



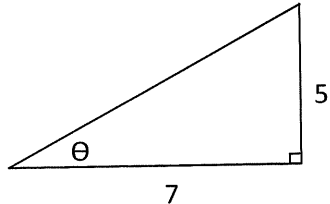
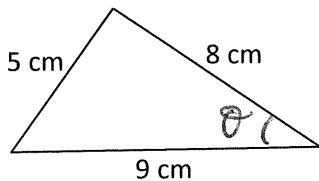
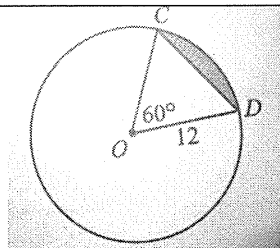
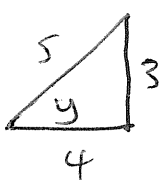
2018 Yr 11 Specialist Mathematics
Sequences, Series & Trigonometry Technology Access Test
 (1 double sided A4 Summary Sheet & 45 min writing time)

Name: SOLUTIONS

Total Marks: / 45

Section A. Multiple Choice Questions. Clearly circle your answer (10 x 2 marks = 20 marks total)

1. In an arithmetic sequence the fourth term is 26 and the eighth term is 2. The first term and common difference respectively are: A 32, -8 B 32, -6 C 50, -8 D 44, -8 ☒ E 44, -6	Working Space $\begin{array}{rcl} a + 3d & = & 26 \\ a + 7d & = & 2 \\ \hline 4d & = & -24 \quad d = -6 \\ \hline a - 18 & = & 26 \\ a & = & 44 \end{array}$
2. A sequence is defined by: $t_n = t_{n-1} - t_{n-2}$, $t_0 = 1$, $t_1 = 2$ t_4 is: ☒ A -2 B 2 C -1 D 1 E 0	$\begin{aligned} t_2 &= 2 - 1 = 1 \\ t_3 &= 1 - 2 = -1 \\ t_4 &= -1 - 1 = -2 \end{aligned}$
3. A geometric sequence is 3, $3\sqrt{3}$, 9, Find n such that $S_n = 363(\sqrt{3} + 1)$ A n = 6 B n = 7 C n = 8 D n = 9 ☒ E n = 10	$\begin{aligned} a &= 3 \quad r = \sqrt{3} \\ S_n &= \frac{3((\sqrt{3})^n - 1)}{\sqrt{3} - 1} \\ 363(\sqrt{3} + 1)(\sqrt{3} - 1) &= 3(3^{n/2} - 1) \\ 726 &= 3(3^{n/2} - 1) \\ 242 &= 3^{n/2} - 1 \\ n &= 10 \end{aligned}$
4. An infinite set of triangles have areas given by the geometric sequence: 1, $2/3$, $4/9$, The total area of all the triangles is: A 6 ☒ B 3 C 2.5 D 2 E 1.5	$\begin{aligned} a &= 1, \quad r = 2/3 \\ S_{\infty} &= \frac{1}{1 - 2/3} \\ &= 1 / \frac{1}{3} \\ &= 3 \end{aligned}$

5. The angle θ in the triangle shown on the right is closest to: A $45^\circ 35'$ B $35^\circ 53'$ C $35^\circ 32'$ D $44^\circ 25'$ E $54^\circ 28'$	 $\tan \theta = \frac{5}{7}$ $\theta = 35^\circ 32'$
6. In the triangle shown on the right the smallest angle is closest to: A 29.7° B 33.6° C 38.1° D 41.3° E 62.2°	 $25 = 64 + 81 - 2 \times 8 \times 9 \times \cos \theta$ $\theta = 33.56^\circ$
7. The shaded area in the diagram to the right is closest to: A 13.04 square units B 74.08 square units C 289.63 square units D 4257.65 square units E 4341.95 square units	 $A = \frac{1}{2} \times 12^2 \left(\frac{\pi}{3} - \sin \frac{\pi}{3} \right)$ ≈ 13.04
8. If $\cos(\theta) = 0.7$, then $\sec(2\theta)$ is: A 0.02 B -0.02 C 50 D -50 E 1.4	$\cos(2\theta) = 2\cos^2(\theta) - 1$ $= 2 \times (7/10)^2 - 1$ $= -1/50$ $\sec(2\theta) = -50$
9. If $\tan(x) = -0.3$ $\pi/2 < x < \pi$ and $\sin(y) = 0.6$ $0 < y < \pi/2$ Then $\tan(x - y)$ is: A $\frac{18}{31}$ B $\frac{18}{49}$ C $-\frac{18}{49}$ D $\frac{42}{31}$ E $-\frac{42}{31}$  $\tan(y) = \frac{3}{4}$	$\tan(x - y) = \frac{\tan(x) - \tan(y)}{1 + \tan(x)\tan(y)}$ $= \frac{-3/10 - 3/4}{1 + (-3/10) \times 3/4}$ $= \frac{-21/20}{31/40}$ $= -42/31$

10. $\sqrt{3} \cos(x) + \sin(x)$ can be expressed as:

- ☒ A $2 \cos(x + \frac{\pi}{6})$
☐ B $2 \cos(x - \frac{\pi}{6})$
☐ C $2 \cos(x + \frac{\pi}{3})$
☐ D $2 \cos(x - \frac{\pi}{3})$
☐ E $2 \cos(x + \frac{\pi}{2})$

$$r = \sqrt{\sqrt{3}^2 + 1^2} = 2$$

$$\alpha = \pi/6$$

Section B. Short Answer Questions (total 25 marks)

1. An arithmetic sequence has $t_5 = 34$ and $S_5 = 100$.

a) Find the first term and common difference.

$$\begin{array}{rcl} a + 4d = 34 & - (1) & \\ 5a + 10d = 100 & - (2) & \end{array} \quad \left. \vphantom{\begin{array}{rcl} a + 4d = 34 \\ 5a + 10d = 100 \end{array}} \right\} 1M$$

$$\textcircled{1} \times 5 \Rightarrow 5a + 20d = 170$$

$$\begin{array}{r} 5a + 20d = 170 \\ \hline 10d = 70 \end{array}$$

$$\begin{array}{rcl} 10d = 70 & d = 7 & \\ \hline a + 28 = 34 & a = 6 & \end{array} \quad \left. \vphantom{\begin{array}{rcl} 10d = 70 \\ a + 28 = 34 \end{array}} \right\} 1M - \text{ANSWER}$$

1M - SOME WORKING

b) Find t_{13}

$$t_{13} = 6 + 12 \times 7 = 90$$

(3 + 1 = 4 marks)

2. Express $3.\dot{1}\dot{9}$ as an improper fraction in simplest form.

$$3 + 0.19 + 0.0019 + 0.000019 + \dots \quad \frac{1}{2}M$$

$$a = 0.19 \quad r = 0.01 \quad \frac{1}{2}M$$

$$= 3 + \frac{0.19}{1 - 0.01} \quad \frac{1}{2}M$$

$$= 3 + \frac{19}{99} \quad \frac{1}{2}M$$

$$= \frac{316}{99} \quad 3 \quad 1M$$

(3 Marks)

3. A car company in its first year of operation produces 2400 cars. Each year the number of cars produced increases by 24%.

a) By what factor (r) does the number of cars increase?

$$r = 1.24$$

b) How many cars are produced in the tenth year? (Answer to the nearest whole number)

$$\begin{aligned} t_{10} &= 2400 \times 1.24^9 \\ &= \underline{16634} \end{aligned}$$

c) How many cars in total are produced in the first ten years? (Answer to the nearest whole number)

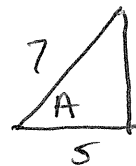
$$\begin{aligned} S_{10} &= \frac{2400 (1.24^{10} - 1)}{1.24 - 1} \\ &= 75944 \end{aligned}$$

(1 + 1 + 2 = 4 marks)

4. If A is an angle in the first quadrant and $\cos(A) = \frac{5}{7}$, find:

a) $\sin(2A)$

$$\sin(A) = \frac{2\sqrt{6}}{7} \quad \frac{1}{2} M$$



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$\frac{1}{2} M$

$$\sin(2A) = 2 \times \frac{2\sqrt{6}}{7} \times \frac{5}{7} \quad \frac{1}{2} M$$

$$= \frac{20\sqrt{6}}{49} \quad \frac{1}{2} M$$

b) $\sin(3A)$

Hint: $\sin(3A) = \sin(2A + A)$

$$\sin(3A) = \sin(2A) \cos(A) + \cos(2A) \sin(A)$$

$$\cos(2A) = \cos^2(A) - \sin^2(A) = \frac{25}{49} - \frac{24}{49} = \frac{1}{49} \quad 1 M$$

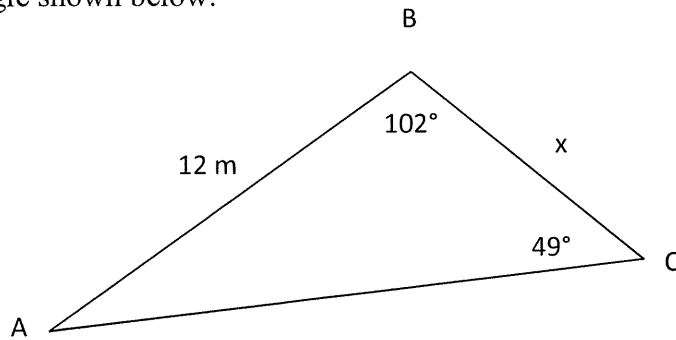
$$\sin(3A) = \frac{20\sqrt{6}}{49} \times \frac{5}{7} + \frac{1}{49} \times \frac{2\sqrt{6}}{7} \quad 1 M$$

$$= \frac{102\sqrt{6}}{343}$$

(2 + 3 = 5 marks)

1 M

5. For the triangle shown below:



a) Find the length x in metres. (Answer to two decimal places)

$$\angle A = 180^\circ - 102^\circ - 49^\circ = 29^\circ \quad \frac{1}{2} \text{ M}$$

$$\frac{x}{\sin 29^\circ} = \frac{12}{\sin 49^\circ} \quad \frac{1}{2} \text{ M}$$

$$x = 7.71 \text{ m} \quad 1 \text{ M}$$

b) Hence find the area of the triangle. (Answer to two decimal places)

$$A = \frac{1}{2} \times 12 \times 7.71 \times \sin 102^\circ \quad \frac{1}{2} \text{ M}$$

$$= 45.24 \text{ m}^2 \quad \frac{1}{2} \text{ M}$$

(2 + 1 = 3 marks)

6. Prove that $\operatorname{cosec}^2(\theta) + \sec^2(\theta) = \frac{1}{\sin^2(\theta) - \sin^4(\theta)}$

$$\text{LHS} = \frac{1}{\sin^2(\theta)} + \frac{1}{\cos^2(\theta)} \quad \frac{1}{2} \text{ M}$$

$$= \frac{\cos^2(\theta) + \sin^2(\theta)}{\sin^2(\theta) \cos^2(\theta)} \quad \frac{1}{2} \text{ M}$$

$$= \frac{1}{\sin^2(\theta) (1 - \sin^2(\theta))} \quad 1 \text{ M}$$

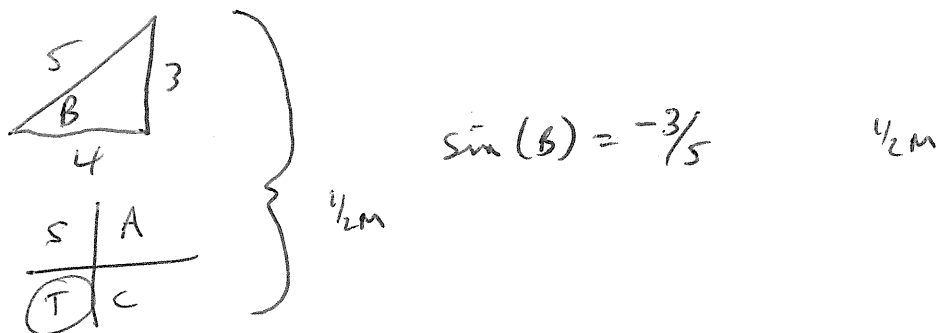
$$= \frac{1}{\sin^2(\theta) - \sin^4(\theta)} = \text{RHS} \quad 1 \text{ M}$$

(3 marks)

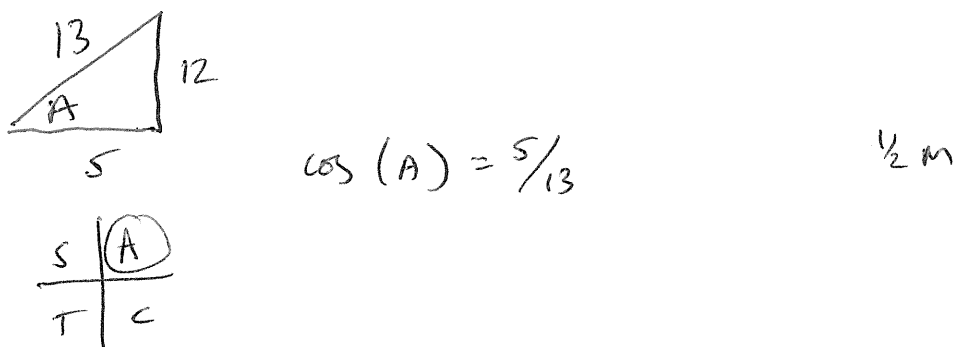
7. If $\sin(A) = \frac{12}{13}$, $0 < A < \frac{\pi}{2}$ and $\cos(B) = -\frac{4}{5}$, $\pi < B < \frac{3\pi}{2}$

Find:

a) $\sin(B)$



b) $\cos(A-B)$



$$\cos(A-B) = \cos(A)\cos(B) + \sin(A)\sin(B) \quad \frac{1}{2} m$$

$$= \frac{5}{13} \times \frac{-4}{5} + \frac{12}{13} \times \frac{-3}{5} \quad 1 m \quad (1+2=3 \text{ marks})$$

$$= \frac{-56}{65} \quad 1 m$$