



Glen Waverley Secondary College 2017

Specialist Mathematics Unit 1 Technology Access Examination



Student Name: _____

Student ID Code: _____ Form: _____

Teacher (circle): CAL PLG GMN

QUESTION AND ANSWER BOOK

Reading Time: 10 minutes
Writing Time: 120 minutes

Structure of Book

Section	Number of questions	Number of questions to be answered	Number of marks
MULTIPLE CHOICE	20	20	40
SHORT ANSWER	8	8	60

- Students are permitted to bring into the room: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring into the exam room: blank sheets of paper and/or white out liquid/tape.
- A CAS calculator, a scientific calculators and Bound Reference are allowed in this exam

Materials supplied

- Question and Answer booklet.
- Formula sheet (attached).

Instructions

- Write your **student name and student ID code** in the space provided above on this page.
- All written responses must be in English.
- Write your answers to the **MULTIPLE CHOICE questions** in the table provided on page 7.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

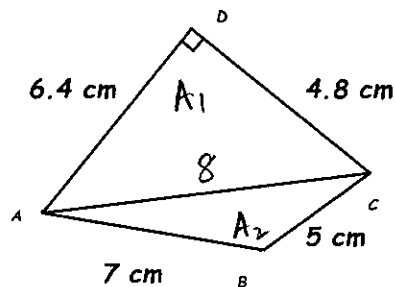
Question 1 The rational function $\frac{6x-8}{x^2-3x+2}$ expressed in terms of partial fractions is: A. $\frac{2}{x-1} + \frac{4}{x-2}$ B. $\frac{2}{x+1} + \frac{4}{x-2}$ C. $\frac{2}{x-1} + \frac{4}{x+2}$ D. $\frac{4}{x+1} + \frac{2}{x-2}$ E. $\frac{4}{x-1} + \frac{2}{x-2}$	<u>CAS</u>
Question 2 If $\frac{\frac{x-a}{a}-1}{a} = x+a$, where $a \neq 0, \pm 1$, then x is equal to: A. a B. $-a$ C. $\frac{a(a+2)}{a-1}$ D. $\frac{a^2+2}{1-a}$ E. $\frac{a(a^2+2)}{1-a^2}$	CAS
Question 3 If $z = x + iy$ is any non-zero complex number, which of the following is a false statement? A. $z + \bar{z}$ is a real number ✓ B. $z\bar{z}$ is a real number ✓ C. $z^{-1} = \frac{\bar{z}}{z\bar{z}}$ ✓ D. $i(z - \bar{z})$ is a real number ✓ E. $\frac{z}{\bar{z}}$ is a real number ✗	$z = x + yi$ $\bar{z} = x - yi$ $i(z - \bar{z}) = 2yi \times i = -2y.$ $\frac{x+yi}{x-yi} \times \frac{x+yi}{x+yi}$ $= \frac{x^2 - y^2 + 2xyi}{x^2 + y^2}$

Question 4 $\frac{1}{\sin x} + \frac{1}{i \cos x}$ is equal to: A. $\frac{1}{\operatorname{cis} x}$ ☒ B. $\frac{\operatorname{cis}(-x)}{\sin x \cos x}$ C. $\frac{\operatorname{cis}(-x)}{\sin(2x)}$ D. $\operatorname{cis}(-x)$ E. $\frac{\operatorname{cis} x}{i \sin x \cos x}$	$\frac{i \cos x + \sin x}{i \sin x \cos x}$ $= \frac{i(\cos x - \sin x)}{i \sin x \cos x}$ $= \frac{\operatorname{cis}(-x)}{\sin x \cos x}$
Question 5 When $\sqrt{3} - i$ is divided by $-1 - i$, the modulus and Argument of the quotient are, respectively: A. $2\sqrt{2}$ and $\frac{7\pi}{12}$ B. $\sqrt{2}$ and $\frac{-11\pi}{12}$ ☒ C. $\sqrt{2}$ and $\frac{7\pi}{12}$ D. $2\sqrt{2}$ and $\frac{-11\pi}{12}$ E. $\sqrt{2}$ and $\frac{11\pi}{12}$	$\sqrt{3} - i = 2 \operatorname{cis}\left(-\frac{\pi}{6}\right)$ $-1 - i = \sqrt{2} \operatorname{cis}\left(-\frac{3\pi}{4}\right)$ $\frac{2 \operatorname{cis}\left(-\frac{\pi}{6}\right)}{\sqrt{2} \operatorname{cis}\left(-\frac{3\pi}{4}\right)} = \sqrt{2} \operatorname{cis}\left(-\frac{\pi}{6} + \frac{3\pi}{4}\right)$ $= \sqrt{2} \operatorname{cis}\left(\frac{7\pi}{12}\right)$ OR <u>CAS</u>
Question 6 The modulus of the complex number $\frac{(\operatorname{cis}(\frac{2\pi}{5}))^3}{2i(1 + \sqrt{3}i)}$ is: A. 4 B. 2 C. 1 D. $\frac{1}{2}$ ☒ E. $\frac{1}{4}$	$\frac{1 \operatorname{cis}\left(\frac{6\pi}{5}\right)}{4 \operatorname{cis}\left(\frac{5\pi}{6}\right)}$ $\operatorname{Mod}(z) = \frac{1}{4}$ $\operatorname{Arg}(z) = \frac{11\pi}{30}$

Question 7

The area of the quadrilateral ABCD (shown below) is closest to which one of the following?

- A. 24 cm^2
- B. 27 cm^2
- C. 28 cm^2
- D. 31 cm^2
- ☒ E. 33 cm^2



$$s = \frac{1}{2}(5+7+8) = 10$$

$$AC = \sqrt{6.4^2 + 4.8^2} = 8$$

$$A_1 = \frac{1}{2} \times 6.4 \times 4.8$$

$$= \frac{384}{25}$$

$$A_2 = \sqrt{10(2)(3)(5)}$$

$$= \sqrt{300}$$

$$\text{Total} = \sqrt{300} + \frac{384}{25}$$

$$\approx 32.68$$

Question 8

The expression $\frac{\sin(\pi + \theta) \times \cos(3\pi - \theta)}{\cos(-\theta) \times \cos(\pi + \theta)}$ can be simplified to which of the following?

- A. $-\sin \theta \times \cos \theta$
- B. $\sin \theta \times \cos \theta$
- C. $\tan \theta$
- ☒ D. $-\tan \theta$
- E. $\frac{\cos \theta}{\sin \theta}$

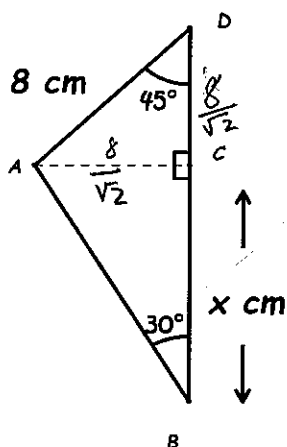
$$\frac{-\sin(\theta) \cdot (-\cos \theta)}{\cos(\theta) \cdot (-\cos \theta)}$$

$$= -\tan(\theta)$$

Question 9

The value of x in the diagram above is which of the following?

- A. $4\sqrt{2} \text{ cm}$
- ☒ B. $4\sqrt{6} \text{ cm}$
- C. 6 cm
- D. $8\sqrt{6} \text{ cm}$
- E. $8\sqrt{2} \text{ cm}$



$$\tan 30^\circ = \frac{8/\sqrt{2}}{x}$$

$$x = \frac{8/\sqrt{2}}{\tan 30^\circ}$$

$$= \frac{8}{\sqrt{2}} \times \frac{3}{\sqrt{3}}$$

$$= \frac{24}{\sqrt{6}}$$

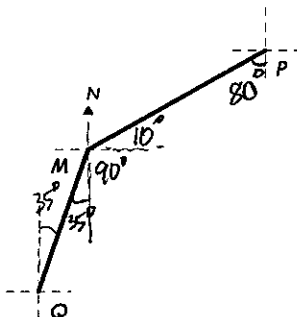
$$= \frac{24\sqrt{6}}{6}$$

$$= 4\sqrt{6}$$

Question 10

The diagram shows the position of two points P and Q relative to M . If the bearing of M from P is $260^\circ T$ and the bearing of M from Q is $35^\circ T$, the size of the obtuse angle PMQ is:

- A. 115°
B. 120°
C. 135°
D. 150°
E. 165°

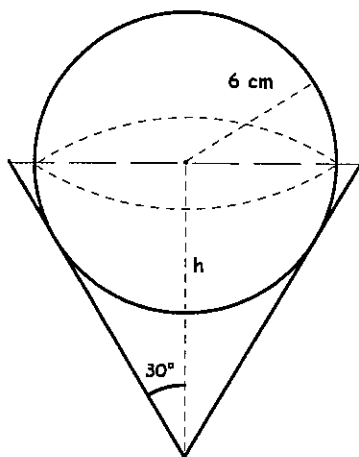


$$\begin{aligned} \angle PMQ &= 10^\circ + 90^\circ + 35^\circ \\ &= 135^\circ \end{aligned}$$

Question 11

A sphere of radius 6 cm fits into an inverted cone with a semi-vertex angle of 30° . If the centre of the sphere and the centre of the base of the cone coincide, the height of the cone is which one of the following:

- A. 6 cm
B. 8 cm
C. 9 cm
D. 12 cm
E. 15 cm



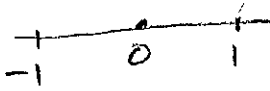
$$\begin{aligned}\sin 30^\circ &= \frac{6}{h} \\ h &= \frac{6}{\sin 30^\circ} \\ &= 12\end{aligned}$$

Question 12

The graph of $y = \frac{1}{x^2 + 2x + k}$ will have two vertical asymptotes provided

- A. $k = 1$
B. $k > 1$
C. $k \leq 1$
D. $k < 1$
E. $k \geq 1$

$$\begin{aligned}\Delta &= b^2 - 4ac \\ &= 4 - (4 \times 1 \times k) \\ &= 4 - 4k \\ \Delta &> 0 \\ 4(1-k) &> 0 \\ 4k &< 4 \\ k &< 1\end{aligned}$$

Question 13 An ellipse has parametric equations $x = 4 \cos t$ and $y = 3 \sin t$. The coordinates of the x-axis intercepts of its graph are A. $(-4, 0)$ and $(-3, 0)$ B. $(-3, 0)$ and $(3, 0)$ C. $(0, -4)$ and $(0, 4)$ ☒ D. $(-4, 0)$ and $(4, 0)$ E. $(4, 0)$ and $(3, 0)$	$\left(\frac{x}{4}\right)^2 + \left(\frac{y}{3}\right)^2 = 1$ $\frac{x^2}{16} + \frac{y^2}{9} = 1$ $\frac{x^2}{16} = 1$ $x = \pm 4$
Question 14 The locus of the points $P(x, y)$ which satisfies the property that $PA + PB = 3$ where A is the point $(1, 0)$ and B is the point $(-1, 0)$ is: A. a straight line segment ☒ B. an ellipse C. a circle D. a hyperbola E. a parabola	
Question 15 What is the difference between the 4th and the 10th term of the arithmetic sequence $t: \{6, 11, 16, \dots\}$? A. 20 B. 21 ☒ C. 30 D. 51 E. 72	$a = 6$ $d = 5$ $t_4 = 6 + (3 \times 5) = 21$ $t_{10} = 6 + (9 \times 5) = 51$ $t_{10} - t_4 = 30$
Question 16 For a geometric sequence, the fourth term is 4 and the seventh term is 32. The second term in the sequence is: A. 0 B. $\frac{1}{2}$ ☒ C. 1 D. 2 E. 4	$t_4 = 4 = ar^3$ $t_7 = 32 = ar^6$ $\frac{r^6}{r^3} = \frac{32}{4}$ $r^3 = 8$ $r = 2$ $a = \frac{1}{2}$ $t_2 = ar = \frac{1}{2} \times 2 = 1$

Question 17

The sum of an infinite geometric sequence is 7.5, with the common ratio equal to 0.5. The sum of the first three terms of the geometric sequence is closest to:

- A. 6.5
- ☒ B. 6.6
- C. 7.4
- D. 7.5
- E. 7.6

$$S_{\infty} = \frac{a}{1-r} = 7.5 \quad r = 0.5$$

$$\frac{a}{0.5} = 7.5$$

$$a = 3.75$$

$$S_3 = \frac{a(r^3 - 1)}{r - 1}$$

$$= \frac{3.75(0.5^3 - 1)}{-0.5}$$

$$= 6.5625$$

Question 18

The sum of the first n terms of the geometric series $1 - 3x + 9x^2 - 27x^3 + \dots$ is:

- A. $3000x$
- B. $(-3x)^{n-1}$
- ☒ C. $\frac{(-3x)^n - 1}{-3x - 1}$
- D. $\frac{(-3x)^n}{3x(3x - 1)}$
- E. $\frac{(-3x)^{n-1}}{3x(3x - 1)}$

$$a = 1$$

$$r = -3x$$

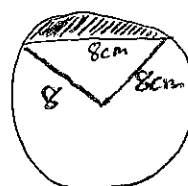
$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$= \frac{1((-3x)^n - 1)}{(-3x) - 1}$$

Question 19

A chord of length 8 cm is drawn in a circle of radius 8 cm. The area of the smaller region inside the circle cut off by the chord is:

- A. $16(2\pi - 1) \text{ cm}^2$
- ☒ B. $32\left(\frac{\pi}{3} - \frac{\sqrt{3}}{2}\right) \text{ cm}^2$
- C. $16\left(\frac{\pi}{6} - \frac{1}{2}\right) \text{ cm}^2$
- D. 13.6 cm^2
- E. 38 cm^2



$$A = \frac{1}{2} r^2 (\theta - \sin \theta)$$

$$= \frac{1}{2} \times 8^2 \left(\frac{\pi}{3} - \sin \frac{\pi}{3} \right)$$

$$= 32 \left(\frac{\pi}{3} - \frac{\sqrt{3}}{2} \right)$$

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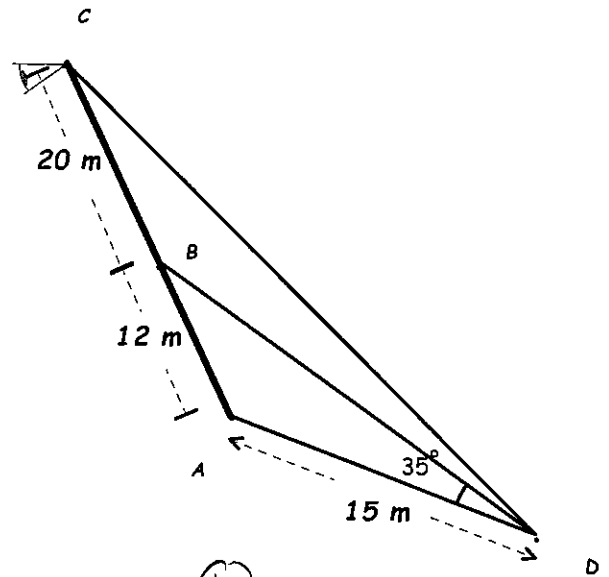
Question 20	
Which one of the following statements is false? A. $\cos \theta = \cos (90^\circ + \theta)$ B. $\cos \theta = \cos (-\theta)$ ✓ C. $\cos \theta = \frac{1}{\sec \theta}$ ✓ D. $\cos \theta = \sin \theta \cot \theta$ ✓ E. $\sec^2 \theta = 1 + \tan^2 \theta$ ✓	$\cos \theta = \cancel{\sin \theta} \times \frac{\cos \theta}{\cancel{\sin \theta}}$

Multiple Choice Answer Table									
1	2	3	4	5	6	7	8	9	10
A	E	E	B	C	E	E	D	B	C
11	12	13	14	15	16	17	18	19	20
D	D	D	B	C	C	B	C	B	A

SHORT ANSWER/EXTENDED RESPONSE QUESTIONS

Question 1 (6 marks)

A new style of street-light is being installed on sloping ground (as shown). It will be supported by two wires, one attached at a point 12 m from the base of the pole (at ground level), and the other at the top of the pole, 32 m from the base. The lower wire forms an angle of 35° with the ground, and is secured at a point 15 m along the sloping ground from the base to the pole.



- a) Show that $\angle ABD = 45^\circ 48'$ to the nearest minute.

$$\frac{15}{\sin B} = \frac{12}{\sin 35^\circ}$$

$$\theta = \sin^{-1} \left(\frac{15 \times \sin 35^\circ}{12} \right)$$

$$= 45^\circ 48' \text{ as required.}$$

(1)

(1)

- b) Find the length of the lower cable BD correct to three decimal places.

$$\angle BAD = 180^\circ - 35^\circ - \angle ABD$$

$$= 99.1951^\circ$$

$$BD^2 = 12^2 + 15^2 - 2 \times 12 \times 15 \cos \angle BAD$$

$$BD = 20.653 \text{ m.}$$

(1/2)

(1/2)

(1)

OR

$$\frac{BD}{\sin \angle BAD} = \frac{12}{\sin 35^\circ}$$

$$BD = \frac{12 \sin 99.1951^\circ}{\sin 35^\circ}$$

$$= 20.653 \text{ m}$$

(1)

(1)

- c) Calculate (to an accuracy of three decimal places) the length of the upper cable CD.

$$\angle CBD = 180^\circ - \angle ABD$$

$$= 134.1951^\circ$$

$$CD^2 = 20^2 + BD^2 - 2 \times 20 \times BD \cos \angle CBD$$

$$CD = 37.449 \text{ m}$$

(1/2)

(1/2)

(1)

OR

$$CD^2 = 32^2 + 15^2 - 2 \times 32 \times 15 \times \cos 99.1951^\circ$$

$$CD^2 = 1402.403$$

$$CD = 37.449 \text{ m}$$

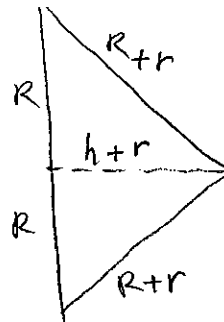
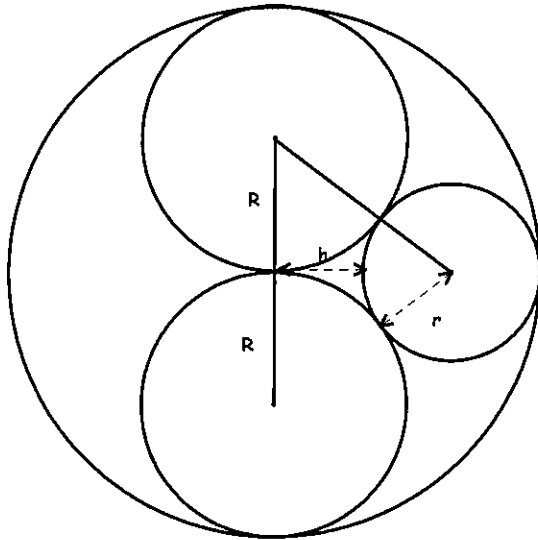
(1)

(1)

(2+2+2=6marks)

Question 2 (5 marks)

When two circles of radius R are drawn inside a larger circle of radius $2R$, a smaller circle (radius r) can also be drawn inside the larger circle. The smaller circle touches the larger circle and the circle with radius R as shown in the diagram.



- a) Use Pythagoras' Theorem to write an equation connecting R , h and r .

$$(R+r)^2 = R^2 + (h+r)^2 \quad (1)$$

$$R^2 + 2Rr + r^2 = R^2 + r^2 + 2rh + h^2$$

$$2Rr = 2rh + h^2 \quad \text{or} \quad R = \frac{2rh + h^2}{2r} \quad (1)$$

- b) Write another equation expressing the radius of the large circle R in terms of h and r .

Large Circle radius = $2R$

$$2R = h + 2r \quad \text{or} \quad R = \frac{1}{2}(h + 2r) \quad (1)$$

- c) Hence calculate in terms of R , the value of the radius of the small circle.

$$h = 2R - 2r$$

Sub into $2Rr = 2rh + h^2$ (1/2)

$$2Rr = 2r(2R - 2r) + (2R - 2r)^2$$

$$2Rr = 4Rr - 4r^2 + 4R^2 - 8rR + 4r^2$$

$$4R^2 - 6Rr = 0 \quad (1/2)$$

$$2R(2R - 3r) = 0$$

As $R > 0$ $2R - 3r = 0$ (1)

$$\therefore r = \frac{2}{3}R$$

(Other method is possible)

(2 + 1 + 2 = 5 marks)

Question 3 (13 marks)

Given $z = 1 + 2i$ and $w = 2 - i$

a) Show that

i) $\overline{z - w} = \bar{z} - \bar{w}$

$$\begin{aligned} \text{LHS} &= \overline{(1+2i) - (2-i)} \\ &= \overline{-1+3i} \\ &= -1-3i \quad (1) \end{aligned}$$

$$\begin{aligned} \text{RHS} &= (1-2i) - (2+i) \\ &= -1-3i \quad (1) \end{aligned}$$

$\therefore \text{LHS} = \text{RHS}$ as required

ii) $\overline{\left(\frac{z}{w}\right)} = \frac{\bar{z}}{\bar{w}}$

$$\begin{aligned} \frac{\bar{z}}{\bar{w}} &= \frac{1-2i}{2+i} \\ &= i \end{aligned}$$

$$\text{LHS} = \overline{\left(\frac{z}{w}\right)} = -i \quad (1)$$

$$\begin{aligned} \text{RHS} &= \frac{\bar{z}}{\bar{w}} \\ &= \frac{1-2i}{2+i} \\ &= -i \quad (1) \end{aligned}$$

$\text{LHS} = \text{RHS}$ as required.

b) Express $\frac{z}{w}$ in polar form.

$$\frac{z}{w} = \frac{1+2i}{2-i} \times \frac{2+i}{2+i}$$

(1)

$$= i$$

$$= \text{cis} \left(\frac{\pi}{2} \right)$$

(1)

c) Hence express $\frac{\bar{z}}{\bar{w}}$ in polar form.

$$\frac{\bar{z}}{\bar{w}} = \overline{\left(\frac{z}{w}\right)}$$

$$= \text{cis} \left(-\frac{\pi}{2} \right)$$

(1)

- d) If $z = r_1 \operatorname{cis} \theta$ and $w = r_2 \operatorname{cis} \phi$ show that $r_1 = r_2$ and that $\theta - \phi = \frac{\pi}{2}$

$$\frac{z}{w} = \frac{r_1 \operatorname{cis} \theta}{r_2 \operatorname{cis} \phi} = \operatorname{cis} \left(\frac{\pi}{2} \right) \quad \left(\frac{1}{2} \right)$$

$$\frac{r_1}{r_2} = 1 \quad \left(\frac{1}{2} \right)$$

$$\therefore r_1 = r_2$$

and

$$\theta - \phi = \frac{\pi}{2} \quad (1)$$

as required

$$\text{OR } \left. \begin{aligned} z &= \sqrt{5} \operatorname{cis} (\tan^{-1}(2)) \\ w &= \sqrt{5} \operatorname{cis} (\tan^{-1}(\frac{1}{2})) \end{aligned} \right\} r_1 = r_2 = \sqrt{5} \quad (1)$$

$$\tan \theta = 2 \quad \text{and} \quad \tan \phi = \frac{1}{2}$$

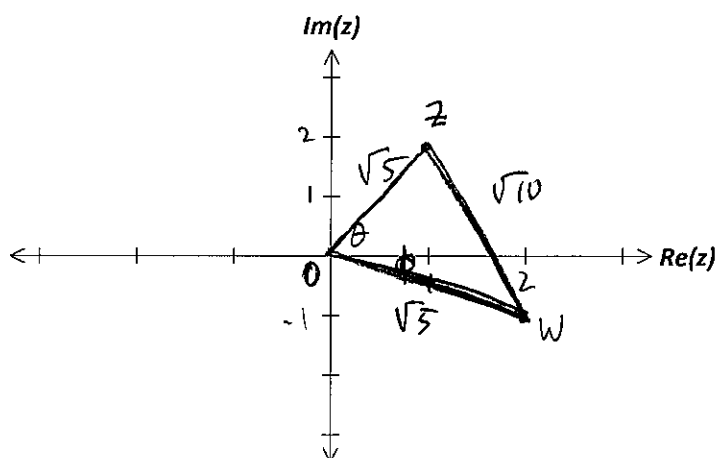
$$\tan(\theta - \phi) = \frac{\tan \theta - \tan \phi}{1 + \tan \theta \tan \phi}$$

$$= \frac{2 - \frac{1}{2}}{1 + 2 \times \frac{1}{2}}$$

$$= \text{undefined}$$

$$\therefore \theta - \phi = \frac{\pi}{2} \quad \text{as required}$$

- e) On the complex plane below draw the triangle OZW whose vertices are the complex numbers $0+0i$, z and w .



1m for correct points

1m for drawing as triangle

(deduct mark if scale of axis not shown)

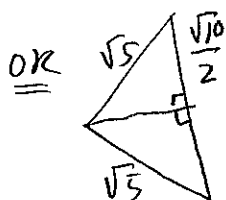
- f) Calculate the area of triangle OZW

$$\overline{OZ} = |z| = \sqrt{5}$$

$$\overline{OW} = |w| = \sqrt{5}$$

$$\angle ZOW = \theta - \phi = \frac{\pi}{2}$$

$$\left. \begin{aligned} \overline{OZ} &= |z| = \sqrt{5} \\ \overline{OW} &= |w| = \sqrt{5} \\ \angle ZOW &= \theta - \phi = \frac{\pi}{2} \end{aligned} \right\} \begin{aligned} A &= \frac{1}{2} \times \sqrt{5} \times \sqrt{5} \times \sin \frac{\pi}{2} \quad (1) \\ &= \frac{5}{2} \text{ unit}^2 \quad (1) \end{aligned}$$



$$h = \sqrt{5 - \frac{10}{4}} = \sqrt{\frac{5}{2}}$$

$$A = \frac{1}{2} \times b \times h$$

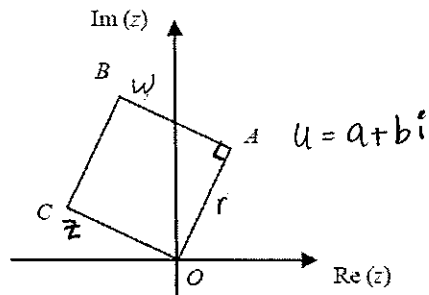
$$= \frac{1}{2} \times \sqrt{10} \times \sqrt{\frac{5}{2}}$$

$$= \frac{5}{2} \text{ unit}^2$$

((2+2)+2+1+2+2+2=13 marks)

Question 4 (5 marks)

On the Argand diagram, $OABC$ forms a square. A , B and C represent the complex numbers u , w and z respectively.

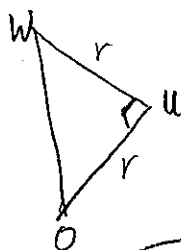


a) By letting $u = a + bi$, show that $w = (1 + i)u$.

$$(1+i)u = (1+i)(a+bi) \\ = (a-b) + (a+b)i \quad \left(\frac{1}{2}\right)$$

$$|(1+i)u| = \sqrt{(a-b)^2} + \sqrt{(a+b)^2} \\ = \sqrt{2(a^2+b^2)} \quad \left(\frac{1}{2}\right)$$

$$|u| = r = \sqrt{a^2+b^2} \quad \left(\frac{1}{2}\right) \\ \overline{AB} = r \text{ (square)}$$



$$r^2 + r^2 = |w|^2 \\ (\sqrt{a^2+b^2})^2 + (\sqrt{a^2+b^2})^2 = |w|^2 \\ 2(a^2+b^2) = |w|^2 \quad \left(\frac{1}{2}\right)$$

$$|w| = \sqrt{2(a^2+b^2)} \quad \left(\frac{1}{2}\right)$$

$$\text{Since } |w| = |(1+i)u|$$

$$w = (1+i)u \text{ as required.}$$

Other method is possible

b) If $u = 5 + 12i$ find;

(i) w

$$w = (1+i)(5+12i) \\ = 5 + 12i + 5i - 12 \\ = -7 + 17i \quad \textcircled{1}$$

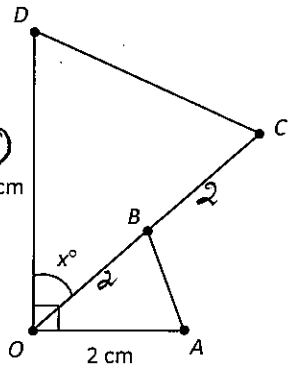
(ii) the complex number representing M , the midpoint of AB .

$$\text{midpoint } AB = \left(\frac{-7+5}{2}\right) + \left(\frac{17+12}{2}\right)i \quad \left(\frac{1}{2}\right) \\ = -1 + \frac{29}{2}i \quad \left(\frac{1}{2}\right)$$

(3+1+1=5 marks)

Question 5 (9 marks)

The diagram shows an isosceles triangle AOB , with $OA = OB = 2$ cm, and an isosceles triangle COD , with $OC = OD = 4$ cm. Angle $AOD = 90^\circ$ and angle $COD = x^\circ$. The sum of the areas of the triangles AOB and COD is s cm².



- a) Show that $s = 2 \cos x + 8 \sin x$.

$$\text{Area } \triangle OCB = \frac{1}{2} \times 4 \times 4 \sin x = 8 \sin x$$

$$\text{Area } \triangle OAB = \frac{1}{2} \times 2 \times 2 \sin(90^\circ - x) = 2 \cos x$$

$$\therefore S = \triangle OAB + \triangle OCB = 2 \cos x + 8 \sin x$$

as required

- b) Express s in the form $R \sin(x + \alpha)$, stating the value of R and α in exact values.

$$R = \sqrt{8^2 + 2^2} = 2\sqrt{17}$$

$$\left(\frac{1}{2}\right)$$

$$\therefore 2 \cos x + 8 \sin x = 2\sqrt{17} \sin\left(x + \tan^{-1}\left(\frac{1}{4}\right)\right)$$

$$\tan \alpha = \frac{2}{8} = \frac{1}{4}$$

$$\alpha = \tan^{-1}\left(\frac{1}{4}\right)$$

$$\left(\frac{1}{2}\right)$$

$$\text{OR } 2\sqrt{17} \sin\left(x + \sin^{-1}\left(\frac{1}{\sqrt{17}}\right)\right)$$

$$\text{OR } 2\sqrt{17} \sin\left(x + \cos^{-1}\left(\frac{4\sqrt{17}}{17}\right)\right)$$

- c) Given that x can vary, find the following and give your answers **in degrees** to 2 decimal places.

- (i) that value of x for which $s = 6$.

$$2\sqrt{17} \sin\left(x + \tan^{-1}\left(\frac{1}{4}\right)\right) = 6$$

$$x = 32.65^\circ$$

$$\left(1\right)$$

- (ii) the maximum value of s and the corresponding value of x

$$s = 2\sqrt{17} \text{ max.}$$

$$\left(1\right)$$

$$\sin\left(x + \tan^{-1}\left(\frac{1}{4}\right)\right) = 1$$

$$x = 75.96^\circ$$

$$\left(1\right)$$

- (iii) the value of x for which the area of triangle AOB is half the area of triangle COD .

$$\triangle AOB = \frac{1}{2} \triangle COD$$

$$2 \cos x = \frac{1}{2} \times 8 \sin x$$

$$\left(\frac{1}{2}\right)$$

$$2 \cos x = 4 \sin x$$

$$\left(\frac{1}{2}\right)$$

$$\tan x = \frac{1}{2}$$

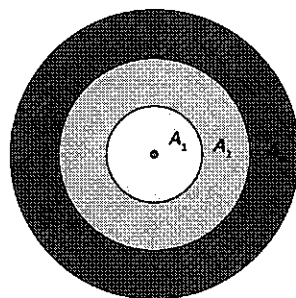
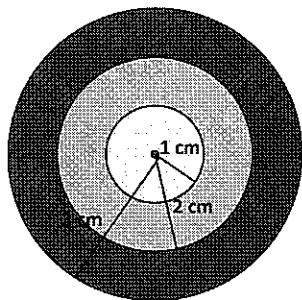
$$x = 26.57^\circ$$

$$\left(1\right)$$

(2+2+1+2+2=9 marks)

Question 6 (10 marks)

- a) A circular board is divided into concentric circles of radii 1cm, 2cm, 3cm, etc as shown. Show that the areas of the regions form an arithmetic sequence. Note that A_2 does not include A_1 and A_3 does not include A_2 .



$$A_1 = \pi \times 1^2 = \pi$$

$$A_2 = (\pi \times 2^2) - A_1 = 3\pi$$

$$A_3 = (\pi \times 3^2) - (\pi \times 2^2) = 5\pi$$

$$a = \pi$$

$$d = 3\pi - \pi = 5\pi - 3\pi = 2\pi.$$

Hence, Area forms arithmetic sequence.

① must show both

$$\text{OR } A_{n+1} = \pi(n+1)^2 - \pi(n)^2 = \pi(2n+1)$$

$$A_n = \pi n^2 - \pi(n-1)^2 = \pi(2n-1)$$

$$d = \pi[(2n+1) - (2n-1)] = 2\pi.$$

- b) Another board is designed such that its areas form a geometric sequence.

- i) Find the common ratio.

$$A_1 = \pi$$

$$A_2 = \pi \times \left(\frac{\sqrt{6}}{2}\right)^2 - \pi = \frac{\pi}{2}$$

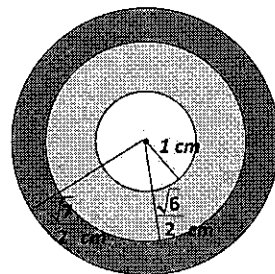
$$A_3 = \pi \times \left(\frac{\sqrt{7}}{2}\right)^2 - \pi \left(\frac{\sqrt{6}}{2}\right)^2 = \frac{\pi}{4}$$

(must show both)

$$r = \frac{A_3}{A_2} = \frac{A_2}{A_1} = \frac{1}{2} \quad \text{①}^m \quad \therefore \text{common ratio} = \frac{1}{2}$$

- ii) Find the total area of the board if this pattern continues forever.

$$S_{\infty} = \frac{a}{1-r} = \frac{\pi}{1-\frac{1}{2}} = 2\pi.$$



① A

- c) A third board is designed such that each area is an equilateral triangle. The inner triangle has side length 2 cm . The next triangle has side length $x\text{ cm}$. Given that the area of an equilateral triangle can be found using $A = \frac{\sqrt{3}}{4} a^2$, where a is the side length, then find the value of x if the **areas** have a common ratio of $\frac{1}{2}$.

$$A_1 = \frac{\sqrt{3}}{4} \times 2^2 = \sqrt{3} \quad \left(\frac{1}{2}\right)$$

$$A_2 = \left(\frac{\sqrt{3}}{4} \cdot x^2\right) - \sqrt{3} = \sqrt{3} \left(\frac{x^2}{4} - 1\right) \quad \left(\frac{1}{2}\right)$$

$$r = \frac{A_2}{A_1} = \frac{1}{2}$$

$$A_2 = \frac{1}{2} A_1 \quad \left(\frac{1}{2}\right)$$

Hence,

$$\sqrt{3} \left(\frac{x^2}{4} - 1\right) = \frac{\sqrt{3}}{2}$$

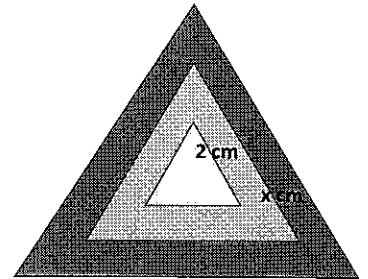
$$\frac{x^2}{4} = 1 + \frac{1}{2}$$

$$x^2 = 6$$

$$x = \sqrt{6}$$

$$\text{as } x > 0 \quad (1)$$

(3 + 3 + 1 + 3 = 10 marks)



Question 7 (8 marks)

In this question, all distances are given in million km.

A particular planet P_1 's orbit around its star has the locus definition $SP + BP = 12$, where the star has co-ordinates $S = (-3, 0)$, and $B = (3, 0)$.

- a) Find a Cartesian equation for the orbit of the planet.

$$\sqrt{(x+3)^2 + y^2} + \sqrt{(x-3)^2 + y^2} = 12 \quad (1)$$

$$x^2 + 6x + 9 + y^2 = 144 - 24\sqrt{(x-3)^2 + y^2} + x^2 - 6x + 9 + y^2$$

$$24\sqrt{(x-3)^2 + y^2} = -12x + 144$$

$$2\sqrt{(x-3)^2 + y^2} = -x + 12 \quad (1)$$

$$4x^2 - 24x + 36 + 4y^2 = x^2 - 24x + 144$$

$$3x^2 + 4y^2 = 108$$

$$\frac{x^2}{36} + \frac{y^2}{27} = 1 \quad (1)$$

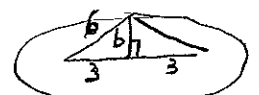
$$a = 6$$

$$b = \sqrt{27} \approx 5.2$$

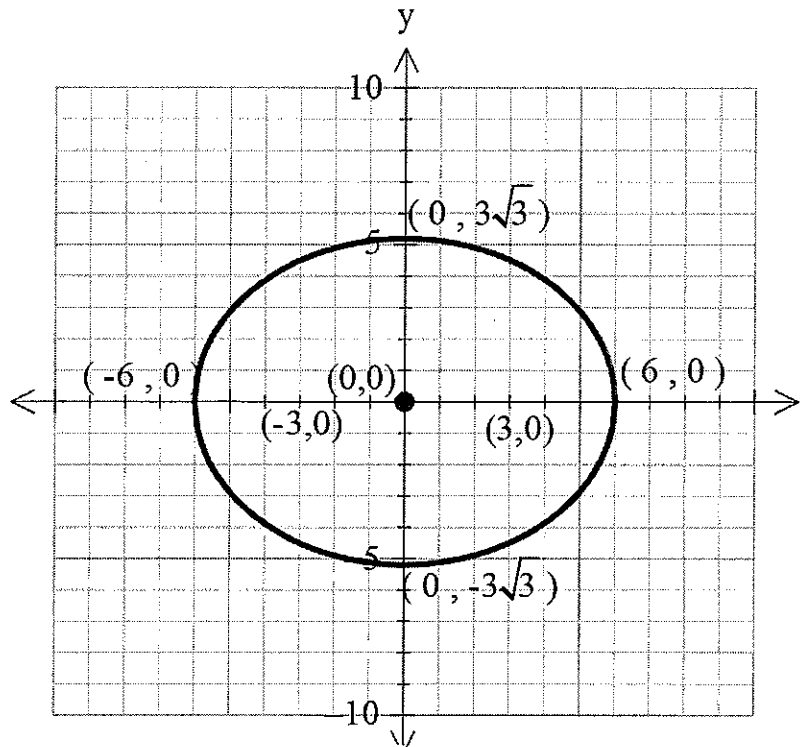
OR $d + 3 + d - 3 = 12$
 $d = 6$

$$b^2 + 9 = 36$$

$$b = \pm\sqrt{27} = \pm 3\sqrt{3}$$



b) Sketch the planet's orbit. Label all intercept points, foci and centre.



1m - shape

1m - intercepts

1m - foci + centre

c) Another planet P_2 has a circular orbit where $SP = 1.5$. It takes 300 days for P_2 to complete its orbit. Assuming it travels at a constant speed, what distance will it travel in 100 days? Give your answer to the nearest kilometre.

300 days on full circle

$$\begin{aligned} l &= 2\pi r \\ &= 2\pi \times 1.5 \\ &= 3\pi \end{aligned} \quad (1)$$

100 days is $\frac{1}{3}$ of circle

$$l = \pi \text{ million km} \quad (1)$$

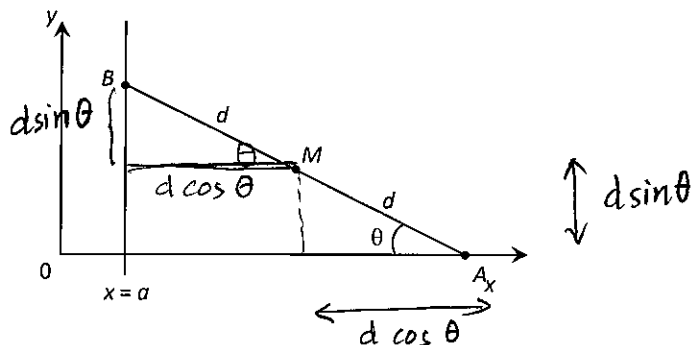
(3+3+2=8 marks)

OR

$$\begin{aligned} l &= \frac{2\pi r}{3} \\ &= \frac{2 \times \pi \times 1.5}{3} \\ &= \pi \text{ million km} \end{aligned}$$

Question 8 (4 marks)

The diagram shows a rod AB of length $2d$ units. The end A moves along the x axis and the end B moves along the line $x = a$. AB makes an angle θ with the x axis.



- a) Show that the coordinates of M are $(a + d \cos \theta, d \sin \theta)$.

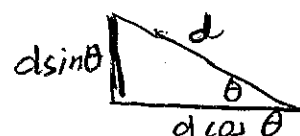
$$M_x = a + d \cos \theta$$

$$M_y = d \sin \theta$$

$$\therefore M = (a + d \cos \theta, d \sin \theta)$$

(1)

(1)



(Diagram can be used as workings)

- b) As the rod slips, θ changes. Let $x = a + d \cos \theta$ and $y = d \sin \theta$ be the changing coordinates of M . Make $\cos \theta$ and $\sin \theta$ the subject of these equations, and square and add. Find the Cartesian equation of the locus of M as θ varies.

$$\cos \theta = \frac{x - a}{d}$$

$$\sin \theta = \frac{y}{d}$$

$$\cos^2 \theta + \sin^2 \theta = 1$$

$$\frac{(x - a)^2}{d^2} + \frac{y^2}{d^2} = 1$$

$$(x - a)^2 + y^2 = d^2$$

$$C = (a, 0)$$

$$r = d$$

(1/2)

(1/2)

(1/2)

(1/2)

(2+2=4 marks)

End of Exam