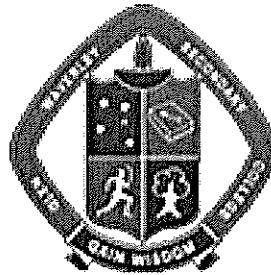


NAME: SOLUTIONS

MARK: / 45

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Glen Waverley Secondary College



SPECIALIST MATHEMATICS – UNIT 2 2018

Vectors Analysis Task

Writing time: 45 minutes (no reading time)

1 CAS and 1 scientific calculator allowed

1 A4 double-sided reference sheet permitted

- This SAC involves completing 6 Analysis questions
- Working must be shown for questions worth more than 1 mark

QUESTION 1

Two students are playing "tiggy" around Glen Waverley Secondary College. The first student Anna is defined by the position vector $\underline{a} = \underline{i} + \underline{j}$. The second student Barry is defined by the position vector $\underline{b} = 3\underline{i} - \underline{j}$. Their positions are relative to the middle school office, O . If M is the midpoint between Anna and Barry, find:

a) $\overrightarrow{OM}$

$$\overrightarrow{AB} = 2\underline{i} - 2\underline{j}$$

$$\overrightarrow{OM} = \overrightarrow{OA} + \frac{1}{2}\overrightarrow{AB} \quad (1)$$

$$= \underline{i} + \underline{j} + (\underline{i} - \underline{j})$$

$$= 2\underline{i} \quad (1)$$

[2 marks]

b) The distance between Anna and Barry.

$$|\overrightarrow{AB}| = \sqrt{2^2 + 2^2} \quad (1)$$

$$= \sqrt{8}$$

$$= 2\sqrt{2} \quad (1)$$

[2 marks]

c) The angle Anna and Barry make with the middle school office ($\angle AOB$), correct to one decimal place.

$$|\overrightarrow{OA}| = \sqrt{2} \quad |\overrightarrow{OB}| = \sqrt{10} \quad (1)$$

$$\overrightarrow{OA} \cdot \overrightarrow{OB} = (\underline{i} + \underline{j}) \cdot (3\underline{i} - \underline{j})$$

$$= 3 - 1$$

$$= 2 \quad (1)$$

[3 marks]

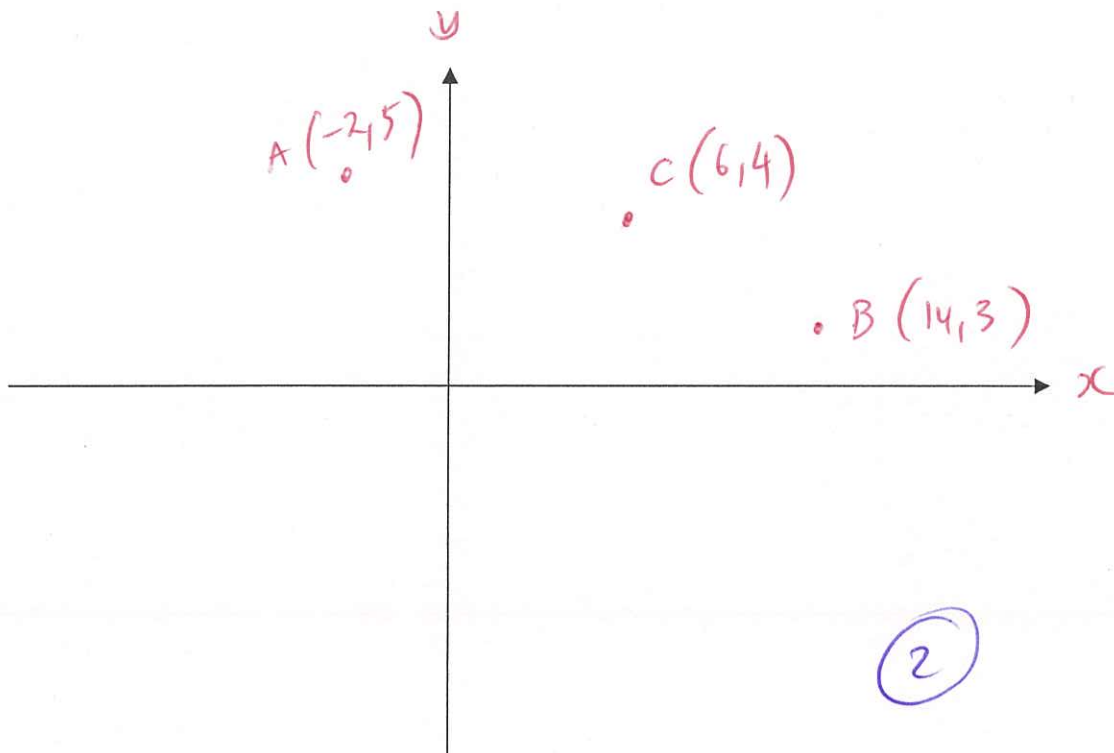
$$\cos Q = \frac{2}{\sqrt{2} \times \sqrt{10}}$$

$$Q = 63.4^\circ \quad (1)$$

QUESTION 2

Three ships, A, B, and C have position vectors $\underline{a} = -2\hat{i} + 5\hat{j}$, $\underline{b} = 14\hat{i} + 3\hat{j}$ and $\underline{c} = 6\hat{i} + 4\hat{j}$ respectively. All units are in kilometres.

- a) Sketch the position of each ship on the graph below. (Show coordinates)



[2 marks]

- b) Find the distance between ship A and ship B.

$$\begin{aligned} \text{DISTANCE} &= \sqrt{16^2 + 2^2} \quad (1) \\ &= \sqrt{260} \\ &= 2\sqrt{65} \text{ km} \quad (1) \end{aligned}$$

[2 marks]

- c) A safety vessel, V, is located at another position. Find $\underline{v}$ if the safety vessel is located in the direction of the vector $\underline{v} = \underline{a} + \underline{b} - \underline{c}$.

$$\begin{aligned} \underline{v} &= (-2\hat{i} + 5\hat{j}) + (14\hat{i} + 3\hat{j}) - (6\hat{i} + 4\hat{j}) \quad (1) \\ &= 6\hat{i} + 4\hat{j} \quad (1) \end{aligned}$$

[2 marks]

d) Show that $\underline{a} + \underline{b}$ is parallel to $\underline{c}$.

$$\underline{a} + \underline{b} = 12\underline{i} + 8\underline{j}$$

$$\underline{c} = 6\underline{i} + 4\underline{j} \quad (1)$$

$$\therefore \underline{a} + \underline{b} \parallel \underline{c} \quad \left(\frac{1}{2}\right)$$

$$\underline{a} + \underline{b} = 2\underline{c} \quad \left(\frac{1}{2}\right)$$

[2 marks]

QUESTION 3

Given the vectors $\underline{a} = 2\underline{i} - 4\underline{j}$ and $\underline{b} = 3\underline{i} - \underline{j}$, find:

a) A unit vector parallel to $\underline{b}$.

$$\hat{\underline{b}} = \frac{1}{\sqrt{10}} (3\underline{i} - \underline{j}) \quad (1)$$

[2 marks]

b) The scalar resolute of $\underline{a}$ in the direction of $\underline{b}$.

$$\begin{aligned} |\underline{a}| &= \underline{a} \cdot \hat{\underline{b}} = (2\underline{i} - 4\underline{j}) \times \frac{1}{\sqrt{10}} (3\underline{i} - \underline{j}) \quad (1) \\ &= \frac{1}{\sqrt{10}} (6 + 4) \\ &= \frac{10}{\sqrt{10}} = \sqrt{10} \quad (1) \end{aligned}$$

[2 marks]

c) The vector resolute of $\underline{a}$ in the direction of $\underline{b}$.

$$\begin{aligned} (\underline{a} \cdot \hat{\underline{b}}) \hat{\underline{b}} &= \sqrt{10} \times \frac{1}{\sqrt{10}} (3\underline{i} - \underline{j}) \quad (1) \\ &= 3\underline{i} - \underline{j} \quad (1) \end{aligned}$$

[2 marks]

d) The vector resolute of $\underline{a}$ perpendicular to $\underline{b}$.

$$\begin{aligned} \underline{a} \perp \underline{b} &= \underline{a} - (\underline{a} \cdot \hat{\underline{b}}) \hat{\underline{b}} \quad (1) \\ &= (2\underline{i} - 4\underline{j}) - (3\underline{i} - \underline{j}) \\ &= -\underline{i} - 3\underline{j} \quad (1) \end{aligned}$$

[2 marks]

e) The angle between the vectors $\underline{a}$ and $\underline{b}$.

$$\underline{a} \cdot \underline{b} = 6 + 4 = 10$$

$$\cos \theta = \frac{10}{\sqrt{20} \times \sqrt{10}}$$

$$\theta = 45^\circ \quad (1)$$

$$|\underline{a}| = \sqrt{2^2 + 4^2} = \sqrt{20} \quad (1/2)$$

$$|\underline{b}| = \sqrt{3^2 + 1^2} = \sqrt{10} \quad (1/2)$$

[2 marks]

QUESTION 4

Let $\underline{a} = 3\underline{i} + \underline{j} + 4\underline{k}$, $\underline{b} = \underline{i} - 3\underline{j} - 2\underline{k}$, $\underline{c} = 2\underline{i} + 4\underline{j} + 6\underline{k}$ and $\underline{d} = 5\underline{i} - 5\underline{j}$.

a) Find the value of the scalar k such that $\underline{a} + 5\underline{b} + \underline{c} = k\underline{d}$.

$$(3\underline{i} + \underline{j} + 4\underline{k}) + 5(\underline{i} - 3\underline{j} - 2\underline{k}) + (2\underline{i} + 4\underline{j} + 6\underline{k}) = k(5\underline{i} - 5\underline{j})$$

$$10\underline{i} - 10\underline{j} = k(5\underline{i} - 5\underline{j}) \quad (1)$$

$$\therefore k = 2 \quad (1)$$

[2 marks]

b) Find the scalars x and y such that $x\underline{a} + y\underline{c} = \underline{d}$.

$$x(3\underline{i} + \underline{j} + 4\underline{k}) + y(2\underline{i} + 4\underline{j} + 6\underline{k}) = 5\underline{i} - 5\underline{j} \quad (1)$$

$$3x + 2y = 5 \quad (1)$$

$$x + 4y = -5 \quad (2)$$

$$4x + 6y = 0 \quad (3)$$

(1)

$$\begin{aligned} & (1) - 3 \times (2) \\ & \begin{cases} 3x + 2y = 5 \\ 3x + 12y = -15 \end{cases} \end{aligned}$$

$$-10y = 20$$

$$y = -2 \quad (1/2)$$

$$\text{If } y = -2,$$

$$x - 8 = -5$$

$$x = 3.$$

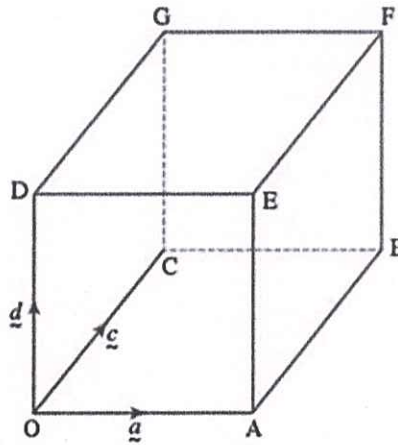
$$(1/2)$$

[3 marks]

$$\underline{x = 3}, \underline{y = -2}$$

QUESTION 5

$OABCDEFG$ is a cuboid. $\vec{OA} = \underline{a}$, $\vec{OC} = \underline{c}$, $\vec{OD} = \underline{d}$



- a) List all the vectors equal to $\vec{OA} = \underline{a}$.

$$\vec{CB} \quad \vec{DE} \quad \vec{GF}$$

(1)

[1 mark]

- b) List all the vectors equal to $\vec{OC} = \underline{c}$.

$$\vec{DG} \quad \vec{EF} \quad \vec{AB}$$

(1)

[1 mark]

- c) List all the vectors equal to $\vec{OD} = \underline{d}$.

$$\vec{AE} \quad \vec{BF} \quad \vec{CG}$$

(1)

[1 mark]

- d) Express each of the following in terms of $\underline{a}$, $\underline{c}$ and $\underline{d}$.

i) $\vec{DF}$

$$= \underline{a} + \underline{c}$$

(1)

[1 mark]

ii) $\vec{EB}$

$$= -\underline{d} + \underline{c}$$

(1)

[1 mark]

iii) $\vec{FO}$

$$= -\underline{c} - \underline{d} - \underline{a}$$

(1)

[1 mark]

iv) $\vec{DB}$

$$= -\underline{d} + \underline{a} + \underline{c}$$

(1)

[1 mark]

QUESTION 6

$\vec{OA} = \underline{a}$, $\vec{OB} = \underline{b}$, $\vec{OP} = \frac{3}{5} \vec{OA}$ and Q is the midpoint of $\vec{AB}$.

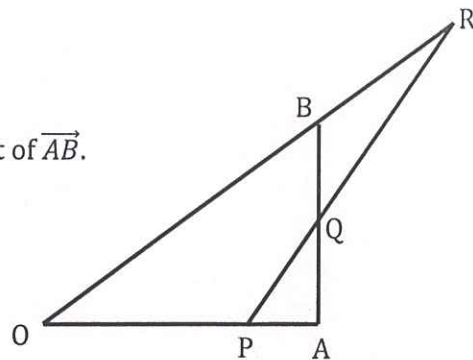


Diagram not to scale

- a) Express $\vec{AB}$ and $\vec{PQ}$ in terms of $\underline{a}$ and $\underline{b}$.

$$\vec{AB} = \underline{b} - \underline{a} \quad (1)$$

$$\begin{aligned} \vec{PQ} &= \frac{2}{5} \underline{a} + \frac{1}{2} (\underline{b} - \underline{a}) \\ &= -\frac{1}{10} \underline{a} + \frac{1}{2} \underline{b} \quad (1) \end{aligned}$$

[2 marks]

- b) The line segments $\vec{PQ}$ and $\vec{OB}$ are extended to meet at R , with $\vec{QR} = n\vec{PQ}$ and $\vec{BR} = k\vec{b}$.

Express the vector $\vec{QR}$ in terms of:

- i) n , $\underline{a}$ and $\underline{b}$

$$\begin{aligned} \vec{QR} &= n \left(-\frac{1}{10} \underline{a} + \frac{1}{2} \underline{b} \right) \quad (1) \\ &= -\frac{n}{10} \underline{a} + \frac{n}{2} \underline{b} \quad (1) \end{aligned}$$

[2 marks]

- i) k , $\underline{a}$ and $\underline{b}$ HINT: Use $\vec{QR} = \vec{QB} + \vec{BR}$

$$\begin{aligned} \vec{OR} &= \vec{OB} + \vec{BR} \\ &= \frac{1}{2} (\underline{b} - \underline{a}) + k \underline{b} \quad (1) \\ &= \frac{1}{2} \underline{b} - \frac{1}{2} \underline{a} + k \underline{b} \\ &= -\frac{1}{2} \underline{a} + \left(k + \frac{1}{2} \right) \underline{b} \quad (1) \end{aligned}$$

[2 marks]

c) By equating your answers to (i) and (ii), find the values of n and k .

$$-\frac{n}{10}a + \frac{n}{2}b = -\frac{1}{2}a + \left(k + \frac{1}{2}\right)b \quad (1)$$

$$-\frac{n}{10} = -\frac{1}{2}$$

$$n = 5$$

$$\frac{1}{2}$$

$$\frac{n}{2} = k + \frac{1}{2}$$

$$\frac{5}{2} = k + \frac{1}{2}$$

$$k = 2 \cdot \frac{1}{2}$$

[2 marks]

END OF ANALYSIS TASK