

Hypergeometric distribution Test

Name: _____

Section A Multiple Choice

- 1 The value of ${}^{10}C_8$ is:
- A 10
 - B 80
 - C 45
 - D 720
 - E 1000
- 2 A bag contains 10 licorice candies and 8 chocolate candies. Two are drawn at random and eaten. The probability that both were chocolate is
- A $\frac{2}{9}$
 - B $\frac{4}{9}$
 - C $\frac{16}{81}$
 - D $\frac{28}{153}$
 - E $\frac{45}{153}$
- 3 From the same bag of candies as in Question 2, the probability that there was one of each type eaten is:
- A $\frac{5}{17}$
 - B $\frac{10}{17}$
 - C $\frac{40}{153}$
 - D $\frac{80}{153}$
 - E $\frac{1}{2}$
- 4 In a classroom there are 5 blondes, 7 brunettes and 3 redheads. A group of three students is chosen at random. The probability that exactly one of them is a redhead is:
- A $\frac{1}{5}$
 - B $\frac{1}{5} \times \frac{2}{14} \times \frac{1}{13}$
 - C $\frac{1}{5} \times \frac{2}{14} \times \frac{1}{13} \times 3$
 - D $\frac{1}{5} \times \frac{12}{14} \times \frac{11}{13} \times 3$
 - E $\frac{1}{5} \times \frac{12}{14} \times \frac{11}{13}$
- 5 In the Hypergeometric formula $\Pr(X = x) = \frac{({}^DC_x)({}^{N-D}C_{n-x})}{{}^NC_n}$ the value of D represents
- A The total number of 'successes' in the population.
 - B The total number of 'successes' in the sample.
 - C The size of the sample.
 - D The size of the population
 - E None of the above.
- 6 In a group of 20 students, 12 study Mathematical Methods. Six students are chosen at random from the group. The probability that exactly three of them study Mathematical Methods is:
- A $\frac{({}^8C_3)({}^{12}C_3)}{{}^{20}C_6}$
 - B $\frac{({}^6C_3)({}^6C_3)}{{}^{20}C_6}$
 - C $\frac{({}^{14}C_3)({}^{20}C_3)}{{}^{20}C_6}$
 - D $\frac{({}^6C_0)({}^{14}C_6)}{{}^{20}C_6}$
 - E $\frac{({}^6C_3)({}^{14}C_3)}{{}^{20}C_3}$

- 7 Using the Hypergeometric Distribution formula with $N = 12$, $n = 4$, $D = 5$ and $x = 3$ yields a probability of:
- A 0.04545
 - B 0.1414
 - C 0.3535
 - D 0.4242
 - E 0.5555

Section B Short/Extended answer

- 1 In a large school there are 111 students studying in Year 12. The principal wishes to select 2 students at random to represent the school at a local sporting carnival. If 49 of the students are boys, what is the probability that the principal selects 1 boy and 1 girl?

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- 2 From same class in the previous question, the vice-principal wishes to choose 3 students at random for yard duty. Find the probability that all of them are boys.

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- 3 A custom car manufacturer makes 24 cars each day. Unfortunately, 6 of them were defective during yesterday's run. By taking a random sample of 6, what is the probability that exactly one of the defective cars will be in the sample?

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- 4 Find the probability of being dealt 5 hearts out of 7 cards from a standard 52 card deck.

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- 5 In a lottery there are 32 numbered balls. Six are drawn at random. Your ticket has 4 numbers on it and you win if you have 4 matches. Determine your probability of winning.

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- 6 In a group of students working in the library, there are 3 left handers and 7 right handers. Find the probability that if a random group of 4 students is selected, that 2 are left-handed.

- 7 On a bookshelf there are 7 hardcover and 5 softcover books. Four books are drawn at random. Find the probability distribution for the number of softcover books drawn.

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- 8 From the results of the previous question, find:
- a) the probability that at least one of the books was a softcover.
 - b) the probability that between 1 and 3 (inclusive) of the books were softcover.
 - c) the probability that all of the books were softcover.

3

- 9 A manufacturer of precision tools makes an equal number of calipers and dividers. From a combined batch of 20, a sample of 3 is drawn.
- Find the probability distribution for the number of calipers.
 - Find the probability that at least 1 was a caliper.

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- 10 A manufacturer of integrated circuits knows that 10% of them are defective. From the day's batch of 50, she decides to take a sample of 10 and if more than 1 is defective, then the entire batch is destroyed. Find the probability that this occurs.

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- 11 In the game of poker, getting a number of the same card is important. For example, one can get 2 aces (a pair), 3 aces (3-of-a-kind) or 4 aces(4-of-a-kind).

a) Find the probability distribution for the number of aces dealt in a 5 card poker hand.

Answers:

MC:

1) C 2) D 3) D 4) D 5) A 6) A 7) B

SA:

1) Pr(boy-girl) =

$$\frac{49}{111} \times \frac{62}{110} + \frac{62}{111} \times \frac{49}{110} = 0.4976$$

$$2) \Pr(3 \text{ boys}) = \frac{49}{111} \times \frac{48}{110} \times \frac{47}{109} = 0.0831$$

3) Apply the Hypergeometric formula

$$\Pr(X = x) = \frac{({}^D C_x) ({}^{N-D} C_{n-x})}{{}^N C_n}$$

with $N = 24$

$D = 6$

$n = 6$

$x = 1$

$$\Pr(1 \text{ defective car}) = \frac{({}^6 C_1) ({}^{18} C_5)}{{}^{24} C_6} = \frac{6 \times 8568}{134596}$$

$$= 0.3819$$

4) Apply the Hypergeometric formula

$$\Pr(X = x) = \frac{({}^D C_x) ({}^{N-D} C_{n-x})}{{}^N C_n}$$

with $N = 52$ (the number of cards in the deck)

$D = 13$ (the number of hearts in the deck)

$n = 7$ (the number of cards dealt)

$x = 5$ (the number of hearts dealt)

$$\begin{aligned} \Pr(5 \text{ Hearts}) &= \frac{({}^{13} C_5) ({}^{39} C_2)}{{}^{52} C_7} \\ &= \frac{1287(741)}{133\,784\,560} = 0.00713 \end{aligned}$$

5) Apply the Hypergeometric formula

$$\Pr(X = x) = \frac{({}^D C_x) ({}^{N-D} C_{n-x})}{{}^N C_n}$$

with $N = 32$ (the number of balls in the draw)

$D = 6$ (the number of winning balls)

$n = 4$ (the number of numbers on your ticket)

$x = 4$ (the number of matches)

$$\Pr(4 \text{ matches}) = \frac{({}^6 C_4) ({}^{26} C_0)}{{}^{32} C_4}$$

$$= \frac{(15)(1)}{35\,960} = 0.000\,417$$

6) Apply the Hypergeometric formula

$$\Pr(X = x) = \frac{({}^D C_x) ({}^{N-D} C_{n-x})}{{}^N C_n}$$

with $N = 10$ (the number of students)

$D = 3$ (the number of left handers)

$n = 4$ (the number in the group)

$x = 2$ (the number of left handers in the group)

$$\Pr(2 \text{ left handers}) = \frac{({}^3 C_2) ({}^7 C_2)}{{}^{10} C_4}$$

$$= \frac{(3)(21)}{210} = 0.3$$

7) Define X , N , D and n .

Let X = the number of soft cover books

$N = 12$ (the total number of books on the shelf)

$D = 5$ (the number of softcover books on the shelf)

$n = 4$ (the number of books in the sample)

For a probability distribution, $x = 0, 1, 2, 3, 4$

Apply the Hypergeometric formula

$$\Pr(X = x) = \frac{{}^D C_x ({}^{N-D} C_{n-x})}{{}^N C_n}$$

$$\Pr(X = 0) = \frac{{}^5 C_0 ({}^7 C_4)}{{}^{12} C_4} = \frac{(1)(35)}{495} = 0.0707$$

$$\Pr(X = 1) = \frac{{}^5 C_1 ({}^7 C_3)}{{}^{12} C_4} = \frac{(5)(35)}{495} = 0.3535$$

$$\Pr(X = 2) = \frac{{}^5 C_2 ({}^7 C_2)}{{}^{12} C_4} = \frac{(10)(21)}{495} = 0.4242$$

$$\Pr(X = 3) = \frac{{}^5 C_3 ({}^7 C_1)}{{}^{12} C_4} = \frac{(10)(7)}{495} = 0.1414$$

$$\Pr(X = 4) = \frac{{}^5 C_4 ({}^7 C_0)}{{}^{12} C_4} = \frac{(5)(1)}{495} = 0.0101$$

$$\begin{aligned} 8) \text{ a) } \Pr(\text{at least } 1) &= 1 - \Pr(0) \\ &= 1 - 0.0707 = 0.9293 \end{aligned}$$

$$\begin{aligned} \text{b) } \Pr(\text{between } 1 \text{ and } 3) &= \Pr(1) + \Pr(2) + \Pr(3) \\ &= 0.3535 + 0.4242 + 0.1414 = 0.9191 \end{aligned}$$

$$\text{c) } \Pr(\text{all}) = \Pr(4) = 0.0101$$

9) a) Define X , N , D and n .

Let X = the number of calipers.

$N = 20$ (the total number of tools)

$D = 10$ (the number of calipers)

$n = 3$ (the number in the sample)

For a probability distribution, $x = 0, 1, 2, 3$

Apply the Hypergeometric formula

$$\Pr(X = x) = \frac{{}^D C_x ({}^{N-D} C_{n-x})}{{}^N C_n}$$

$$\Pr(X = 0) = \frac{{}^{10} C_0 ({}^{10} C_3)}{{}^{20} C_3} = \frac{(1)(120)}{1140} = 0.1053$$

$$\Pr(X = 1) = \frac{{}^{10} C_1 ({}^{10} C_2)}{{}^{20} C_3} = \frac{(10)(45)}{1140} = 0.3947$$

$$\Pr(X = 2) = \frac{{}^{10} C_2 ({}^{10} C_1)}{{}^{20} C_3} = \frac{(45)(10)}{1140} = 0.3947$$

$$\Pr(X = 3) = \frac{{}^{10} C_3 ({}^{10} C_0)}{{}^{20} C_3} = \frac{(120)(1)}{1140} = 0.1053$$

$$\begin{aligned} \text{b) } \Pr(\text{at least } 1) &= 1 - \Pr(0) = 1 - 0.1053 \\ &= 0.8947 \end{aligned}$$

10) Define X , N , D and n .

Let X = the number of defective circuits.

$N = 50$ (the total number of circuits)

$D = 5$ (= 10% of 50, the number of defectives)

$n = 10$ (the number in the sample)

$$\Pr(\text{more than } 1) = 1 - \Pr(0) - \Pr(1)$$

Apply the Hypergeometric formula

$$\Pr(X = x) = \frac{{}^D C_x ({}^{N-D} C_{n-x})}{{}^N C_n}$$

$$\Pr(X = 0) = \frac{{}^5 C_0 ({}^{45} C_{10})}{{}^{50} C_{10}} = 0.3106$$

$$\Pr(X = 1) = \frac{{}^5 C_1 ({}^{45} C_9)}{{}^{50} C_{10}} = 0.4313$$

$$\begin{aligned} \Pr(\text{batch destroyed}) &= 1 - 0.3106 - 0.4313 \\ &= 0.2581 \end{aligned}$$

11) a) Define X , N , D and n .

Let X = the number of aces.

$N = 52$ (the number of cards in the deck)

$D = 4$ (the number of aces)

$n = 5$ (the number in the sample)

For a probability distribution, $x = 0, 1, 2, 3, 4$

Apply the Hypergeometric formula

$$\Pr(X = x) = \frac{{}^D C_x ({}^{N-D} C_{n-x})}{{}^N C_n}$$

$$\begin{aligned} \Pr(X = 0) &= \frac{{}^4 C_0 ({}^{48} C_5)}{{}^{52} C_5} = \frac{(1)(1\,712\,304)}{2\,598\,960} \\ &= 0.6588 \end{aligned}$$

$$\begin{aligned} \Pr(X = 1) &= \frac{{}^4 C_1 ({}^{48} C_4)}{{}^{52} C_5} = \frac{(4)(194\,580)}{2\,598\,960} \\ &= 0.2995 \end{aligned}$$

$$\begin{aligned} \Pr(X = 2) &= \frac{{}^4 C_2 ({}^{48} C_3)}{{}^{52} C_5} = \frac{(6)(17\,296)}{2\,598\,960} \\ &= 0.03993 \end{aligned}$$

$$\begin{aligned} \Pr(X = 3) &= \frac{{}^4 C_3 ({}^{48} C_2)}{{}^{52} C_5} = \frac{(4)(1\,128)}{2\,598\,960} \\ &= 0.00174 \end{aligned}$$

$$\begin{aligned} \Pr(X = 4) &= \frac{{}^4 C_4 ({}^{48} C_1)}{{}^{52} C_5} = \frac{(1)(48)}{2\,598\,960} \\ &= 0.000\,018\,468\,9 \end{aligned}$$

(about 2 times in a hundred thousand!)