

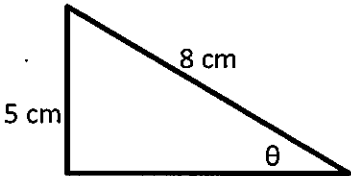
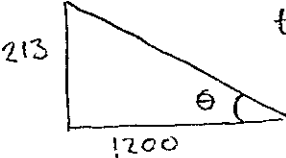


2017 Yr 11 Specialist Mathematics
Applications of Trigonometry Technology Access
(1 double sided A4 Summary Sheet & 45 min writing time)

Name: Solutions

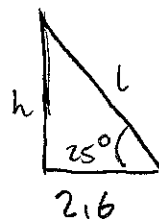
Total Marks: / 45

Section A. Multiple Choice Questions. Clearly circle your answer (10 x 2 marks = 20 marks total)

1. The angle 1.754 radians, when converted to its degree equivalent (to the nearest minute) is: A $120^{\circ} 50'$ B $93^{\circ} 27'$ C $110^{\circ} 7'$ D $113^{\circ} 10'$ (E) $100^{\circ} 30'$	Working Space $\frac{1.754 \times 360}{2\pi} = 100.497$
2. The value of θ is: A $51^{\circ} 20'$ (B) $38^{\circ} 41'$ C $51^{\circ} 19'$ D $38^{\circ} 40'$ E 32° 	Working Space $\sin(\theta) = \frac{5}{8}$ $\theta = 38.68$
3. If $\sin \alpha = -0.8$ and $\cos \alpha = -0.6$, the value of $\cos\left(\frac{\pi}{2} - \alpha\right)$ is: A $\frac{\pi}{2} + 0.6$ B -0.6 (C) -0.8 D 0.6 E 0.8	Working Space $\cos\left(\frac{\pi}{2} - \alpha\right) = \sin(\alpha)$ $= -0.8$
4. The longest slide in the world is in Vermont USA. It starts at a vertical height of 213 metres, with a horizontal distance of 1200 metres. What is its angle of inclination? A 31° (B) 10° C 15° D 22° E 28°	Working Space  $\tan(\theta) = \frac{213}{1200}$ $\theta = 10.07$

5. A ladder leans up against a wall with its foot 2.6 m from the foot of the wall. If the angle that it makes with the ground is 25° , the length of the ladder and the height that it reaches up the wall respectively are in metres:

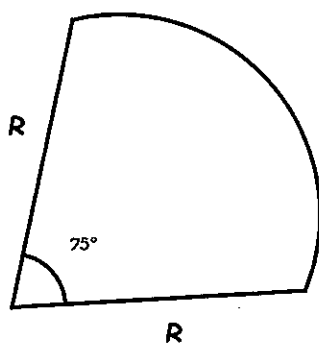
- A $\frac{2.6}{\tan(25^\circ)}, 2.6 \cos(25^\circ)$
 B $2.6 \tan(25^\circ), 2.6 \cos(25^\circ)$
☒ C $\frac{2.6}{\cos(25^\circ)}, 2.6 \tan(25^\circ)$
 D $2.6 \tan(25^\circ), \frac{2.6}{\cos(25^\circ)}$
 E $\frac{2.6}{\tan(25^\circ)}, 2.6 \sin(25^\circ)$



$$l = \frac{2.6}{\cos(25^\circ)}, h = 2.6 \tan(25^\circ)$$

6. A sector of a circle of radius R cm has a total perimeter (including the two straight sides) of M cm. If the angle of the sector is 75° , the value of R can be determined by which of the following expressions?

- A $\frac{24M}{5\pi + 12}$
 B $\frac{5M}{12\pi + 12}$
☒ C $\frac{12M}{5\pi + 24}$
 D $\frac{5M}{24\pi + 12}$
 E $\frac{12M}{24\pi + 5}$



$$l = 2\pi R \times \frac{75}{360}$$

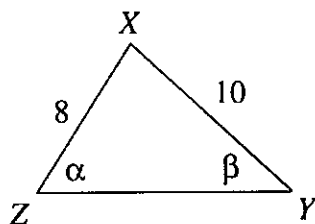
$$= \frac{5\pi R}{12}$$

$$M = 2R + \frac{5\pi R}{12}$$

$$R = \frac{12M}{5\pi + 24}$$

7. Given that $XZ = 8$, $XY = 10$ and $\sin \alpha = \frac{2}{3}$, $\sin \beta$ equals:

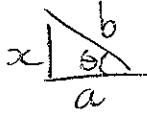
- A $\frac{4}{15}$
☒ B $\frac{8}{15}$
 C $\frac{5}{12}$
 D $\frac{4}{5}$
 E $\frac{5}{6}$



$$\frac{\sin(\alpha)}{10} = \frac{\sin(\beta)}{8}$$

$$\frac{2}{30} = \frac{\sin(\beta)}{8}$$

$$\sin(\beta) = \frac{8}{15}$$

8. $-\cos x + \sin x$ in the form $r \sin(x + \alpha)$, where $r > 0$, is: A $\sqrt{2} \sin\left(x + \frac{\pi}{4}\right)$ B $-\sin\left(x + \frac{\pi}{4}\right)$ C $\sqrt{2} \sin\left(x + \frac{5\pi}{4}\right)$ D $\sqrt{2} \sin\left(x + \frac{7\pi}{4}\right)$ E $\sqrt{2} \sin\left(x + \frac{3\pi}{4}\right)$	$a \cos(x) + b \sin(x) = r \sin(x + \alpha)$ $r = \sqrt{a^2 + b^2}$, $\sin(\alpha) = \frac{a}{r}$, $\cos(\alpha) = \frac{b}{r}$ $r = \sqrt{2}$, $\sin(\alpha) = -\frac{1}{2}$, $\cos(\alpha) = \frac{1}{2}$ $\Rightarrow \alpha = \frac{7\pi}{4}$  $\sqrt{2} \sin\left(x + \frac{7\pi}{4}\right)$
9. A clock has an hour hand 40 cm long. The hour hand takes 12 hours to complete one full revolution. The area swept after 30 minutes is nearest to: A 5 cm² B 10 cm² C 209 cm² D 5026 cm² E 1838 cm²	$\frac{1}{24} \times 2\pi = \frac{\pi}{12}$ $A = \frac{1}{2} (40)^2 \left(\frac{\pi}{12}\right)$
10. If $\cos \theta = \frac{a}{b}$ and $0 < \theta < \frac{\pi}{2}$, then $\cot \theta$ is: A $\frac{a}{\sqrt{b^2 - a^2}}$ B $\frac{b}{\sqrt{b^2 - a^2}}$ C $\frac{\sqrt{a^2 + b^2}}{a}$ D $\frac{a}{\sqrt{a^2 - b^2}}$ E $\frac{a}{\sqrt{b^2 + a^2}}$	 $a = \sqrt{b^2 - a^2}$ $\cot(\theta) = \frac{a}{\sqrt{b^2 - a^2}}$

Section B. Short Answer Questions (total 25 marks)

1. A swarm of locusts begin a path of destruction in the shape of a triangle as they spread. The sides of the triangle formed are 8 km and 6.3 km and the angle of spread is 28° . Find the length of the third side of this shape and the area devastated by the pest. Give answers rounded to one decimal place.

$$a^2 = b^2 + c^2 - 2bc \cos(A)$$

$$a^2 = 8^2 + 6.3^2 - 2(8)(6.3) \cos(28^\circ) \quad (1M)$$

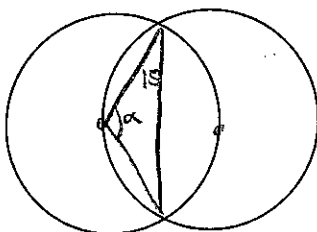
$$a^2 = 14.69 \quad (1M)$$

$$a = 3.83 \text{ km} \quad (1A)$$

$$\begin{aligned} \text{Area} &= \frac{1}{2}(8)(6.3) \sin(28^\circ) \\ &= 11.83 \text{ km}^2 \end{aligned} \quad (1A)$$

(4 marks)

2. Two circles of equal radius (15 cm) overlap so that the centre of each lies on the circumference of the other. (See diagram below).



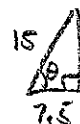
- a. Find the angle at the centre of one of the circles that is subtended by the chord that joins the two points of intersection.

Let α be the required angle, $\theta = \frac{\alpha}{2}$

$$\cos(\theta) = \frac{7.5}{15}$$

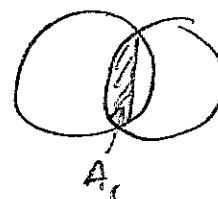
$$\theta = 60^\circ \quad \left(\text{or } \frac{\pi}{3}\right) \quad (1M)$$

$$\Rightarrow \alpha = 120^\circ \quad \left(\text{or } \frac{2\pi}{3}\right) \quad (1A)$$



- b. Hence or otherwise, calculate the exact total area of the overlapping region (express your final answer with a common denominator).

$$\begin{aligned} A_1 &= \frac{1}{2}(15)^2 \left(\frac{2\pi}{3} - \sin\left(\frac{2\pi}{3}\right) \right) \\ &= \frac{225 \left(\frac{2\pi}{3} - \frac{\sqrt{3}}{2} \right)}{2} \quad (1M) \end{aligned}$$



$$A = 2A_1$$

$$= 225 \left(\frac{2\pi}{3} - \frac{\sqrt{3}}{2} \right)$$

$$= \frac{75(4\pi - 3\sqrt{3})}{2} \quad (1A)$$

(or using formula for degrees)

(2+2=4 Marks)

3. Given $\sin x = \frac{1}{5}$ and $\sin y = -\frac{2}{3}$, angle x is in the second quadrant and angle y is in the third quadrant, find the exact value of $\sin(x+y)$

$$\begin{aligned}\sin(x+y) &= \sin(x)\cos(y) + \cos(x)\sin(y) \\ &= \frac{1}{5}\cos(y) - \frac{2}{3}\cos(x) \quad (1M)\end{aligned}$$

$$\cos(\theta) = \pm\sqrt{1-\sin^2(\theta)}$$

$$\begin{aligned}\cos(y) &= -\sqrt{1-\frac{4}{9}} \\ &= -\frac{\sqrt{5}}{3} \quad \left(\frac{1}{2}M\right)\end{aligned}$$

$$\begin{aligned}\cos(x) &= -\sqrt{1-\frac{1}{25}} \\ &= -\frac{\sqrt{24}}{5} \quad \left(\frac{1}{2}M\right)\end{aligned}$$

$$\text{So } \sin(x+y) = \frac{1}{5} \times -\frac{\sqrt{5}}{3} - \frac{2}{3} \times -\frac{\sqrt{24}}{5} = \frac{2\sqrt{24} - \sqrt{5}}{15} \quad \left(\frac{1}{2}A\right)$$

(3 marks)

4. Verify the identity $\frac{\cot x - \tan x}{\sin x \cos x} = \operatorname{cosec}^2 x - \sec^2 x$. $= \frac{4\sqrt{6} - \sqrt{5}}{15} \quad \left(\frac{1}{2}A\right)$

$$\frac{\cot(x) - \tan(x)}{\sin(x)\cos(x)} = \frac{\frac{\cos(x)}{\sin(x)}}{\sin(x)\cos(x)} - \frac{\frac{\sin(x)}{\cos(x)}}{\sin(x)\cos(x)} \quad (1M)$$

$$= \frac{\cos(x)}{\sin^2(x)\cos(x)} - \frac{\sin(x)}{\sin(x)\cos^2(x)} \quad (1M)$$

$$= \frac{1}{\sin^2(x)} - \frac{1}{\cos^2(x)} \quad (1M)$$

$$= \operatorname{cosec}^2(x) - \sec^2(x) \quad \square$$

(3 marks)

5. Write $\sqrt{3}\sin x - \cos x$ in the form $r \sin(x-\alpha)$

$$a \cos(x) + b \sin(x) = r \sin(x+\alpha)$$

$$\begin{aligned}a &= -1, \quad b = \sqrt{3}, \quad r = \sqrt{1+3} \\ &= 2 \quad \left(\frac{1}{2}M\right)\end{aligned}$$

$$\sin(\alpha) = -\frac{1}{2}, \quad \cos(\alpha) = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \alpha = -\frac{\pi}{6} \quad \left(\frac{1}{2}M\right)$$

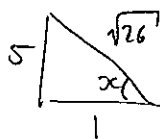
$$\text{So, } \sqrt{3}\sin(x) - \cos(x) = 2 \sin\left(x - \frac{\pi}{6}\right) \quad (1A)$$

(2 marks)

6. x is an angle in the fourth quadrant and $\tan x = -5$. Find $\sin x$.

$$\tan(x) = 5 \Rightarrow \sin(x) = \frac{5}{\sqrt{26}} \quad (1M)$$

$$4\text{th Quadrant} \Rightarrow \sin(x) = -\frac{5}{\sqrt{26}} \quad (1A)$$

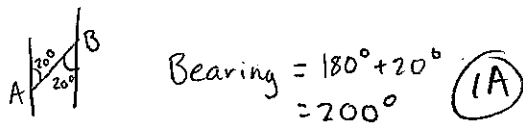


(Students may also solve algebraically)

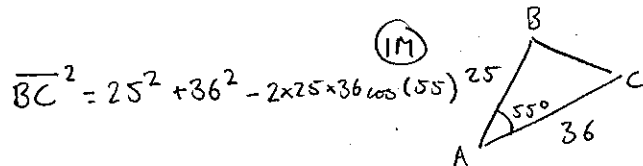
(2 marks)

7. In the diagram, A, B and C represent three islands. B is 25 km from A on a bearing of 020° and C is 36 km from A on a bearing of 075° . Give your answers correct to 2 decimal places.

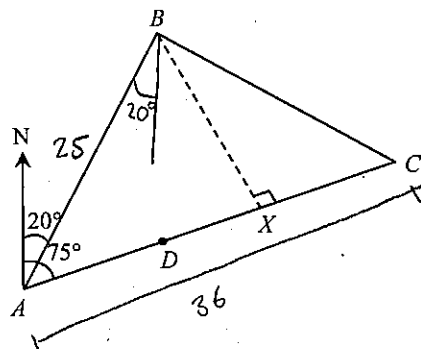
a. Calculate the bearing of A from B.



b. Calculate the distance of C from B.



$BC = 29.81 \text{ km}$ (1A)

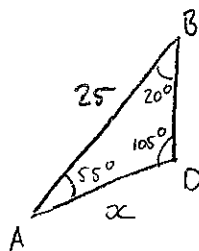


c. A ship leaves A and sails directly to C. When the ship is at D it is due south of B.

i. Calculate the distance AD.

$\frac{25}{\sin(105^\circ)} = \frac{x}{\sin(20^\circ)}$ (1M)

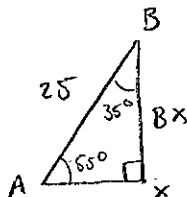
$x = 8.85 \text{ km}$ (1A)



ii Find the distance BX, where X is the point on AC closest to B.

$25 = \frac{BX}{\sin(55^\circ)}$ (1M)

$BX = 20.48$ (1A)



(1+2+2+2=7 marks)