

Arithmetic and Geometric Series Questions

Exercise A

1. An object, falling freely from a height, travels 5 meters in the first second, 15 meters in the second second, and 25 meters in the third second. How far has it fallen:
 - a. after 6 seconds?
 - b. during the sixth second?
2. The rungs of a ladder decreases uniformly in length, from 40 cm at the bottom rung to 30 cm at the top rung. How many rungs are there if their total length is 5.25m?
3. The track of a gramophone record is in the shape of a spiral curve and may be considered as a number of concentric circles of inner radius 5.25 cm and outer radius 10.5 cm. The record rotates at $33\frac{1}{3}$ revolutions per minute and takes 18 minutes to play. Find the length of the track. Use $\pi = \frac{22}{7}$
4. Logs of wood are stacked in a pile so that there are 15 logs on the top row, 16 on the next, 17 on the next, and so on. If there are 246 logs altogether:
 - a. how many rows are there?
 - b. how many logs are on the bottom row?
5. The lengths of the rungs of a ladder increases uniformly, from 40 cm for the top rungs to 75 cm for the bottom rung. If 13.8 m of wood are used to make the rungs, how many rungs are there?
6. The lengths of the sides of a right-angle triangle form the terms of an arithmetic sequence. If the hypotenuse is 15 cm in length, what are the lengths of the other two sides?
7. The angles of a hexagon form the terms of an arithmetic sequence. IF the smallest angle is 95° , find the size of each of each of the other angles.
8. An angler caught 50 fish in five days. Each day the angler caught three more than on the previous day. How many were caught on each of the five days?
9.
 - a. In order to purchase a car, Daniel decided to put aside a certain amount of money each week for 100 weeks; \$1 the first week, \$2 the second week, \$3 the third week and so on. How much did he put aside?
 - b. Tony decided to put aside \$1 the first week, \$3 the second week, \$5 the third week and so on. How many weeks would Tony need before he has put aside the same amount as Daniel?
 - c. How much would Tony put aside in the last week?
10. A clock strikes the particular hour-every hour and it strikes once every half hour. How many strikes does it make in a day (24 hours)?
11. Logs of wood are stacked in a pile so that there are 10 on the top row, 11 on the next row, 12 on the next, and so on. If there are 25 on the bottom row:
 - a. how many rows are there,
 - b. how many logs are there all together?
12. An employer has just completed her twenty-fourth year of service at a particular company. She started at an annual salary of \$5500 and each year her salary increased by \$500. Find the total amount she earned in her 24 years with the company.
13. A builder wishes to tile a section of a roof in the shape of an isosceles triangle. In order to fix the tiles it is necessary to lay battens across the roof parallel to one another. If the shortest batten is 1 metre long, the longest batten 5 metres long and 16 battens are required, find:
 - a. the amount by which adjacent batten differ in length
 - b. the total length of the batten required.
14. The total cost of 100 articles is \$10 400. Find the cost of the cheapest and the dearest articles if the prices of consecutive articles differ by \$2.
15. An object is projected vertically upwards from the ground and rises 95 metres in the first second and 10 metres less each second thereafter. Find;
 - a. the highest-point reached above the ground
 - b. the time taken to reach this height

16. A circular dart-board consists of five concentric circles whose radii are $r, 2r, 3r, 4r, 5r$. Show that the circles divided the board into regions whose areas form the terms of an arithmetic sequence and find this sequence.
17. The track of a gramophone record is a shape of a spiral curve and maybe considered as a number of concentric circles of inner radius 7 cm and outer radius 14 cm. The record rotates at $33\frac{1}{3}$ revolutions per minute and takes 12 minutes to play. Find the length of the track. Use $\pi = \frac{22}{7}$.
18. A frame in the shape of an equilateral triangle contains a maximum of 210 spheres of equal radius. They are arranged in rows and the numbers in the rows form the terms of an arithmetic sequence with first term 1 and common difference 1. How many rows are there?



Answers Exercise A

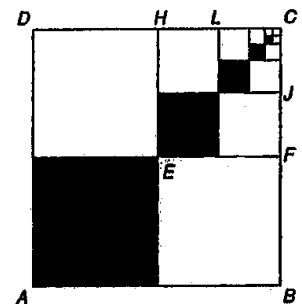
- 1a. 180m b. 55m 2. 15 3. 297 m 4a. 12 b. 26 5. 24 6. 9cm, 12 cm 7. 95°, 105°, 115°, 125°, 135°, 145°
 8. 4, 7, 10, 13, 16 9a. 5050 b. 72 c. \$9 10. 180 11a. 16 b. 280 12. \$270 000
 13a. $\frac{15}{4}$ m b. 48m 14a. \$5 b. \$203 15a. 500m b. 105
 16a. $a = \pi r^2$, $d = 2\pi r^2$ 17. 264m 18. 20

Exercise B

- The value of a car, when new, is \$10 000. If it depreciates at the rate of 15 per cent of its value at the beginning of each year, find its value after 6 years.
- A pump removes one-half of the water in a tank in 10 minutes. What fraction of the original amount of water will be left in the tank after one hour?
- A pump removes one-quarter of the water from a tank every 15 minutes. IF the tank initially holds 256 000 litres, how much water will remain in the tank after one hour?
- Find the fractional equivalent of:
 - $2.3\dot{8}$
 - $4.\dot{6}\dot{2}$
 - $0.417171717\ldots$
- $0.323232 \ldots = \frac{p}{q}$, where p and q are integers. Find the value of p and q .
- Show that $1.2888\ldots$ is a rational number by expressing it in the form $\frac{m}{n}$, where m and n are integers.
- In 1984, 500 students entered for a particular examination. This number increased each year by 20 per cent of the number who entered the previous year. Calculate:
 - the number who entered in 1989
 - the total number who entered between 1984 and 1989 inclusive.
- An employee has been working with a company for 10 years, starting with an annual salary of \$10 000 which increased by 10 per cent each year. Find the total amount the employee earned with the firm.
 - Another employee started with the same salary and each year her increase in salary was 15% of her original salary. Find the total amount she earned in the 10 years.
- A car worth \$10 000 new is worth \$8000 when it is one year old, and \$6400 when it is two year old. Calculate:
 - the percentage rate of depreciation each year
 - the car's value after 5 years, if it continues to depreciate at the same rate.

10. The value of a car when new is \$15 000. If it depreciates at the rate of 15% of its value at the beginning of each year, find:
 - a. its value after 6 years
 - b. the number of years it will take for its value to drop below \$2500
11. A hiker sets out to walk a distance of 100 km by walking 20 km on the first day and, each day thereafter, 75% of the distance walked on the previous day. Will the hiker reach the destination?
12. If the amount of wheat harvested in a certain area in 1982 is 4.2 thousand tonnes, and it is anticipated that this will increase annually by 2.5 per cent, estimate:
 - a. the amount harvested in 1990
 - b. the total amount to be harvested over the nine years from 1982 to 1990 inclusive.
13. At the beginning of 1981, a mining town had a population of 15 000. It was estimated that this would increase each year by 8 per cent of the population at the beginning of that year. What would be the population at the beginning of 1985?
14. A ball is dropped from a height of 20 metres and rebounds to a height of 18 metres. If each time it rebounds it rises to nine-tenths of its previous height, calculate the total distance it could travel.
15. Express $0.233333\dots$ in the form $\frac{m}{n}$, where m and n are integers.
16. a. A clerk was employed at an initial salary of \$12 000 per annum and, after each year of service, received an increase of \$20 per week. What was the clerk's salary in the tenth year of service, and the total earning for the first ten years?
 b. Another clerk had the same initial salary of \$12 000 and after each year of service this salary increased by 10% of the salary of the preceding year. Calculate the salary in the tenth year and the total earnings for the first ten years.

17. In the diagram below, one quarter of the square ABCD is shaded, then one-quarter of the square EFCH, followed by one-quarter of the square IJCL, and so on. Assume that this process is continued an infinite number of times.

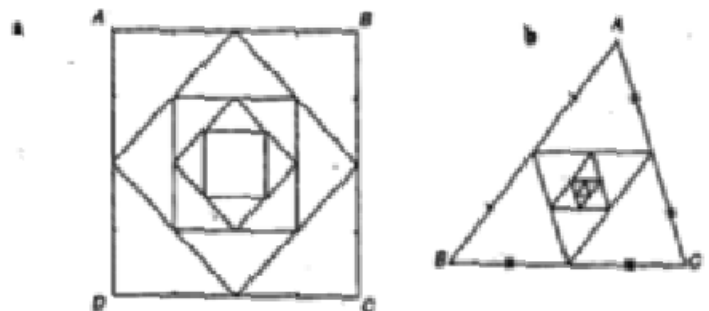


- a. If the area of ABCD is 1 unit, write a series of fractions representing the shaded regions.
 - b. If an infinite number of shading is done, what fraction of the total square ABCD will be shaded?
18. A person with \$1000 on January decided to spend 10% of her monthly bank balance each month. How much does she have 12 months later?

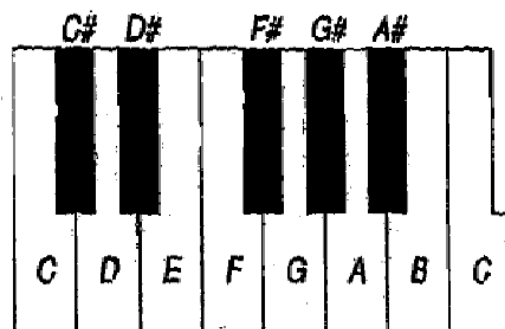
19. In diagram 'a' at the top of the facing page, four squares have been inscribed in the original square ABCD by joining the midpoint each time. In diagram 'b', four triangles have been inscribed in the original triangle ABC by joining the midpoint each time.

For each diagram, write the ratio of:

- a. the area
- b. the perimeter of the inscribed squares or triangles to the original square or triangle.



20. The equal temperament scale is used in all modern music. An octave contains 12 notes and is divided into 12 intervals, each of which is called an equal temperament semitone. Taking A to be the key note, having a frequency of 440 cycles per second (c.p.s), the frequencies of the other notes in the scale form the terms of a geometric sequence with a common ratio of the twelfth root of 2.



- Write the frequencies of the 12 notes of the octave.
 - Show that the frequency of upper C is twice that of middle C.
 - Show that the frequency of each note is approximately 6% higher than the frequency of the note below it.
21. Two companies expand their production of cement as follows. The company A, the number of tonnes made each month increases by 80 from one month to the next one. For company B, production increases by 4 per cent per month, so that the number of tonnes made each month is 4 per cent greater than the number made in the immediately preceding month.
- If each of the companies produces 1000 tonnes in January, find the total number of tonnes produced during the year January – December for each company.
 - Calculate the constant monthly increase by company A that would suffice for its total production of company B.
 - Suppose that company B, working on a 4 per cent monthly growth rate for the months January to June, managed to increase its growth rate to 8 per cent per month for the months July to December. Given the same base production figure of 1000 in January, find the amount that the company now produces for the year.

Answers Exercise B

1. \$3771.50 2. $\frac{64}{1}$

3. 81000 L

4. a. $2\frac{18}{7}$

b. $4\frac{99}{62}$

c. $\frac{413}{990}$

5. $p=32, q=99$

6. $\frac{45}{58}$ 7. a 1245

10. a \$5657.24 b. 12 years

13. 20 407

14. 380m 15. $\frac{30}{7}$

16a. \$21 360

b. \$28 295 \$ 191 249

17a. $\frac{1}{1}, \frac{4}{16}, \frac{1}{1}, \frac{64}{64}, \frac{1}{1}$

b. $\frac{3}{1}$

18. \$282.43

19a. (i) $\frac{1}{1}, \frac{1}{1}, \frac{1}{1}, \frac{1}{1}, \frac{1}{1}, \frac{1}{1}, \frac{1}{1}, \frac{1}{1}, \frac{1}{1}, \frac{1}{1}, \frac{1}{1}, \frac{1}{1}$

(ii) $\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$

b. (i) $\frac{1}{1}, \frac{4}{16}, \frac{1}{1}, \frac{64}{64}, \frac{1}{1}$ (ii) $\frac{1}{1}, \frac{1}{1}, \frac{1}{1}, \frac{1}{1}, \frac{1}{1}, \frac{1}{1}, \frac{1}{1}, \frac{1}{1}, \frac{1}{1}, \frac{1}{1}, \frac{1}{1}, \frac{1}{1}$

21a. A: 17280, B: 15026

b. 45.85

c. 16272