

2017 Analysis Task (23 marks)

Question 3 (7 marks)

The acceleration (in m/s^2) as a function of time (in seconds) of a speed boat is: $a = 10 - t$ $0 \leq t \leq 10$

Initially the speed boat is at rest at the origin.

a) Find expressions for the velocity and displacement of the speed boat as functions of time.

$$v = \int 10 - t \, dt = 10t - \frac{t^2}{2} + C \quad t=0, v=0 \Rightarrow C=0 \quad \therefore v = 10t - \frac{t^2}{2}, 0 \leq t \leq 10$$

$$x = \int 10t - \frac{t^2}{2} \, dt = 5t^2 - \frac{t^3}{6} + C \quad t=0, x=0 \Rightarrow C=0 \quad \therefore x = 5t^2 - \frac{t^3}{6}, 0 \leq t \leq 10$$

b) How long does the boat take to travel 200 metres? (Answer to two decimal places)

$$5t^2 - \frac{t^3}{6} = 200 \quad \text{CAS} \Rightarrow t = -5.79, 7.27, 28.53$$

$$\text{SINCE } 0 \leq t \leq 10 \quad \underline{t = 7.27 \text{ s}}$$

c) What is the average speed from $t = 2$ to $t = 5$?

$$x(2) = 5 \times 4 - \frac{8}{6} = \frac{56}{3}$$

$$x(5) = 5 \times 25 - \frac{125}{6} = \frac{625}{6}$$

$$\int V_{av} = \frac{\frac{625}{6} - \frac{56}{3}}{5-2} = \frac{57}{2} \quad \underline{\text{OR } 28.5 \text{ m/s}}$$

d) What is the speed boat's speed after traveling 300 metres? (Answer to two decimal places)

$$5t^2 - \frac{t^3}{6} = 300 \quad \text{CAS} \Rightarrow t = 9.33234$$

$$v = 10 \times 9.33234 - \frac{9.33234^2}{2} = \underline{49.78 \text{ m/s}}$$

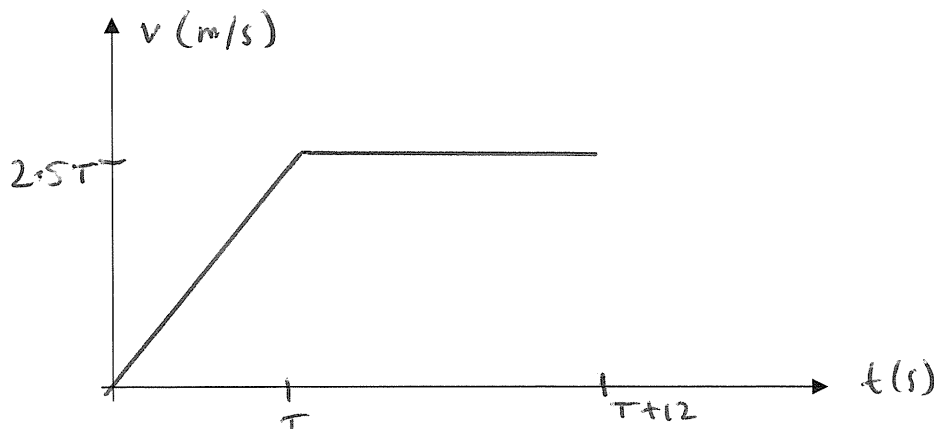
(2 + 1 + 2 + 2 = 7 marks)

Question 4 (7 marks)

A car accelerates uniformly from rest at an acceleration of 2.5 m/s^2 for T seconds and then continues at constant velocity for 12 seconds.

The total distance traveled is 320 metres.

2016) Draw a velocity – time graph showing all relevant information.



$\frac{1}{2} \text{ m} = \text{AXES LABELED}$

$\frac{1}{2} \text{ m} = \text{X-AXIS VALUES}$

$\frac{1}{2} \text{ m} = \text{Y-AXIS VALUE}$

$\frac{1}{2} \text{ m} = \text{SHAPE}$

b) Find an expression for the distance travelled in terms of T only.

$$d = \frac{1}{2} \times T \times 2.5T + 2.5T \times 12$$

$$= 1.25T^2 + 30T$$

c) Use your CAS to find all possible values of T .

$$\text{CAS} \Rightarrow T = -32.8$$

SINCE $T \geq 0$

$$\underline{T = 8}$$

d) Use equations of motion to find the distance travelled in the fourth second (from $t = 3$ to $t = 4$).

$$\boxed{0-3}$$

$$s = ut + \frac{1}{2}at^2$$

$$s = 0 + \frac{1}{2} \times 2.5 \times 3^2$$

$$= 11.25$$

$$\boxed{0-4}$$

$$s = ut + \frac{1}{2}at^2$$

$$s = 0 + \frac{1}{2} \times 2.5 \times 4^2$$

$$= 20 \text{ m}$$

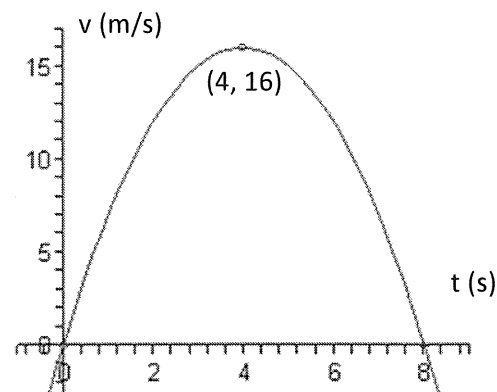
$$d = 20 - 11.25 = \underline{8.75 \text{ m}}$$

(2 + 2 + 1 + 2 = 7 marks)

Question 5 (9 marks)

The graph to the right represents the velocity – time graph of an object:

- a) Assuming the graph is parabolic, find the velocity function $v(t)$ for the object. (Use your CAS if necessary)



$$v = a t(8-t) \quad (\text{using intercepts})$$

$$t=4, v=16 \Rightarrow 16 = a \times 4 \times 4 \Rightarrow a=1$$

$$v = 8t - t^2$$

- b) If the initial position of the object is $x = 4$ metres, find the position function $x(t)$ for the object.

$$x = \int 8t - t^2 dt = 4t^2 - \frac{t^3}{3} + C \quad x(0) = 4 \Rightarrow C = 4$$

$$\therefore x = 4t^2 - \frac{t^3}{3} + 4$$

- c) Find the total distance travelled from $t = 0$ to $t = 10$.

$$x(0) = 4$$

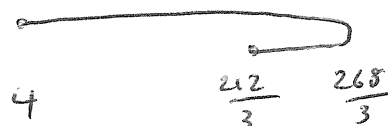
$$x(10) = 4 \times 100 - \frac{1000}{3} + 4 = \frac{212}{3}$$

$$x(8) = 4 \times 64 - \frac{512}{3} + 4 = \frac{268}{3}$$

- d) Hence, find the average speed from $t = 0$ to $t = 10$.

$$v_{av} = \frac{104}{10} = \underline{10.4 \text{ m/s}}$$

$$d = \frac{268}{3} - 4 + \frac{268}{3} - \frac{212}{3} = \underline{104 \text{ m}}$$



- e) Would you expect the average velocity to be higher or lower than your answer to d). Explain your answer.

• DISPLACEMENT IS LESS THAN DISTANCE

• THEREFORE AV. VELOCITY IS LOWER THAN AV. SPEED

(2 + 2 + 2 + 1 + 2 = 9 marks)

2016 Analysis Task (20 marks)

Question 3

An object starts at the origin and moves in a straight line with a velocity given by:

$$v(t) = t^2 - 3t + 2, \quad t \geq 0$$

- a) Find an expression for the position of the object at any time, $x(t)$.

$$x(t) = \int t^2 - 3t + 2 dt = \frac{1}{3}t^3 - \frac{3}{2}t^2 + 2t + C \quad t=0, x=0 \Rightarrow C=0$$

$$\therefore x(t) = \frac{1}{3}t^3 - \frac{3}{2}t^2 + 2t$$

(1 mark)

- b) Find the displacement of the object after four seconds.

$$x(0) = 0 \quad x(4) = \frac{1}{3} \times 4^3 - \frac{3}{2} \times 4^2 + 2 \times 4 = \frac{16}{3}$$

$$s = \frac{16}{3} - 0 = \underline{\underline{\frac{16}{3}}}$$

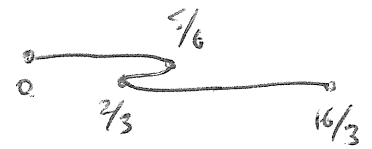
(1 mark)

c) Find the total distance travelled by the object during the first four seconds.

CMS: T.P. @ $t=1, x=5/6$ $t=2, x=2/3$

$$d = (5/6 - 0) + (5/6 - 2/3) + (16/3 - 2/3)$$

$$= 17/3$$



(2 marks)
(total 4 marks)

Question 4

- Two balls are thrown upwards.
- Ball 1 is thrown from the top of a 19.6m high building with an initial velocity of 4.9 m/s **upwards**.
- Ball 2 is thrown upwards with an unknown velocity (u) from the ground.
- The balls collide at a height of 9.8m above the ground.

a) Find the time taken for Ball 1 to reach the point of collision.

Diagram: A building of height 19.6m. Ball 1 is thrown upwards from the top. They collide at a height of 9.8m from the ground.

$s = -9.8, u = 4.9, a = -9.8, t = ?$

$$s = ut + \frac{1}{2}at^2$$

$$-9.8 = 4.9t - 4.9t^2$$

$$-2 = t - t^2$$

$$t^2 - t - 2 = 0$$

$$(t+1)(t-2) = 0$$

$t > 0 \Rightarrow t = 2 \text{ s}$ (2 marks)

b) Considering that Ball 2 will take the same time to reach the same height, find its initial velocity (u).

$s = 9.8, t = 2, a = -9.8, u = ?$

$$s = ut + \frac{1}{2}at^2$$

$$9.8 = 2u - 4.9 \times 2^2$$

$$u = \frac{9.8 + 19.6}{2}$$

$$u = 14.7 \text{ m/s UP}$$
 (2 marks)

c) If the balls did not collide, how long would Ball 1 have taken to reach the ground? (Give answer as exact value)

$u = 4.9, a = -9.8, s = -19.6, t = ?$

$$s = ut + \frac{1}{2}at^2$$

$$-19.6 = 4.9t - 4.9t^2$$

CMS $\Rightarrow t = \frac{\sqrt{17} + 1}{2} \text{ s}$ (2 marks)

c) At what speed would Ball 1 strike the ground? (Answer to three decimal places)

$u = 4.9, a = -9.8, s = -19.6, t = ?$

$$v^2 = u^2 + 2as$$

$$v^2 = 4.9^2 + 2 \times -9.8 \times -19.6$$

(2 marks)
(total 8 marks)

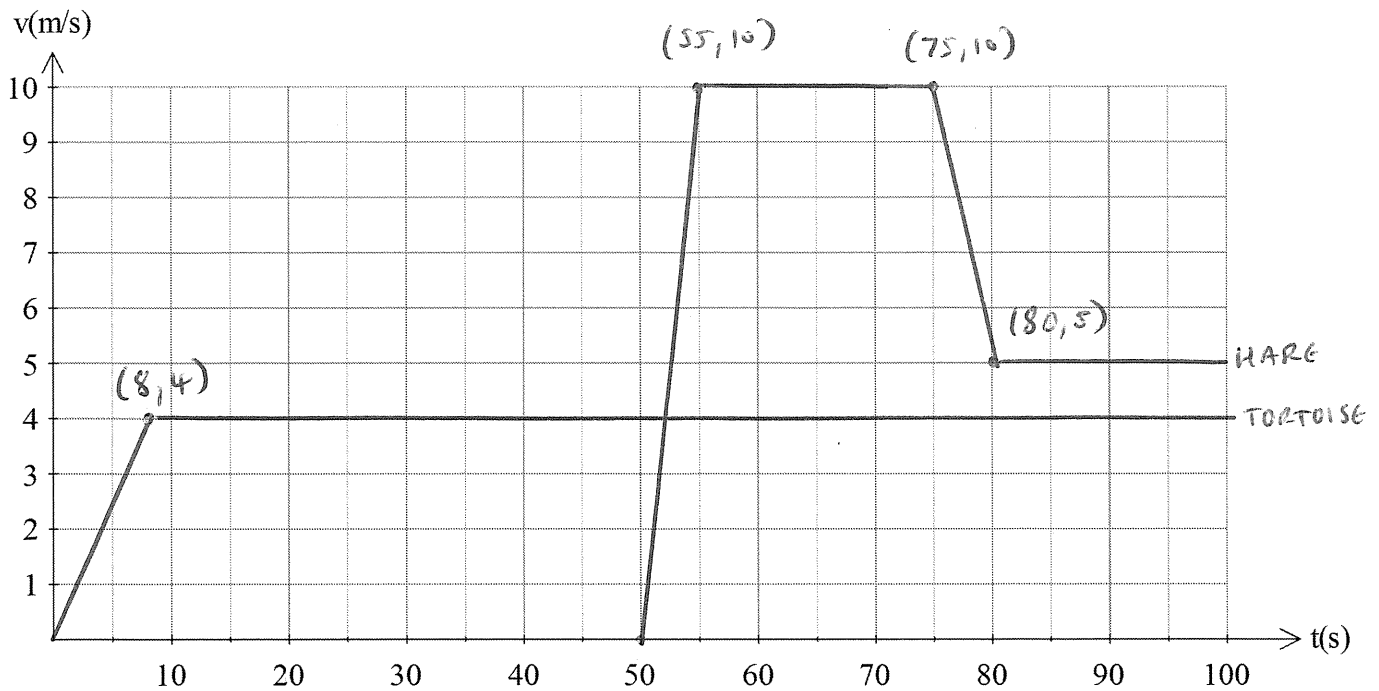
$$v = 20.203 \text{ m/s}$$

Question 5

The hare and the tortoise are having a race over one kilometre.

- The tortoise initially accelerates at 0.5 m/s^2 until it reaches a speed of 4 m/s.
- Then maintains a constant speed for the rest of the race.
- The hare finishes a snack and then starts 50 seconds after the tortoise.
- He accelerates at 2 m/s^2 until he reaches a speed of 10 m/s.
- He then maintains this speed for 20 seconds.
- Then he gets tired and decelerates at 1 m/s^2 until he reaches a speed of 5 m/s.
- Which he then maintains for the rest of the race.

a) On the axes below, draw the velocity – time graphs for both the hare and the tortoise. Label the graphs appropriately.



(2 marks)

b) Find the time taken for the tortoise to finish the race.

$$\frac{1}{2} \times 8 \times 4 + (t-8) \times 4 = 1000$$

$$16 + 4t - 32 = 1000$$

$$t = \underline{254} \text{ s}$$

(2 marks)

c) Find the time taken, from $t = 0$, for the hare to finish the race.

$$\frac{1}{2} \times 5 \times 10 + 20 \times 10 + \frac{1}{2} \times 5 \times 5 + 5 \times 5 + (t-80) \times 5 = 1000$$

$$25 + 200 + 12.5 + 25 + 5t - 400 = 1000$$

$$t = \underline{227.5} \text{ s}$$

(2 marks)

d) At what time and at what distance, from the start, does the hare catch up to the tortoise?

$$\frac{1}{2} \times 8 \times 4 + 4(t-8) = \frac{1}{2} \times 10 \times 5 + 20 \times 10 + \frac{1}{2} \times 5(10+5) + 5(t-80)$$

$$4t - 16 = 5t - 137.5$$

$$t = \underline{121.5}$$

$$d = 4 \times 121.5 - 16$$

$$= \underline{470 \text{ m}}$$

(2 marks)
(total 8 marks)

2015 Analysis Task (20 marks)

Question 1 (8 marks)

A missile is fired vertically upwards from a point on the ground, level with the foot of a tower 64m high. The missile is level with the top of the tower 0.8 seconds after being fired.

a) If acceleration due to gravity is $g \text{ m/s}^2$, find the initial velocity of the missile in terms of g .

$$x = ut + \frac{1}{2}at^2$$

$$64 = 0.8u + \frac{1}{2} \times g \times 0.8^2$$

$$u = 80 + 0.4g$$

(2 marks)

b) Find the time taken to reach the greatest height in terms of g .

$$v = u + at$$

$$0 = 80 + 0.4g - gt \Rightarrow t = \frac{80 + 0.4g}{g}$$

(2 marks)

- c) If acceleration due to gravity is 9.8 m/s^2 , calculate the greatest height of the missile. Express your answer correct to 2 decimal places.

$$v^2 = u^2 + 2as$$

$$0 = 83.42^2 + 2 \times -9.8 \times s$$

$$s = \underline{359.31 \text{ m}}$$

(2 marks)

- d) Calculate the length of time the missile is above the top of the tower ($g = 9.8 \text{ m/s}^2$). Express your answer correct to 2 decimal places.

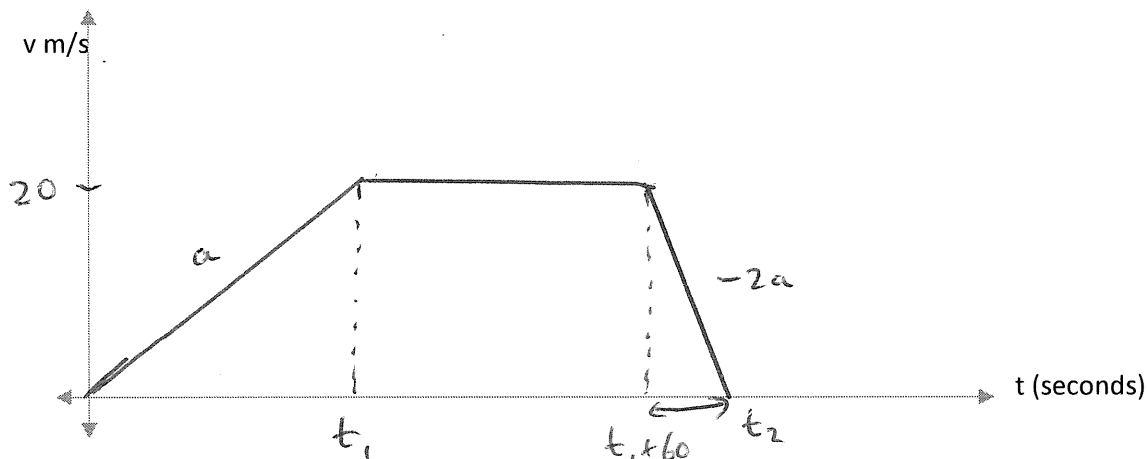
GROUND TO TOP: $v = u + at$ $t = \frac{0 - 83.42}{-9.8} = 8.5633$
 TOWER TO TOP: $t = 8.5633 - 0.8$
 $= 7.7633$

(2 marks)

Question 2 (12 marks) ABOVE TOWER: $t = 2 \times 7.7633 = \underline{15.53 \text{ s}}$

A train which is initially at rest at one station, accelerates uniformly to a speed of 20 m/s . It maintains this speed for 1 minute, then decelerates uniformly at twice the magnitude of the original acceleration to rest at next station. The two stations are 1420 m apart.

- a) Sketch a velocity-time graph showing the motion of the train between the two stations.



(2 marks)

- b) How long does it take the train to travel between the two stations? $a = \frac{20}{t_1}$ AND $t_2 = \frac{1}{2} t_1$

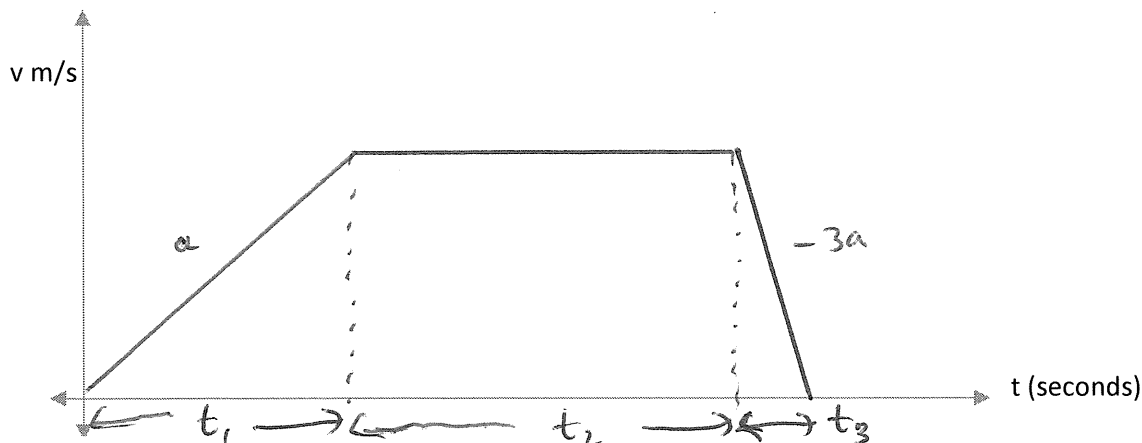
$$\left(\frac{1}{2} \times t_1 \times 20\right) + (20 \times 60) + \left(\frac{1}{2} \times \frac{1}{2} t_1 \times 20\right) = 1420$$

$$t_1 = \underline{44/3}$$

$$\text{TOTAL TIME} = 60 + \frac{44}{3} + \frac{1}{2} \times \frac{44}{3} = \underline{82 \text{ s}} \quad (4 \text{ marks})$$

The train company decides that they should upgrade their trains, as a new "Super Deluxe" model has been released. They realise that this will enable them to save time and in turn, increase the number of passengers that they can move throughout the day. They test the train between the existing stations, which are 1420 m apart. The new "Super Deluxe" can accelerate uniformly at the same rate as part (a) to a speed of 30 m/s (due to an upgraded engine and aerodynamics), travel at a constant speed of 30 m/s and then decelerate at 3 times the magnitude at which it accelerates (due to upgraded brakes).

- c) Sketch a velocity-time graph showing the motion of the train between the two stations.



(2 marks)

- d) How much time will the "Super Deluxe" save travelling between the stations? NOTE: ACCELERATION SAME AS (c)

$$a = \frac{20}{44/3} = 15/11 = \frac{30}{t_1} \Rightarrow t_1 = 22$$

$$t_3 = t_1/3 = 22/3$$

$$(\frac{1}{2} \times 22 \times 30) + (t_2 \times 30) + (\frac{1}{2} \times 22/3 \times 30) = 1420 \Rightarrow t_2 = 98/3$$

(4 marks)

$$\text{2014 Analysis Task (8 marks)} \quad \text{TOTAL} = 22 + 98/3 + 22/3 = 62 \text{ s}$$

1. A drag car has an acceleration given by:

$$a(t) = 25 - 5t \quad 0 \leq t \leq 6$$

It starts with zero displacement and zero velocity.

- (a) Find an expression for the displacement $x(t)$

$$v = \int 25 - 5t \, dt = 25t - \frac{5}{2}t^2 + c$$

$$t=0, v=0 \Rightarrow v = 25t - \frac{5}{2}t^2$$

$$x = \int 25t - \frac{5}{2}t^2 \, dt = \frac{25}{2}t^2 - \frac{5}{6}t^3 + c$$

$$t=0, x=0 \Rightarrow x(t) = \frac{25}{2}t^2 - \frac{5}{6}t^3$$

- (b) At what time(s) does the drag car reach its maximum speed?

$$a=0 \Rightarrow t=5$$

2 marks

1 mark

- (c) What is the total distance travelled over six seconds?

$$x(0) = 0 \quad x(6) = \frac{25}{2} \times 6^2 - \frac{5}{6} \times 6^3 = 270$$

$$d = 270 - 0 = 270$$

$$\text{CHECK: } v=0 \Rightarrow t=0, 10$$

1 mark

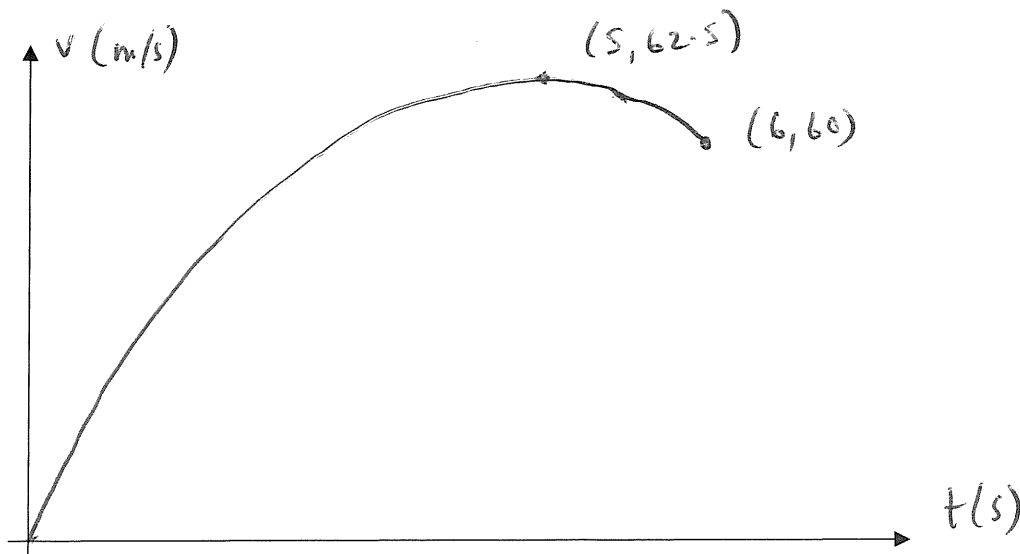
- (d) What is the average velocity over the first five seconds?

$$x(5) = \frac{25}{2} \times 5^2 - \frac{5}{6} \times 5^3 = \frac{625}{3}$$

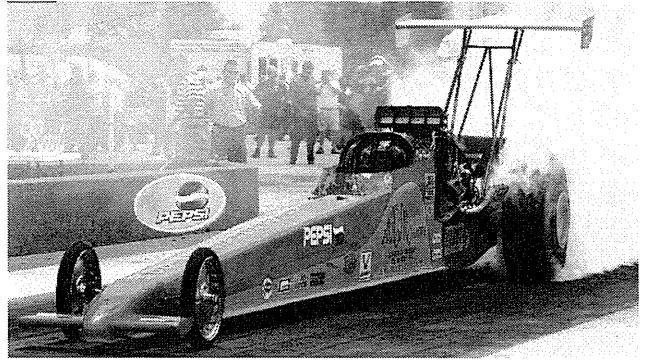
$$v_{AV} = \frac{\frac{625}{3} - 0}{5 - 0} = \frac{125}{3}$$

2 marks

- (e) Sketch a labelled graph of velocity vs time on the axes below for the six second time interval.



2 marks

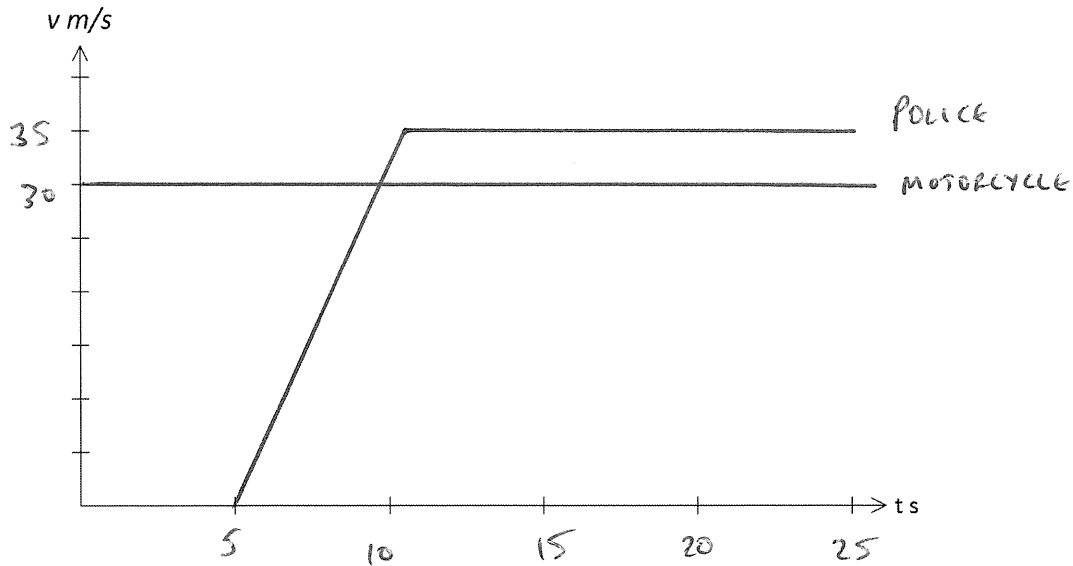


2012 Analysis Task (10 marks)

Question 1

A motorcyclist travelling at 30 m/s passes a stationary police car. 5 seconds later, the police car takes off, accelerating uniformly for 5 seconds until it reaches a speed of 35 m/s which it then maintains.

- a) Draw a velocity time graph on the diagram below.



(3 marks)

- b) If the police car catches up with the motorcyclist T seconds after it is first passed,

- i) write an expression, d_m , in terms of T , for the distance travelled by the motorcyclist.

$$d_m = 30T$$

(1 mark)

- ii) write an expression, d_p , in terms of T , for the distance travelled by the police car.

$$\begin{aligned} d_p &= \frac{1}{2} \times 35 \times 5 + 35(T - 10) \\ &= 35T - 262.5 \end{aligned}$$

(2 marks)

- c) Hence find the time the police car takes to catch the motorcyclist.

$$30T = 35T - 262.5$$

$$T = 52.5 \text{ s}$$

$$\text{TIME TO CATCH UP} = 47.5 \text{ s}$$

(3 marks)

- d) Find the total distance travelled in the pursuit.

$$\begin{aligned} d &= 30 \times 52.5 \\ &= 1575 \text{ m} \end{aligned}$$

(1 mark)