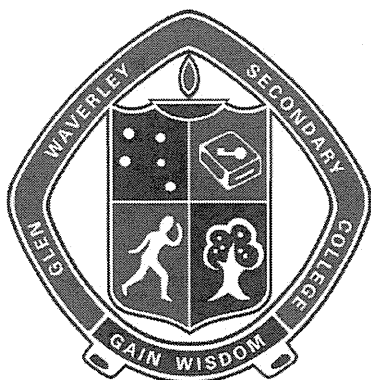




Glen Waverley Secondary College 2018

VCE Specialist Mathematics – Tech Free

Unit 1 Examination

Student Name: _____ SOLUTIONS _____

Student ID Code: _____ Form: _____

Teacher: CAL GMN RTO

QUESTION AND ANSWER BOOK

Reading Time: 10 minutes

Writing Time: 60 minutes

Structure of Book

Section	Number of questions	Number of questions to be answered	Number of marks
Short Answer	15	15	50

- Students are permitted to bring into the room: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring into the exam room: blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and Answer book.
- Formula sheet.

Instructions

- Write your **student name and student ID code** in the space provided above on this page.
- CAS / Scientific calculators, a bound reference and/or summary notes are **NOT** allowed.
- Marks are allocated for neat and clear working as well as answers.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Question 1

Express $\frac{2x^2 + x + 5}{x^2 - 1}$ in partial fractions form.

$$= 2 + \frac{x+7}{(x+1)(x-1)} = 2 + \frac{A}{x-1} + \frac{B}{x+1} \quad (1)$$

$$A(x+1) + B(x-1) = x+7$$

$$x=1 \quad ; \quad 2A = 8 \quad A = 4$$

$$x=-1 \quad ; \quad -2B = 6 \quad B = -3$$

} (1)

$$\frac{2x^2 + x + 5}{x^2 - 1} = 2 + \frac{4}{x-1} - \frac{3}{x+1} \quad (1)$$

[3 marks]

Question 2

Solve the following simultaneous equations to find all possible solutions for x and y:

$$x^2 + y^2 = 25 \quad - (1)$$

$$x + y = 1 \quad - (2)$$

$$(2) \Rightarrow x = 1 - y \quad (1/2)$$

$$\text{Sub in (1)} \Rightarrow (1-y)^2 + y^2 = 25 \quad (1/2)$$

$$1 - 2y + y^2 + y^2 = 25$$

$$2y^2 - 2y - 24 = 0$$

$$y^2 - y - 12 = 0 \quad (1/2)$$

$$(y-4)(y+3) = 0 \quad (1/2)$$

$$y = 4, -3 \quad (1)$$

$$x = -3, 4 \quad (4, -3)$$

2

$$(-3, 4)$$

[3 marks]

6

Question 3

Solve the following equation to find x:

$$\frac{x+3}{5} + \frac{3x-13}{4} = x-3$$

$$\times 20 \Rightarrow 4x + 12 + 15x - 65 = 20x - 60 \quad (1)$$

$$x = 7 \quad (1)$$

[2 marks]

Question 4

Find the sum of the first 100 multiples of 3. (i.e. the sum: $3 + 6 + 9 + \dots + 300$)

$$\begin{aligned} S_n &= \frac{n}{2} (2a + (n-1)d) \\ &= 50 (6 + 99 \times 3) \\ &= 50 \times 303 \\ &= 15150 \end{aligned}$$

or

$$\begin{aligned} S_n &= \frac{n}{2} (a + l) \quad (1/2) \\ &= 50 (3 + 300) \quad (1/2) \\ &= 50 \times 303 \quad (1/2) \\ &= 15150 \quad (1/2) \end{aligned}$$

[2 marks]

Question 5

A geometric series has $t_2 = \frac{2}{3}$ and $S_\infty = 3$, find all possible values of a and r

$$ar = \frac{2}{3} \quad (1) \quad \frac{a}{1-r} = 3 \quad (2) \quad (1/2)$$

$$(1) \div (2) \Rightarrow r(1-r) = \frac{2}{9} \quad (1/2)$$

$$9r - 9r^2 - 2 = 0 \quad (1/2)$$

$$(3r-2)(3r-1) = 0 \quad (1/2)$$

$$r = \frac{2}{3}, \frac{1}{3}$$

$$a = 1, 3/2$$

(1) [3 marks]

Question 6

The fourth term of an arithmetic sequence is 2 and the sum of the first six terms is 24. Find the second term of the sequence.

$$a + 3d = 2 \quad - (1)$$

$$3(2a + 5d) = 24$$

$$2a + 5d = 8 \quad - (2)$$

$$(1) \times 2 \Rightarrow 2a + 6d = 4 \quad - (3)$$

$$(3) - (2) \Rightarrow d = -4 \quad (1/2)$$

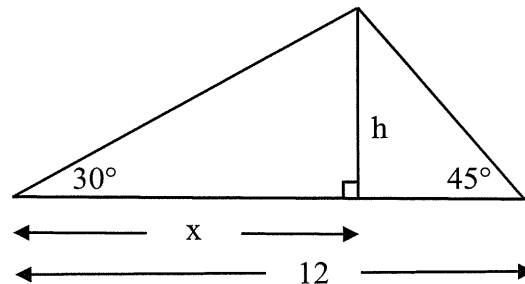
$$\text{SUB IN } (1) \Rightarrow a - 12 = 2 \quad (1/2)$$

$$a = 14 \quad (1/2)$$

$$t_2 = 10 \quad (1/2)$$

[3 marks]

Question 7



a) In the diagram above, show that $x = \frac{12\sqrt{3}}{\sqrt{3}+1}$

$$\tan 30^\circ = h/x = 1/\sqrt{3} \quad (1)$$

$$x = \sqrt{3} h \quad - (1)$$

$$\tan 45^\circ = \frac{h}{12-x} = 1$$

$$h = 12 - x \quad - (2) \quad (1)$$

$$\text{SUB } (2) \text{ IN } (1) \Rightarrow x = \sqrt{3} (12 - x)$$

$$x (\sqrt{3} + 1) = 12\sqrt{3} \quad (1)$$

$$x = \frac{12\sqrt{3}}{\sqrt{3}+1} \quad \text{AS REQ'D}$$

b) Hence, or otherwise find the area of the largest triangle.

$$h = \frac{12}{\sqrt{3}+1} \quad (1/2)$$

$$A = \frac{1}{2} \times 12 \times \frac{12}{\sqrt{3}+1} \quad (1/2)$$

$$= \frac{72}{\sqrt{3}+1} \quad \text{OR} \quad 36(\sqrt{3}-1) \quad (1)$$

OK $h = 12 - \frac{12\sqrt{3}}{\sqrt{3}+1} \quad (1/2)$

$$A = \frac{1}{2} \times 12 \times \left(12 - \frac{12\sqrt{3}}{\sqrt{3}+1}\right) \quad (1/2)$$

$$= 72 - \frac{72\sqrt{3}}{\sqrt{3}+1} \quad (1)$$

[3 + 2 = 5 marks]

Question 8

Find the area of the minor segment subtended by an angle of 90° in a circle of radius 2.

$$A = \frac{1}{2}r^2(\theta - \sin\theta)$$

$$= \frac{1}{2} \times 2^2 \left(\frac{\pi}{2} - \sin\frac{\pi}{2}\right)$$

$$= 2\left(\frac{\pi}{2} - 1\right)$$

$$= (\pi - 2) \text{ unit}^2$$

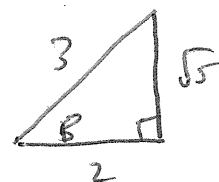
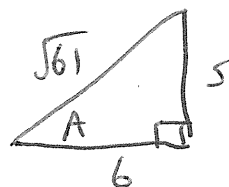
[2 marks]

Question 9.

If $\tan A = \frac{5}{6}$ where $0 \leq A \leq \frac{\pi}{2}$ and $\cos B = -\frac{2}{3}$ where $\frac{\pi}{2} \leq B \leq \pi$

Find:

a) $\sin A = \frac{5}{\sqrt{61}}$



b) $\sin B = \frac{\sqrt{5}}{3}$

c) $\cos(A+B) = \cos(A)\cos(B) - \sin(A)\sin(B)$

$$= \frac{6}{\sqrt{61}} \times \frac{-2}{3} - \frac{5}{\sqrt{61}} \times \frac{\sqrt{5}}{3} \quad (1)$$

$$= \frac{-12 - 5\sqrt{5}}{3\sqrt{61}} \quad (1)$$

$$\begin{aligned}
 \text{d) } \tan 2A &= \frac{2 \tan(A)}{1 - \tan^2(A)} \\
 &= \frac{2 \times 5/6}{1 - 25/36} \quad (1) \\
 &= \frac{5/3}{11/36} = \frac{60}{11} \quad (1)
 \end{aligned}$$

[1 + 1 + 2 + 2 = 6 marks]

Question 10.

Two complex numbers are $z_1 = \sqrt{3} + i$ and $z_2 = \sqrt{2} - \sqrt{2}i$

Find:

a) z_1 and z_2 in polar form.

$$z_1 = 2 \operatorname{cis}(\pi/6) \quad (1)$$

$$z_2 = 2 \operatorname{cis}(-\pi/4) \quad (1)$$

b) $\frac{z_1}{z_2}$ in polar form

$$\frac{z_1}{z_2} = \frac{2 \operatorname{cis}(\pi/6)}{2 \operatorname{cis}(-\pi/4)} = \operatorname{cis}\left(\frac{5\pi}{12}\right)$$

(1)

(1)

[2 + 2 = 4 marks]

Question 11.

Find all solutions to the equation: $z^3 + 8i = 0$ over $\mathbb{C}$

a) In polar form.

$$r^3 \operatorname{cis} 3\theta = 8 \operatorname{cis}(-\pi/2) \quad (1)$$

$$r = 2 \quad (1/2)$$

$$3\theta = -5\pi/2, -\pi/2, 3\pi/2 \quad (1/2)$$

$$\theta = -5\pi/6, -\pi/6, \pi/2 \quad (1)$$

6

$$z = 2 \operatorname{cis}(-5\pi/6), 2 \operatorname{cis}(-\pi/6), 2 \operatorname{cis}(\pi/2)$$

10

b) In Cartesian form

$$z_1 = 2\left(-\frac{\sqrt{3}}{2} + \frac{1}{2}i\right) = -\sqrt{3} + i$$

$$z_2 = 2\left(\frac{\sqrt{3}}{2} - \frac{1}{2}i\right) = \sqrt{3} - i$$

$$z_3 = 2i$$

(-1M EACH WRONG ANSWER)

[4 + 2 = 6 marks]

Question 12.

Find all solutions to the equation: $(z + i)^2 + 1 = 0$ over $\mathbb{C}$

$$(z + i + i)(z + i - i) = 0 \quad \left(\frac{1}{2}\right)$$

$$z(z + 2i) = 0 \quad \left(\frac{1}{2}\right)$$

$$z = 0, -2i \quad (1)$$

[2 marks]

Question 13.

A locus is defined as the distance from the point $P(x, y)$ to $F(4, 0)$ is equal to the perpendicular distance to the line with equation $x = -2$. Find the Cartesian equation of the locus.

$$\sqrt{(x-4)^2 + y^2} = x + 2 \quad (1)$$

$$\cancel{x^2} - 8x + 16 + y^2 = \cancel{x^2} + 4x + 4 \quad (1)$$

$$y^2 = 12x - 12 \quad (1)$$

[3 marks]

7

Question 14.

A curve is defined by the Cartesian equations:

$$x = 3 \sin(t) + 2 \quad y = 2 \cos(t) - 1$$

- a) Find the Cartesian equation of the curve.

$$\left(\frac{x-2}{3}\right)^2 + \left(\frac{y+1}{2}\right)^2 = 1 \quad (1)$$

$$\frac{(x-2)^2}{9} + \frac{(y+1)^2}{4} = 1 \quad (1)$$

- b) Find exact values for the x - and y - intercepts of the curve.

$$y=0 \quad \frac{(x-2)^2}{9} + \frac{1}{4} = 1 \quad (1/2)$$

$$(x-2)^2 = \frac{27}{4}$$

$$x = 2 \pm \frac{3\sqrt{3}}{2} \quad (1/2)$$

$$x=0 \quad \frac{4}{9} + \frac{(y+1)^2}{4} = 1 \quad (1/2)$$

$$(y+1)^2 = \frac{20}{9}$$

$$y = -1 \pm \frac{2\sqrt{5}}{3} \quad (1/2)$$

[2 + 2 = 4 marks]

Question 15.

Express the Cartesian equation: $x^2 - y^2 = 4$ as a polar equation. Simplify using double angle formulas as necessary.

$$r^2 \cos^2 \theta - r^2 \sin^2 \theta = 4 \quad (1/2)$$

$$r^2 \cos 2\theta = 4 \quad (1/2)$$

$$r^2 = \frac{4}{\cos 2\theta} \quad \text{OR} \quad r = \pm \frac{2}{\sqrt{\cos(2\theta)}} \quad (1)$$

[2 marks]

End of Exam

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