneous equations for x and y :

a $axy + b = (a+c)y$

b $x(b-c) + by - c = 0$

$bxy + a = (b+c)y$

$y(c-a) - ax + c = 0$



Exercise 5A

1 If $ax^2 + bx + c = 10x^2 - 7$, find the values of a , b and c .

Example 1 2 If $(2a - b)x^2 + (a + 2b)x + 8 = 4x^2 - 3x + 8$, find the values of a and b .

3 If $(2a - 3b)x^2 + (3a + b)x + c = 7x^2 + 5x + 7$, find the values of a , b and c .

4 If $2x^2 + 4x + 5 = a(x + b)^2 + c$, find the values of a , b and c .

Example 2 5 Express x^2 in the form $c(x + 2)^2 + a(x + 2) + d$.

6 Express x^3 in the form $(x + 1)^3 + a(x + 1)^2 + b(x + 1) + c$.

Example 3 7 Find the values of a , b and c such that $x^2 = a(x + 1)^2 + bx + c$.

Example 4 8 a Show that $3x^3 - 9x^2 + 8x + 2$ cannot be expressed in the form $a(x + b)^3 + c$.

b If $3x^3 - 9x^2 + 9x + 2$ can be expressed in the form $a(x + b)^3 + c$, then find the values of a , b and c .

9 Show that constants a , b , c and d can be found such that

$$n^3 = a(n + 1)(n + 2)(n + 3) + b(n + 1)(n + 2) + c(n + 1) + d$$

10 a Show that no constants a and b can be found such that

$$n^2 = a(n + 1)(n + 2) + b(n + 2)(n + 3)$$

b Express n^2 in the form $a(n + 1)(n + 2) + b(n + 1) + c$.

11 a Express $a(x + b)^2 + c$ in expanded form.

b Express $ax^2 + bx + c$ in completed-square form.

12 Prove that, if $ax^3 + bx^2 + cx + d = (x - 1)^2(px + q)$, then $b = d - 2a$ and $c = a - 2d$.

13 If $3x^2 + 10x + 3 = c(x - a)(x - b)$ for all values of x , find the values of a , b and c .

14 For any number n , show that n^2 can be expressed as $a(n - 1)^2 + b(n - 2)^2 + c(n - 3)^2$, and find the values of a , b and c .

15 If $x^3 + 3x^2 - 9x + c$ can be expressed in the form $(x - a)^2(x - b)$, show that either $c = 5$ or $c = -27$, and find a and b for each of these cases.

16 A polynomial P is said to be **even** if $P(-x) = P(x)$ for all x . A polynomial P is said to be **odd** if $P(-x) = -P(x)$ for all x .



a Show that, if $P(x) = ax^4 + bx^3 + cx^2 + dx + e$ is even, then $b = d = 0$.

b Show that, if $P(x) = ax^5 + bx^4 + cx^3 + dx^2 + ex + f$ is odd, then $b = d = f = 0$.

Exercise 5D

Example 13

1 Resolve the following rational expressions into partial fractions:

a $\frac{5x+1}{(x-1)(x+2)}$

b $\frac{-1}{(x+1)(2x+1)}$

c $\frac{3x-2}{x^2-4}$

d $\frac{4x+7}{x^2+x-6}$

e $\frac{7-x}{(x-4)(x+1)}$

Example 14

2 Resolve the following rational expressions into partial fractions:

a $\frac{2x+3}{(x-3)^2}$

b $\frac{9}{(1+2x)(1-x)^2}$

c $\frac{2x-2}{(x+1)(x-2)^2}$

Example 15

3 Resolve the following rational expressions into partial fractions:

a $\frac{3x+1}{(x+1)(x^2+x+1)}$

b $\frac{3x^2+2x+5}{(x^2+2)(x+1)}$

c $\frac{x^2+2x-13}{2x^3+6x^2+2x+6}$

Example 16

4 Resolve $\frac{3x^2-4x-2}{(x-1)(x-2)}$ into partial fractions.

5 Show that it is not possible to find values of A and C such that

$$\frac{2x+10}{(x+1)(x-1)^2} = \frac{A}{x+1} + \frac{C}{(x-1)^2}$$

6 Express each of the following as partial fractions:

a $\frac{1}{(x-1)(x+1)}$

b $\frac{x}{(x-2)(x+3)}$

c $\frac{3x+1}{(x-2)(x+5)}$

d $\frac{1}{(2x-1)(x+2)}$

e $\frac{3x+5}{(3x-2)(2x+1)}$

f $\frac{2}{x^2-x}$

g $\frac{3x+1}{x^3+x}$

h $\frac{3x^2+8}{x(x^2+4)}$

i $\frac{1}{x^2-4x}$

j $\frac{x+3}{x^2-4x}$

k $\frac{x^3-x^2-1}{x^2-x}$

l $\frac{x^3-x^2-6}{2x-x^2}$

m $\frac{x^2-x}{(x+1)(x^2+2)}$

n $\frac{x^2+2}{x^3-3x-2}$

o $\frac{2x^2+x+8}{x(x^2+4)}$

p $\frac{1-2x}{2x^2+7x+6}$

q $\frac{3x^2-6x+2}{(x-1)^2(x+2)}$

r $\frac{4}{(x-1)^2(2x+1)}$

s $\frac{x^3-2x^2-3x+9}{x^2-4}$

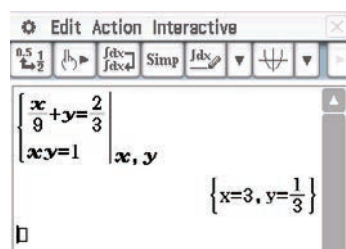
t $\frac{x^3+3}{(x+1)(x-1)}$

u $\frac{2x-1}{(x+1)(3x+2)}$



Using the Casio ClassPad

- In $\sqrt{\square}$, select the simultaneous equations template by tapping $\left\{ \begin{array}{l} \square \\ \square \end{array} \right\}$ from the (Math1) keyboard.
- Enter the two equations and the variables x, y in the spaces provided; tap (EXE).



Example 20

Find the coordinates of the points of intersection of the graphs of $y = -3x^2 - 4x + 1$ and $y = 2x^2 - x - 1$.

Solution

$$-3x^2 - 4x + 1 = 2x^2 - x - 1$$

$$-5x^2 - 3x + 2 = 0$$

$$5x^2 + 3x - 2 = 0$$

$$(5x - 2)(x + 1) = 0$$

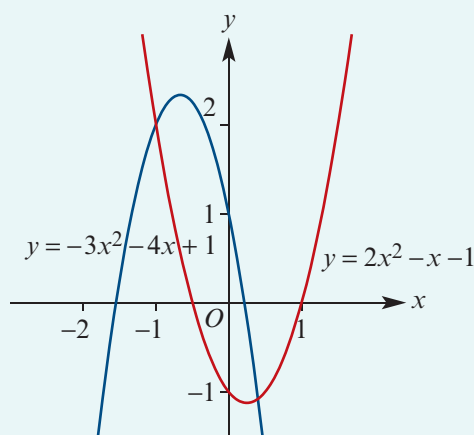
$$\therefore x = \frac{2}{5} \text{ or } x = -1$$

Substitute in $y = 2x^2 - x - 1$:

When $x = -1$, $y = 2$.

When $x = \frac{2}{5}$, $y = 2 \times \frac{4}{25} - \frac{2}{5} - 1 = -\frac{27}{25}$.

The points of intersection are $(-1, 2)$ and $(\frac{2}{5}, -\frac{27}{25})$.



Exercise 5E

Skillsheet

- 1 Find the coordinates of the points of intersection for each of the following:

a $y = x^2$
 $y = x$

b $y = 2x^2 = 0$
 $y - x = 0$

c $y = x^2 - x$
 $y = 2x + 1$

Example 17

- 2 Find the coordinates of the points of intersection for each of the following:

a $x^2 + y^2 = 178$
 $x + y = 16$

b $x^2 + y^2 = 125$
 $x + y = 15$

c $x^2 + y^2 = 185$
 $x - y = 3$

d $x^2 + y^2 = 97$
 $x + y = 13$

e $x^2 + y^2 = 106$
 $x - y = 4$

Example 19**3** Find the coordinates of the points of intersection for each of the following:

a $x + y = 28$

b $x + y = 51$

c $x - y = 5$

$xy = 187$

$xy = 518$

$xy = 126$

4 Find the coordinates of the points of intersection of the straight line with equation $y = 2x$ and the circle with equation $(x - 5)^2 + y^2 = 25$.**5** Find the coordinates of the points of intersection of the curves with equations $y = \frac{1}{x-2} + 3$ and $y = x$.**6** Find the coordinates of the points A and B where the line with equation $x - 3y = 0$ meets the circle with equation $x^2 + y^2 - 10x - 5y + 25 = 0$.**7** Find the coordinates of the points of intersection of the line with equation $\frac{y}{4} - \frac{x}{5} = 1$ and the circle with equation $x^2 + 4x + y^2 = 12$.**8** Find the coordinates of the points of intersection of the curve with equation $y = \frac{1}{x+2} - 3$ and the line with equation $y = -x$.**9** Find the point where the line $4y = 9x + 4$ touches the parabola $y^2 = 9x$.**10** Find the coordinates of the point where the line with equation $y = 2x + 3\sqrt{5}$ touches the circle with equation $x^2 + y^2 = 9$.**11** Find the coordinates of the point where the straight line with equation $y = \frac{1}{4}x + 1$ touches the curve with equation $y = -\frac{1}{x}$.**12** Find points of intersection of the curve $y = \frac{2}{x-2}$ and the line $y = x - 1$.**Example 20****13** Find the coordinates of the points of intersection of the graphs of the following pairs of quadratic functions:

a $y = 2x^2 - 4x + 1$, $y = 2x^2 - x - 1$

b $y = -2x^2 + x + 1$, $y = 2x^2 - x - 1$

c $y = x^2 + x + 1$, $y = x^2 - x - 2$

d $y = 3x^2 + x + 2$, $y = x^2 - x + 2$

14 In each of the following, use the discriminant of the resulting quadratic equation:**a** Find the possible values of k for which the straight line $y = k(1 - 2x)$ touches but does not cross the parabola $y = x^2 + 2$.**b** Find the possible values of c for which the line $y = 2x + c$ intersects the circle $x^2 + y^2 = 20$ in two distinct points.**c** Find the value of p for which the line $y = 6$ meets the parabola $y = x^2 + (1 - p)x + 2p$ at only one point.

Partial Fractions

Miscellaneous questions

Express each of the following as partial fractions.

Examples:

Type 1: Ordinary fractions: $\frac{ax+b}{(cx+d)(ex+f)} \equiv \frac{A}{(cx+d)} + \frac{B}{(ex+f)}$

Type 2: Repeated linear factor: $\frac{ax+b}{(cx+d)^2} \equiv \frac{A}{(cx+d)^2} + \frac{B}{(cx+d)}$

Type 3: Quadratic factor (irreducible):
 $\frac{ax^2+bx+c}{(dx^2+ex+f)(gx+h)} \equiv \frac{Ax+B}{(dx^2+ex+f)} + \frac{C}{gx+h}, \text{ where } e^2-4df < 0$

Type 4: Numerator is same or higher order as denominator:
 $\frac{ax^2+bx+c}{(dx+e)(fx+g)} \equiv A + \frac{B}{(dx+e)} + \frac{c}{(fx+g)}$

Note: These forms of the fractions are not exhaustive.

Questions:

1. $\frac{-5x^2+2x-6}{(x^2+2)(x+2)}$

2. $\frac{-5x^2+2x-26}{(x^2-4)(x+3)}$

3. $\frac{-5x+12}{(x+2)(x-2)}$

4. $\frac{x^3-x^2-7x+12}{(x+1)(x-2)}$

5. $\frac{3}{x^3-1}$

6. $\frac{x^5+7x^4+18x^3+36x^2+57x+57}{(x^2+3)(x+3)^2}$

7. $\frac{2ax-bx+10a-2b}{(x+5)(x+2)}$

8. $\frac{bx-ax-a^2-b^2}{(x+a)(x-b)}$

ANSWERS

Exercise 1H

- 1 **a** $x = \frac{m-n}{a}$ **b** $x = \frac{b}{b-a}$ **c** $x = -\frac{bc}{a}$
d $x = \frac{5}{p-q}$ **e** $x = \frac{m+n}{n-m}$ **f** $x = \frac{ab}{1-b}$
g $x = 3a$ **h** $x = -mn$ **i** $x = \frac{a^2 - b^2}{2ab}$
j $x = \frac{p-q}{p+q}$ **k** $x = \frac{3ab}{b-a}$ **l** $x = \frac{1}{3a-b}$
m $x = \frac{p^2 + p^2t + t^2}{q(p+t)}$ **n** $x = -\frac{5a}{3}$
- 4 **a** $x = \frac{d-bc}{1-ab}, y = \frac{c-ad}{1-ab}$
b $x = \frac{a^2 + ab + b^2}{a+b}, y = \frac{ab}{a+b}$
c $x = \frac{t+s}{2a}, y = \frac{t-s}{2b}$ **d** $x = a+b, y = a-b$
e $x = c, y = -a$ **f** $x = a+1, y = a-1$
- 5 **a** $s = a(2a+1)$ **b** $s = \frac{2a^2}{1-a}$
c $s = \frac{a^2 + a + 1}{a(a+1)}$ **d** $s = \frac{a}{(a-1)^2}$
e $s = 3a^3(3a+1)$ **f** $s = \frac{3a}{a+2}$
g $s = 2a^2 - 1 + \frac{1}{a^2}$ **h** $s = \frac{5a^2}{a^2 + 6}$

Exercise 5A

- 1 $a = 10, b = 0, c = -7$
2 $a = 1, b = -2$
3 $a = 2, b = -1, c = 7$
4 $a = 2, b = 1, c = 3$
5 $(x+2)^2 - 4(x+2) + 4$
6 $(x+1)^3 - 3(x+1)^2 + 3(x+1) - 1$
7 $a = 1, b = -2, c = -1$
8 **a** It is impossible to find a, b and c such that
 $a = 3, 3ab = -9, 3ab^2 = 8$ and $ab^3 + c = 2$
b $a = 3, b = -1, c = 5$
9 $a = 1, b = -6, c = 7, d = -1$
10 **a** If $a = -\frac{5}{3}b$ and $a = -3b$, then both a and b
are zero, but then $a + b = 1$ is not satisfied
b $(n+1)(n+2) - 3(n+1) + 1$
11 **a** $ax^2 + 2abx + ab^2 + c$
b $a\left(x + \frac{b}{2a}\right)^2 + c - \frac{b^2}{4a}$
13 $a = -3, b = -\frac{1}{3}, c = 3$ or
 $a = -\frac{1}{3}, b = -3, c = 3$
14 $a = 3, b = -3, c = 1$
15 If $c = 5$, then $a = 1$ and $b = -5$;
if $c = -27$, then $a = -3$ and $b = 3$

Exercise 1I

- 1 **a** $x = a - b$ **b** $x = 7$
c $x = -\frac{a \pm \sqrt{a^2 + 4ab - 4b^2}}{2}$ **d** $x = \frac{a+c}{2}$
- 2 **a** $(x-1)(x+1)(y-1)(y+1)$
b $(x-1)(x+1)(x+2)$
c $(a^2 - 12b)(a^2 + 4b)$
d $(a-c)(a-2b+c)$
- 3 **a** $x = \frac{a+b+c}{a+b}, y = \frac{a+b}{c}$
b $x = \frac{-(a-b-c)}{a+b-c}, y = \frac{a-b+c}{a+b-c}$

Exercise 5D

$$1 \text{ a } \frac{2}{x-1} + \frac{3}{x+2} \quad \text{b } \frac{1}{x+1} + \frac{2}{2x+1}$$

$$\text{c } \frac{2}{x+2} + \frac{1}{x-2} \quad \text{d } \frac{1}{x+3} + \frac{3}{x-2}$$

$$\text{e } \frac{3}{5(x-4)} - \frac{8}{5(x+1)}$$

$$2 \text{ a } \frac{2}{x-3} + \frac{9}{(x-3)^2}$$

$$\text{b } \frac{4}{1+2x} + \frac{2}{1-x} + \frac{3}{(1-x)^2}$$

$$\text{c } \frac{-4}{9(x+1)} + \frac{4}{9(x-2)} + \frac{2}{3(x-2)^2}$$

$$3 \text{ a } \frac{-2}{x+1} + \frac{2x+3}{x^2+x+1} \quad \text{b } \frac{x+1}{x^2+2} + \frac{2}{x+1}$$

$$\text{c } \frac{x-2}{x^2+1} - \frac{1}{2(x+3)}$$

$$4 \text{ } 3 + \frac{3}{x-1} + \frac{2}{x-2}$$

5 It is impossible to find A and C such that $A = 0$, $C - 2A = 2$ and $A + C = 10$

$$6 \text{ a } \frac{1}{2(x-1)} - \frac{1}{2(x+1)} \quad \text{b } \frac{2}{5(x-2)} + \frac{3}{5(x+3)}$$

$$\text{c } \frac{1}{x-2} + \frac{2}{x+5}$$

$$\text{d } \frac{2}{5(2x-1)} - \frac{1}{5(x+2)}$$

$$\text{e } \frac{3}{3x-2} - \frac{1}{2x+1}$$

$$\text{f } \frac{2}{x-1} - \frac{2}{x}$$

$$\text{g } \frac{1}{x} + \frac{3-x}{x^2+1}$$

$$\text{h } \frac{2}{x} + \frac{x}{x^2+4}$$

$$\text{i } \frac{1}{4(x-4)} - \frac{1}{4x}$$

$$\text{j } \frac{7}{4(x-4)} - \frac{3}{4x}$$

$$\text{k } x + \frac{1}{x} - \frac{1}{x-1}$$

$$\text{l } -x - 1 - \frac{3}{x} - \frac{1}{2-x}$$

$$\text{m } \frac{2}{3(x+1)} + \frac{x-4}{3(x^2+2)}$$

$$\text{n } \frac{2}{3(x-2)} + \frac{1}{3(x+1)} - \frac{1}{(x+1)^2}$$

$$\text{o } \frac{2}{x} + \frac{1}{x^2+4} \quad \text{p } \frac{8}{2x+3} - \frac{5}{x+2}$$

$$\text{q } \frac{26}{9(x+2)} + \frac{1}{9(x-1)} - \frac{1}{3(x-1)^2}$$

$$\text{r } \frac{16}{9(2x+1)} - \frac{8}{9(x-1)} + \frac{4}{3(x-1)^2}$$

$$\text{s } x - 2 + \frac{1}{4(x+2)} + \frac{3}{4(x-2)}$$

$$\text{t } x - \frac{1}{x+1} + \frac{2}{x-1} \quad \text{u } \frac{3}{x+1} - \frac{7}{3x+2}$$

Exercise 5E

$$1 \text{ a } (1, 1), (0, 0) \quad \text{b } (0, 0), \left(\frac{1}{2}, \frac{1}{2}\right)$$

$$\text{c } \left(\frac{3+\sqrt{13}}{2}, 4+\sqrt{13}\right), \left(\frac{3-\sqrt{13}}{2}, 4-\sqrt{13}\right)$$

$$2 \text{ a } (13, 3), (3, 13) \quad \text{b } (10, 5), (5, 10)$$

$$\text{c } (-8, -11), (11, 8) \quad \text{d } (9, 4), (4, 9)$$

$$\text{e } (9, 5), (-5, -9)$$

$$3 \text{ a } (11, 17), (17, 11) \quad \text{b } (37, 14), (14, 37)$$

$$\text{c } (14, 9), (-9, -14)$$

$$4 (0, 0), (2, 4)$$

$$5 \left(\frac{5+\sqrt{5}}{2}, \frac{5+\sqrt{5}}{2}\right), \left(\frac{5-\sqrt{5}}{2}, \frac{5-\sqrt{5}}{2}\right)$$

$$6 \left(\frac{15}{2}, \frac{5}{2}\right), (3, 1)$$

$$7 \left(\frac{-130+80\sqrt{2}}{41}, \frac{60+64\sqrt{2}}{41}\right),$$

$$\left(\frac{-130-80\sqrt{2}}{41}, \frac{60-64\sqrt{2}}{41}\right)$$

$$8 \left(\frac{1+\sqrt{21}}{2}, \frac{-1-\sqrt{21}}{2}\right), \left(\frac{1-\sqrt{21}}{2}, \frac{-1+\sqrt{21}}{2}\right)$$

$$9 \left(\frac{4}{9}, 2\right)$$