

Binomial distribution Test

Name: _____

Section A Multiple Choice

- 1 A fair coin is tossed 7 times. The probability of getting exactly 3 heads is:
- A ${}^7C_4\left(\frac{3}{7}\right)^3\left(\frac{4}{7}\right)^4$
- B $\left(\frac{1}{2}\right)^7$
- C $\left(\frac{3}{7}\right)^3$
- D $\left(\frac{1}{2}\right)^3$
- E ${}^7C_3\left(\frac{1}{2}\right)^7$
- 2 The value of the probability in the previous question is:
- A 0.2938
- B 0.0787
- C 0.2734
- D 0.125
- E 0.007 81
- 3 A burglar alarm has a probability of failure of 0.02. In a factory where there are 6 such alarms, the probability that more than 1 fails is given by:
- A $1 - {}^6C_1(0.02)^1(0.98)^5$
- B $1 - {}^6C_5(0.02)^5(0.98)^1$
- C $1 - (0.98)^6$
- D $1 - [(0.98)^6 + (0.02)^1(0.98)^5]$
- E $1 - [(0.98)^6 + {}^6C_1(0.02)^1(0.98)^5]$
- 4 The value of $(0.9)^5 + {}^5C_1(0.1)^1(0.9)^4$ is:
- A 0.320 85
- B 0.918 54
- C 0.590 81
- D 0.590 49
- E 0.000 01
- 5 Consider a binomial experiment with the number of trials, $n = 10$ and the probability of success, p , is 0.45. The mean and standard deviation (SD) are given by:
- A mean = 4.5, SD = 2.48
- B mean = 4.5, SD = 1.57
- C mean = 2.48, SD = 1.57
- D mean = 2.48, SD = 2.35
- E mean = 2.48, SD = 4.5
- 6 Of all the cars registered in Victoria, 32% use LPG. The probability that 9 or more of the next 10 cars use LPG is closest to:
- A 0.0211
- B 0.1206
- C 0.0004
- D 0.0003
- E 0.0002
- 7 Consider a binomial distribution with $n = 5$. The probability of exactly 4 successes is given by:
- A ${}^5C_4(p)^4(q)^1$
- B $80q^5$
- C $40q^5$
- D $10p^5$
- E $\frac{5}{8}p^5$

Section B Short/Extended answer

1 A single die is tossed 5 times. Find the probability of getting exactly three 6's. **3**

2 Determine the probability distribution for the number of 6's in the experiment from the previous question. **6**

3 A new drug which cures constipation is successful in 95% of cases. Find the probability that in a group of 20 people:
a) All are cured.
b) None are cured.
c) All except 1 are cured. **4**

- 4 A lottery claims that 10% of tickets win a prize. What is the probability that you win at least one prize if you purchase:
a) 10 tickets
b) 20 tickets
- 5

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- 5 Consider a Binomial Distribution with $n = 4$ and $p = 0.4$.
a) Determine the probability distribution
b) Determine the expected value of the distribution and show that it is equal to np .
- 4

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- 6 A coin is biased so that heads comes up 70% of the time. The coin is tossed 6 times. Find the probability of getting exactly 4 heads.
- 5

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- 7 In manufacturing light bulbs it is known that 2.5% are defective. Bulbs are packaged in cases of 40. Find the probability that in a case;
a) there are no defectives
b) there is more than 1 defective.
- 5

- 8 A first year examination in Medicine is designed so that 75% pass. Find the probability that in a group of 10 students:
- a) exactly 9 pass
 - b) at least 9 pass
 - c) no more than 9 pass
 - d) less than 10 pass

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- 9 Find the mean and standard deviation of the binomial distribution in the previous question.

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- 10 Consider a binomial distribution with $n = 10$, $p = 0.5$, find:
- a) $\Pr(1 \leq X \leq 9)$
 - b) $\Pr(2 \leq X \leq 8)$

5

- 11 A company has very important data to save on its computer system. It decides to backup by storing the data on 3 separate sets of floppy disks. Each set has a probability of failure of 0.05 in the first year. Find the probability that:
- all disks are still working.
 - all disks fail.
 - at least 1 disk is still working.

Answers:

MC:

- 1) E 2) C 3) E 4) B 5) B 6) D 7) A

SA:

1) Let X = the number of sixes.

We have a binomial experiment with $n = 5$,

$p = \frac{1}{6}$, and $x = 3$.

$$\begin{aligned}\Pr(X = 3) &= {}^5C_3 \left(\frac{1}{6}\right)^3 \left(\frac{5}{6}\right)^2 \\ &= (10) \left(\frac{5^2}{6^5}\right) = 0.0322\end{aligned}$$

2) We need to determine all the probabilities, namely, $\Pr(X = 0)$, $\Pr(X = 1)$, ..., $\Pr(X = 5)$

$$\Pr(X = 0) = {}^5C_0 \left(\frac{1}{6}\right)^0 \left(\frac{5}{6}\right)^5 = 0.4019$$

$$\Pr(X = 1) = {}^5C_1 \left(\frac{1}{6}\right)^1 \left(\frac{5}{6}\right)^4 = 0.4019$$

$$\Pr(X = 2) = {}^5C_2 \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^3 = 0.1608$$

$$\Pr(X = 3) = {}^5C_3 \left(\frac{1}{6}\right)^3 \left(\frac{5}{6}\right)^2 = 0.0322$$

$$\Pr(X = 4) = {}^5C_4 \left(\frac{1}{6}\right)^4 \left(\frac{5}{6}\right)^1 = 0.0032$$

$$\Pr(X = 5) = {}^5C_5 \left(\frac{1}{6}\right)^5 \left(\frac{5}{6}\right)^0 = 0.0001$$

3) a) This is a binomial experiment with $n = 20$ and $p = 0.95$.

$$\begin{aligned}\Pr(X = 20) &= {}^{20}C_{20} (0.95)^{20} (0.05)^0 \\ &= (0.95)^{20} = 0.3585\end{aligned}$$

$$\begin{aligned}\text{b) } \Pr(X = 0) &= {}^{20}C_0 (0.95)^0 (0.05)^{20} \\ &= (0.05)^{20} = 9.5 \times 10^{-27}\end{aligned}$$

which is so small it can be considered as 0.

$$\begin{aligned}\text{c) } \Pr(X = 19) &= {}^{20}C_{19} (0.95)^{19} (0.05)^1 \\ &= 20(0.95)^{19} (0.05)^1 = 0.3774\end{aligned}$$

4) a) $\Pr(\text{winning a prize}) = 1 - \Pr(\text{not winning any prizes})$

$$\begin{aligned}&= 1 - {}^{10}C_0 (0.1)^0 (0.9)^{10} \\ &= 1 - (0.9)^{10} \\ &= 1 - 0.3487 = 0.6513\end{aligned}$$

$$\begin{aligned}\text{b) } \Pr(\text{winning}) &= 1 - {}^{20}C_0 (0.1)^0 (0.9)^{20} \\ &= 1 - 0.1216 = 0.8784\end{aligned}$$

5) a) We need to determine all the probabilities, namely, $\Pr(X = 0)$, $\Pr(X = 1)$, ..., $\Pr(X = 4)$

$$\Pr(X = 0) = {}^4C_0 (0.4)^0 (0.6)^4 = 0.1296$$

$$\Pr(X = 1) = {}^4C_1 (0.4)^1 (0.6)^3 = 0.3456$$

$$\Pr(X = 2) = {}^4C_2 (0.4)^2 (0.6)^2 = 0.3456$$

$$\Pr(X = 3) = {}^4C_3 (0.4)^3 (0.6)^1 = 0.1536$$

$$\Pr(X = 4) = {}^4C_4 (0.4)^4 (0.6)^0 = 0.0256$$

$$\begin{aligned}\text{b) } E(X) &= 0 \times \Pr(X=0) + 1 \times \Pr(X = 1) + \\ &\quad 2 \times \Pr(X = 2) + 3 \times \Pr(X = 3) + 4 \times \Pr(X = 4) \\ &= 0 + 0.3456 + 2(0.3456) + 3(0.1536) + \\ &\quad 4(0.0256) \\ &= 1.6\end{aligned}$$

$$\text{Note that } np = 4(0.4) = 1.6$$

6) $n = 6$, $p = 0.7$

$$\begin{aligned}\Pr(4 \text{ Heads}) &= {}^6C_4 (0.7)^4 (0.3)^2 \\ &= 15(0.7)^4 (0.3)^2 \\ &= 0.3241\end{aligned}$$

7) a) Let X = number of defective bulbs.
This is a binomial experiment with $n = 40$ and $p = 0.025$.

$$\begin{aligned}\Pr(X = 0) &= {}^{40}C_0 (0.025)^0 (0.975)^{40} \\ &= (0.975)^{40} = 0.3632\end{aligned}$$

b) $\Pr(\text{more than 1}) = 1 - [\Pr(0) + \Pr(1)]$

$$\begin{aligned}\Pr(X = 1) &= {}^{40}C_1 (0.025)^1 (0.975)^{39} \\ &= 40(0.025)^1 (0.975)^{39} \\ &= 0.3725\end{aligned}$$

Therefore

$$\begin{aligned}\Pr(X > 1) &= 1 - [0.3632 + 0.3725] = \\ &0.2643\end{aligned}$$

8) a) Let X = the number of students that pass.
This is a binomial experiment with $n = 10$ and $p = 0.75$.

$$\begin{aligned}\Pr(X = 9) &= {}^{10}C_9 (0.75)^9 (0.25)^1 \\ &= 10(0.75)^9 (0.25)^1 \\ &= 0.1877\end{aligned}$$

b) $\Pr(\text{at least 9 pass}) = \Pr(X = 9) + \Pr(X = 10)$

$$\begin{aligned}\Pr(X = 10) &= {}^{10}C_{10} (0.75)^{10} (0.25)^0 \\ &= 0.0563\end{aligned}$$

$$\Pr(\text{at least 9 pass}) = 0.1877 + 0.0563 = 0.244$$

c) $\Pr(\text{no more than 9})$

$$\begin{aligned}&= \Pr(X = 0) + \Pr(X = 1) + \dots + \Pr(X = 9) \\ &= 1 - [\Pr(X = 10)] \\ &= 1 - 0.0563 \\ &= 0.9437\end{aligned}$$

d) $\Pr(\text{less than 10}) =$

$$\begin{aligned}&= \Pr(X = 0) + \Pr(X = 1) + \dots + \Pr(X = 9) \\ &= 1 - [\Pr(X = 10)] \\ &= 1 - 0.0563 \\ &= 0.9437\end{aligned}$$

9) The mean of a binomial distribution is given by np , the SD by $\sqrt{npq}$.

$$\text{mean} = np = 10(0.75) = 7.5$$

$$\text{standard deviation} = \sqrt{npq}$$

$$= \sqrt{10(0.75)(0.25)} = 1.369$$

10) a) Determine the probability distribution for X , taking advantage of symmetry ($p = q$)

$$\Pr(X = 0) = {}^{10}C_0 (0.5)^{10} = 0.000\,976\,6$$

$$\Pr(X = 1) = {}^{10}C_1 (0.5)^{10} = 0.009\,766$$

$$\Pr(X = 2) = {}^{10}C_2 (0.5)^{10} = 0.043\,95$$

$$\Pr(X = 10) = \Pr(X = 0) = 0.000\,976\,6$$

$$\Pr(X = 9) = \Pr(X = 1) = 0.009\,766$$

$$\Pr(X = 8) = \Pr(X = 2) = 0.043\,95$$

$$\begin{aligned}\Pr(1 \leq X \leq 9) &= \Pr(X = 1) + \dots + \Pr(X = 9) \\ &= 1 - [\Pr(X = 0) + \Pr(X = 10)] \\ &= 1 - [0.000\,976\,6 + 0.000\,976\,6] \\ &= 0.998\end{aligned}$$

$$\begin{aligned}\text{b) } \Pr(2 \leq X \leq 8) &= \Pr(X = 2) + \dots + \Pr(X = 8) \\ &= 1 - [\Pr(X = 0) + \Pr(X = 1) + \Pr(X = 9) + \Pr(X = 10)] \\ &= 1 - 2[0.000\,976\,6 + 0.009\,766] \\ &= 0.9785\end{aligned}$$

11) a) Let X = the number of disks that fail.
This is a binomial distribution with $n = 3$ and $p = 0.05$.

$$\begin{aligned}\Pr(\text{all disks working}) &= \Pr(X = 0) \\ &= {}^3C_0 (0.05)^0 (0.95)^3 = 0.8574\end{aligned}$$

b) $\Pr(\text{all disks fail}) = \Pr(X = 3)$

$$= {}^3C_3 (0.05)^3 (0.95)^0 = 0.000125$$

$$\begin{aligned}\text{c) } \Pr(\text{at least 1 disk working}) &= 1 - \Pr(X = 3) \\ &= 1 - 0.000\,125 = 0.999\,875\end{aligned}$$