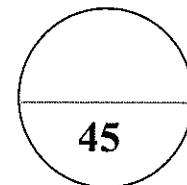




Yr 11 Specialist Maths 2017 Statistics and Probability Test

Technology Allowed Assessment – 45 minutes writing time
One-sided summary sheet



Name: SOLUTIONS.

Multiple Choice (10x2 marks = 20 marks)

- 1 In order to estimate the proportion of teenagers in Australia who smoke, a researcher selected a random sample of 200 teenagers and found that 25% of them smoked. This percentage is an example of a:
- A ratio
 - ☒ B sample statistic
 - C parameter
 - D population statistic
 - E sample parameter
2. A bag contains 4 black balls and three blue balls. If a random sample of three balls is taken from the bag, without replacement, the possible values of the *sample proportion* of blue balls in the sample are:
- A {1, 2, 3}
 - B {0, 1, 2, 3}
 - C $\{0, \frac{1}{7}, \frac{2}{7}\}$
 - D $\{0, \frac{1}{3}, \frac{2}{3}\}$
 - ☒ E $\{0, \frac{1}{3}, \frac{2}{3}, 1\}$
3. A company employs 5 males and 5 females in their technology team. If four people are selected at random for a new training program, and $\hat{P}$ represents the proportion of females chosen, then $\Pr(\hat{P} > 0.5)$ is:
- ☒ A 0.2619
 - B 0.3222
 - C 0.2381
 - D 0.0238
 - E 0.2050
- $$\frac{{}^5C_3 \cdot {}^5C_1 + {}^5C_4}{{}^{10}C_4}$$
4. A sampling distribution can best be described as a distribution which
- A gives the possible range of values of the sample statistic
 - ☒ B describes how a statistic's value will change from sample to sample
 - C describes how samples do not give reliable estimates
 - D gives the distribution of the values observed in particular sample
 - E none of the above

5. In a group of 20 students, 12 study Year 11 Specialist Mathematics. Six students are chosen at random from the group. The probability that exactly three of them study Specialist Maths is:

A $\frac{({}^8C_3)({}^{12}C_3)}{{}^{20}C_6}$

B $\frac{({}^6C_3)({}^6C_3)}{{}^{20}C_6}$

C $\frac{({}^{14}C_3)({}^{20}C_3)}{{}^{20}C_6}$

D $\frac{({}^6C_0)({}^{14}C_6)}{{}^{20}C_6}$

E $\frac{({}^6C_3)({}^{14}C_3)}{{}^{20}C_3}$

6. Which of these statements is true?

- A** The sample proportion is always equal to the population proportion.
B The sample proportion is a good estimate of the population proportion.
C The sample proportion is always less than the population proportion.
D The sample proportion is always greater than the population proportion.
E The sample proportion is unrelated to the population proportion.

7. In a group of 100 Year 11 students, it was found that 25 play hockey, 42 play AFL and 10 swim. No one plays two sports and the remaining students don't play any sport. The sample proportion of students who don't participate in any sporting activity is:

A $\frac{1}{4}$

B $\frac{21}{50}$

C $\frac{1}{10}$

D $\frac{87}{100}$

E $\frac{23}{100}$

25 Hockey

42 AFL

10 swim.

$$100 - 25 - 42 - 10 = 23$$

8. A machine produces fuses, each of which has a probability p of being defective. The fuses are packed four to a box. What is the probability that exactly half of the articles in a box are defective?

A p^2

B $6p^2$

C $p^2(1-p)$

D $6p^2(1-p)^2$

E $12p^2(1-p)^2$

$$\binom{4}{2} p^2 (1-p)^2$$

9. From a population of 10 000, samples of size 100 were selected. If the population mean and standard deviation were 34.2 and 12.3 respectively, what is the standard deviation of the sample means?

- A 3.42
- B 34.2
- C 12.3
- D 1.23**
- E 0.123

10. A set of 21 balls, consisting of 15 red balls and 6 balls of other colours, is placed into a bag. If three balls are selected **without** replacement, find the probability that exactly two of the three selected balls are red.

- A 0.1458
- B 0.1579
- C 0.4373
- D 0.4737**
- E 0.9494

$$\frac{{}^{15}C_2 \cdot {}^6C_1}{{}^{21}C_3}$$

Short Answer Questions (25 marks)

1. Holly and Matthew are interested in the number of people that spend more than \$100 in a single shop. Holly samples 215 people and finds that 108 of them spend more than \$100. Matthew samples 53 people and finds that 32 of them spend more than \$100.

(a) Find the sample proportions.

Holly's : $\hat{p} = \frac{108}{215}$ (1)

Matthew's : $\hat{p} = \frac{32}{53}$ (1)

(b) Comment on who is more likely to be closer to the population proportions.

Holly more likely to be closer to the population proportions as she has a larger sample.

(2+1=3 marks)

2. The following table gives the sampling distribution of sample proportion of heads observed obtained when a coin biased towards tails is tossed five times.

$\hat{p}$	0	0.2	0.4	0.6	0.8	1.0
$\Pr(\hat{P} = \hat{p})$	0.116	0.312	0.336	0.181	0.048	0.005

Use the table to determine:

- (a) The probability that the proportion of heads in a random sample is more than 0.5.

$$\Pr(X > 0.5) = 0.181 + 0.048 + 0.005 = 0.234$$

(b) $\Pr(\hat{P} \geq 0.7 | \hat{P} > 0.2)$. Show your workings.

$$\Pr(\hat{P} \geq 0.7 | \hat{P} > 0.2) = \frac{\Pr(\hat{P} \geq 0.7)}{\Pr(\hat{P} > 0.2)} = \frac{0.048 + 0.005}{0.336 + 0.234} = \frac{53}{570}.$$

$\left(\frac{1}{2}\right)$ $\left(\frac{1}{2}\right)$ (1)

(1+2=3 marks)

3. A population has a mean of 65 and a standard deviation of 22.3. Samples of size 50 are selected.

(a) Find the mean and standard deviation of the distribution of sample means.

$$\begin{aligned} \mu &= 65 & \mu_{\bar{x}} &= \mu = 65 & (1) \\ \sigma &= 22.3 & \sigma_{\bar{x}} &= \frac{\sigma}{\sqrt{n}} & (1) \\ n &= 50 & \sigma_{\bar{x}} &= \frac{22.3}{\sqrt{50}} = 0.446 & \end{aligned}$$

(b) What sample size would be needed to reduce the standard deviation of the distribution of sample means to less than 3?

$$\begin{aligned} \frac{\sigma}{\sqrt{n}} &< 3 & n &> 55.25 & \left(\frac{1}{2}\right) \\ \frac{22.3}{\sqrt{n}} &< 3 & \therefore n &= 56 & (1) \end{aligned}$$

(2+2=4marks)

4. The following is a discrete probability distribution table.

x	1	2	3	4	5
$\Pr(X=x)$	$\frac{3k}{13}$	$\frac{k^2}{13}$	$\frac{5k-4}{13}$	$\frac{k^2}{13}$	$\frac{7}{13}$

(a) Find the value of k for this distribution

$$\begin{aligned} \frac{3k}{13} + \frac{k^2}{13} + \frac{5k-4}{13} + \frac{k^2}{13} + \frac{7}{13} &= 1 & (1) \\ 3k + k^2 + 5k - 4 + k^2 + 7 &= 13 \\ 2k^2 + 8k - 10 &= 0 \\ k^2 + 4k - 5 &= 0 \\ (k+5)(k-1) &= 0 \\ k &= -5 \text{ or } 1 & (1) \end{aligned}$$

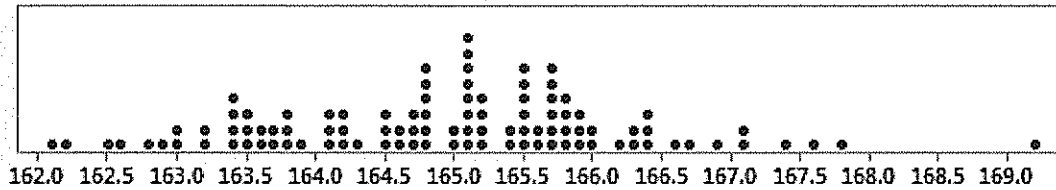
$\therefore \text{As } k > 0, k = 1$

(b) Explain why one of the values of k had to be discarded.

As $\Pr(X=x) > 0$ so $k > 0$
 Negative value of k had to be discarded.

(2+1=3 marks)

5. The mean height of women in a certain country is $\mu = 165$ cm with a standard deviation, $\sigma = 10$ cm. Assuming that the height of women is approximately normally distributed 100 samples each of size 50 were generated, and the following dot plot of the sample means constructed.



Use the dot plot to estimate:

- (a) The probability that the average height of a random sample of 50 women selected from that population will be more than 168 cm.

$$\Pr(\text{Height} > 168) = \frac{1}{100} \text{ or } 0.01$$

- (b) The probability that the average height of a random sample of 50 women selected from that population will be less than 163 cm.

$$\Pr(H < 163) = \frac{6}{100} = \frac{3}{50} \text{ or } 0.06$$

(1+1=2 marks)

6. A spinner has outcomes with equal probability of 1, 2 and 4. The spinner is spun twice and the average taken. Let X be the average of the two spins. Complete the table for the averages of the two numbers and hence, complete the table for the probability distribution of X .

(a)

	1	2	4
1	1	1.5	2.5
2	1.5	2	3
4	2.5	3	4

(1)

x	1	1.5	2	2.5	3	4
$\Pr(X=x)$	$\frac{1}{9}$	$\frac{2}{9}$	$\frac{1}{9}$	$\frac{2}{9}$	$\frac{2}{9}$	$\frac{1}{9}$

(1)

- (b) Find $\Pr(\bar{X} \geq 2)$

$$\Pr(\bar{X} \geq 2) = \frac{1}{9} + \frac{2}{9} + \frac{2}{9} + \frac{1}{9} = \frac{6}{9} = \frac{2}{3}$$

(1)

- (c) Find $\Pr(\bar{X} \geq 2 | \bar{X} \geq 1.5)$

$$\Pr(\bar{X} \geq 2 | \bar{X} \geq 1.5) = \frac{\Pr(\bar{X} \geq 2)}{\Pr(\bar{X} \geq 1.5)} = \frac{\frac{6}{9}}{\frac{8}{9}} = \frac{6}{8} = \frac{3}{4}$$

(1/2)

(2+1+2=5 marks)

7. Assume that dates of birth in a large population are distributed such that the probability of a randomly chosen person's birthday being in any particular month is $\frac{1}{12}$.

(a) Find the probability that of six people chosen at random exactly two will have a birthday in January.

(i). Write the formula required and the substitution of values.

$$Pr(X=2) = \binom{6}{2} \cdot \left(\frac{1}{12}\right)^2 \cdot \left(\frac{11}{12}\right)^4$$

(1)

(ii). Find the probability and give your final answer correct to 4 decimal places.

$$Pr(X=2) = 0.0735$$

(1)

(b) Find the probability that of eight people at least one will have a birthday in January. Give your answer correct to 4 decimal places.

$$\begin{aligned} Pr(X \geq 1) &= 1 - Pr(X=0) \\ &= 1 - \left(\frac{11}{12}\right)^8 \\ &= 0.5015 \end{aligned}$$

(1/2)

(1/2)

(c) N people are chosen at random. Find the least value of N such that the probability that at least one will have a birthday in January exceeds 0.9.

$$Pr(X \geq 1) > 0.9$$

$$\begin{aligned} Pr(X \geq 1) &= 1 - Pr(X=0) \\ &= 1 - \left(\frac{11}{12}\right)^N \end{aligned}$$

(1/2)

$$\left(1 - \frac{11}{12}\right)^N > 0.9$$

$$N > 26.46$$

(1/2)

$$\therefore N = 27$$

(1)

(2+1+2=5 marks)

OR $Pr(X=0) < 0.1$

$$\left(\frac{11}{12}\right)^N < 0.1$$

$$N > 26.46$$