



Glen Waverley Secondary College 2017

VCE Specialist Mathematics – Tech Free

Unit 2 Examination

Student Name: _____

Student ID Code: _____ Form: _____

Teacher: **CAL PLG GMN**
(circle your teacher's code)

QUESTION AND ANSWER BOOK

Reading Time: 15 minutes
Writing Time: 60 minutes

Structure of Book

Section	Number of questions	Number of questions to be answered	Number of marks
Short Answer	11	11	50

- Students are permitted to bring into the room: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring into the exam room: blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and Answer book.
- Formula sheet.

Instructions

- Write your **student name and student ID code** in the space provided above on this page.
- CAS calculators, a bound reference and/or summary notes are **NOT** allowed.
- Marks are allocated for neat and clear working as well as answers.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Question 1

Given $x^2 - 8x - 29 \equiv (x + a)^2 + b$, where a and b are constants,

a) find the value of a and the value of b .

b) Hence, or otherwise, show that the roots of $x^2 - 8x - 29 = 0$ are of the form $c \pm d\sqrt{5}$, where c and d are integers.

(2 + 2 = 4 marks)

Question 2

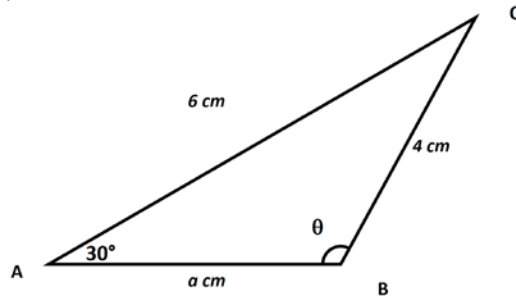
Find the first term and common difference of an arithmetic sequence whose 16th term is 22, where the sum of the first 6 terms is 57.

(3 marks)

Question 3

In triangle ABC side AC is of length 6 cm, side BC is of length 4 cm and $\angle BAC = 30^\circ$. Let $\angle ABC = \theta$ and let side AB be of length a cm.

"Diagram **not** to scale"



- a) Use the sine rule in triangle ABC to evaluate $\sin \theta$.
- b) Hence, obtain two possible values for $\cos \theta$ in triangle ABC.

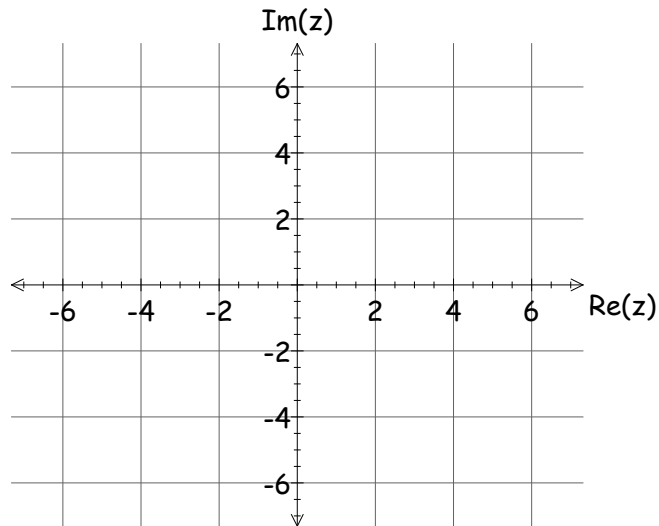
(2 + 2 = 4 marks)

Question 4

Let $z = \sqrt{2} \operatorname{cis} \left(\frac{3\pi}{4} \right)$

- (a) Use De Moivre's theorem to evaluate z^5 . Express your answer in Cartesian form.

(b) Plot z^5 on the Argand diagram below.



(3 + 1 = 4 marks)

Question 5

(a) Show that the matrix $\begin{bmatrix} -2 & -1 \\ 3 & 2 \end{bmatrix}$ is its own inverse.

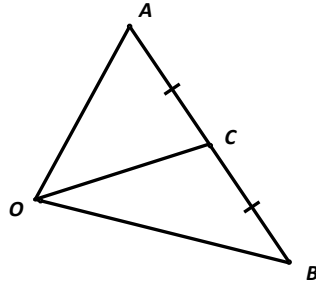
(b) Determine an equation that shows how the values of a , b and c are related if a matrix of the form

$$\begin{bmatrix} a & b \\ c & -a \end{bmatrix} \text{ is such that } \begin{bmatrix} a & b \\ c & -a \end{bmatrix}^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

(2+2=4 marks)

Question 6

In the triangle AOB point C is the midpoint of side AB as shown in the diagram below:



Let $\vec{a}$ be the vector representing the side OA and $\vec{c}$ be the vector representing the median OC .

(a) Show that $\vec{OB}$ is represented by the vector $2\vec{c} - \vec{a}$.

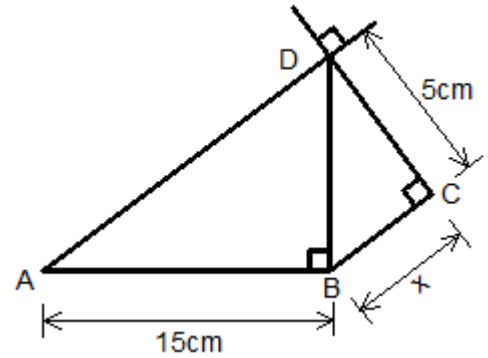
(b) Express $\vec{OB} \cdot \vec{OB}$ in terms of $\vec{a} \cdot \vec{a}$, $\vec{c} \cdot \vec{c}$ and $\vec{a} \cdot \vec{c}$

(c) Hence prove that if triangle AOB is isosceles with side OA of equal length to side OB , then the median OC will be perpendicular to side AB .

(2+2+3=7 marks)

Question 7

- (a) Show that $\triangle ABD \sim \triangle BCD$.



- (b) Calculate the value of x . Express the answer with a rationalized denominator.

(2+4=6 marks)

Question 8

Suppose a fair “six sided” die is rolled three times and the number of “sixes” observed.

- (a) What is the probability p that a ‘six’ is observed on a single roll of the dice?
- (b) What are the possible values of the sample proportion $\hat{p}$ of “sixes” in the sample?
- (c) What is the probability that the sample proportion is equal to $\frac{1}{3}$.

(1+1+2=4 marks)

Question 9

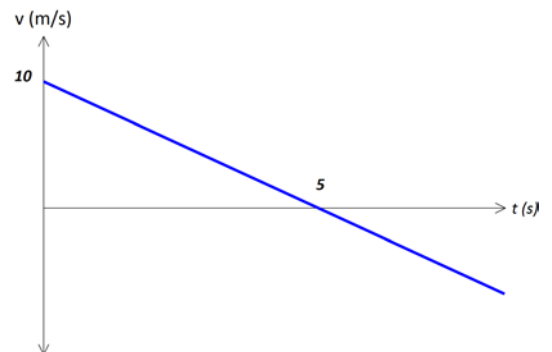
- (a) A sample of n people were asked whether education was important, and 80% said yes. What is the value of the sample proportion $\hat{p}$ for this sample?
- (b) A second sample of m people were asked whether education was important, and 95% said yes. What is the value of the sample proportion $\hat{p}$ for this sample?
- (c) Determine the sample proportion based on the combined results of both samples in terms of n and m .

(1+1+1=3 marks)

Question 10

A body initially at a point 4 metres to the right of the origin, undergoes constant acceleration resulting in the following velocity-time diagram.

- (a) By first deriving an equation for the velocity of the body v m/s at any time t seconds, $t \geq 0$, show that the position of the body x metre at any time t is given by the equation
- $$x = -t^2 + 10t$$



- (b) Calculate the average velocity over the first 8 seconds of motion.

(3 + 2 = 5 marks)

Question 11

The velocity, v m/s of a body moving in a straight line is given by $v(t) = 24 + 6t - 3t^2$, $t \geq 0$ where t is measured in seconds. If the body is initially 5m to the right of the origin, find:

- (a) the acceleration at any time, t
- (b) the position at any time, t
- (c) the time when the body is stationary
- (d) the position when it is stationary

(1+2+2+1=6 marks)

End of Exam

Formula Sheet

Coordinate Geometry

angle to the x-axis: $m = \tan \theta$

angle between two lines: $\tan \theta = \pm \frac{m_2 - m_1}{1 + m_1 m_2}$

distance between two points:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

midpoint: $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$

Vectors

$$\vec{r} = x\vec{i} + y\vec{j} \quad \vec{r} = |\vec{r}| \hat{r} \quad |\hat{r}| = \sqrt{x^2 + y^2}$$

$$\vec{r}_1 \cdot \vec{r}_2 = |\vec{r}_1| |\vec{r}_2| \cos \theta = x_1 x_2 + y_1 y_2$$

$$\vec{V}_{a \text{ rel } b} = \vec{V}_a - \vec{V}_b$$

Complex Numbers

$$z = x + yi = r(\cos \theta + i \sin \theta) = r \operatorname{cis} \theta$$

where $r = \sqrt{x^2 + y^2}$ and $\theta = \tan^{-1} \frac{y}{x}$

$$-\pi < \operatorname{Arg} z < \pi$$

$$z_1 z_2 = r_1 r_2 \operatorname{cis}(\theta_1 + \theta_2) \text{ and } \frac{z_1}{z_2} = \frac{r_1}{r_2} \operatorname{cis}(\theta_1 - \theta_2)$$

$$z^n = r^n \operatorname{cis}(n\theta) \text{ (de Moivre's Theorem)}$$

Trigonometry

$$\text{sine rule: } \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\text{cosine rule: } c^2 = a^2 + b^2 - 2ab \cos C$$

$$\text{area of a triangle: } A = \frac{1}{2}bh = \frac{1}{2}ab \sin C$$

$$\text{Heron's formula: } A = \sqrt{s(s-a)(s-b)(s-c)}$$

$$\text{where } s = \frac{1}{2}(a+b+c)$$

Reciprocal trigonometric functions:

$$\sec \theta = \frac{1}{\cos \theta} \quad \operatorname{cosec} \theta = \frac{1}{\sin \theta} \quad \cot \theta = \frac{1}{\tan \theta}$$

Fundamental trigonometric identities:

$$\cos^2 x + \sin^2 x = 1$$

$$1 + \tan^2 x = \sec^2 x$$

$$\cot^2 x + 1 = \operatorname{cosec}^2 x$$

Compound angle (Addition) formulae:

$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$$

$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$

$$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$$

Double angle formulae:

$$\sin 2x = 2 \sin x \cos x$$

$$\cos 2x = 2 \cos^2 x - 1$$

$$= 1 - 2 \sin^2 x$$

$$= \cos^2 x - \sin^2 x$$

$$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$$

$a \sin \theta \pm b \cos \theta = R \sin(\theta \pm \alpha)$ where:

$$R = \sqrt{a^2 + b^2} \text{ and } \tan \alpha = \frac{b}{a}$$

$$\sin \alpha = \frac{a}{\sqrt{a^2 + b^2}}, \cos \alpha = \frac{b}{\sqrt{a^2 + b^2}}$$

$a \sin \theta \pm b \cos \theta = R \cos(\theta \mp \alpha)$ where:

$$R = \sqrt{a^2 + b^2} \text{ and } \tan \alpha = \frac{b}{a}$$

$$\sin \alpha = \frac{a}{\sqrt{a^2 + b^2}}, \cos \alpha = \frac{b}{\sqrt{a^2 + b^2}}$$

Circle Mensuration

Arc length: $r\theta$

Chord length: $2r \sin\left(\frac{\theta}{2}\right)$

Sector area: $\frac{1}{2}r^2\theta$

Segment area: $\frac{1}{2}r^2(\theta - \sin \theta)$

Matrices

Determinant of the 2×2 matrix:

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$\det A = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

Inverse of a 2×2 matrix:

$$A^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \text{ where } \det(A) \neq 0$$

$$\text{Translation Matrix: } T_{(a,b)} = \begin{bmatrix} a \\ b \end{bmatrix}$$

Rotation Matrix

(anticlockwise about (0,0)):

$$R_\theta = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$$

Matrix for Reflection in the X axis:

$$M_x = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

Matrix for Reflection in the Y axis:

$$M_y = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

Matrix for Reflection in the line $y = x$:

$$M_{y=x} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

Matrix for dilation parallel to X axis by a factor

$$\text{of } m: D_x = \begin{bmatrix} m & 0 \\ 0 & 1 \end{bmatrix}$$

Matrix for dilation parallel to Y axis by a factor

$$\text{of } n: D_y = \begin{bmatrix} 1 & 0 \\ 0 & n \end{bmatrix}$$

Matrix for dilation about the origin by a factor

$$\text{of } k: D_k = \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$$

Kinematics

$$v = u + at$$

$$v^2 = u^2 + 2as$$

$$s = ut + \frac{1}{2}at^2$$

$$s = \frac{1}{2}(u + v)t$$

Statistics & Probability

$$\mu_{\bar{x}} = \mu$$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

Hypergeometric:

$$Pr(X = x) = \frac{{}^D C_x \times {}^{N-D} C_{n-x}}{{}^N C_n}$$

Binomial:

$$Pr(X = x) = {}^n C_x \times p^x \times (1-p)^{n-x}$$

Sequences and Series

Arithmetic:

$$t_n = a + (n-1)d$$

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

Geometric:

$$t_n = ar^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$S_\infty = \frac{a}{1-r}$$