

Multiple choice



- The ratio into which the point $(4, 6)$ divides the line segment between the points $A(-2, -2)$ and $B(28, 38)$ closer to A is:
A 1:5 B 2:3 C 3:2 D 1:4 E 4:1
- The equation of the perpendicular bisector of the line segment between the points $(2, 4)$ and $(-2, -4)$ is:
A $y = \frac{1}{2}x$ B $y = -\frac{1}{2}x + 2$ C $y = \frac{1}{2}x - 2$ D $y = -\frac{1}{2}x$ E $y = \frac{1}{2}x - \frac{1}{2}$
- The point of intersection between the lines with equations $3y - 2x + 15 = 0$ and $4y - x - 24 = 0$ is:
A $(26\frac{3}{5}, 12\frac{3}{5})$ B $(26\frac{4}{5}, 12\frac{3}{5})$ C $(26\frac{2}{5}, 12\frac{4}{5})$ D $(26\frac{2}{5}, 12\frac{3}{5})$ E $(26\frac{1}{5}, 12\frac{4}{5})$
- The x - and y -intercepts of the line with equation $2x - 3y - 4 = 0$ respectively are:
A $(2, 0), (0, -3)$ B $(-2, 0), (0, 3)$ C $(2, 0), (0, -\frac{4}{3})$
D $(-2, 0), (0, \frac{4}{3})$ E $(-2, 0), (0, -\frac{4}{3})$
- The midpoint of the line segment that joins the points $(a, -b)$ and $(2a, -3b)$ is:
A $(\frac{3a}{2}, -2b)$ B $(\frac{3a}{2}, -b)$ C $(\frac{a}{2}, -b)$ D $(\frac{a}{2}, -2b)$ E $(\frac{3a}{2}, -4b)$
- The midpoint of the line segment AB where $A(-1, -4)$ and $B(-4, 6)$ is:
A $(-5, 2)$ B $(-2\frac{1}{2}, -1)$ C $(2\frac{1}{2}, 1)$ D $(-2\frac{1}{2}, 1)$ E $(2\frac{1}{2}, -1)$
- The vertices of a triangle are $A(2, 4)$, $B(-2, 2)$ and $C(3, 6)$. The perimeter of the triangle is:
A $\sqrt{41} + \sqrt{5}$ B $\sqrt{41} + 2\sqrt{5}$ C $\sqrt{41} + 3\sqrt{5}$ D $\sqrt{41} + 4\sqrt{5}$ E $\sqrt{41} + 5\sqrt{5}$
- The acute angle that the line with equation $y = -4x + 2$ makes with the positive direction of the x -axis to the nearest degree is:
A 63° B 75° C 76° D 105° E 104°
- The obtuse angle between lines with equations $2x + 3y - 10 = 0$ and $x - 3y + 10 = 0$ to the nearest degree is:
A 126° B 127° C 128° D 129° E 130°

- 10 The equation of the line with gradient -2 which passes through the point $(-2, -3)$ is:
 A $2x - y - 7 = 0$ B $2x + y - 7 = 0$ C $2x - y + 7 = 0$
 D $2x + y + 7 = 0$ E $-2x + y + 7 = 0$
- 11 The equation of the line that passes through the points $(-2, 4)$ and $(3, -4)$ is:
 A $8x + 5y - 4 = 0$ B $8x - 5y - 4 = 0$ C $8x - 5y + 4 = 0$
 D $-8x + 5y - 4 = 0$ E $-8x - 5y - 4 = 0$
- 12 The equation of the line with x -intercept 4 and y -intercept -10 is:
 A $5x + 2y - 20 = 0$ B $5x - 2y + 20 = 0$ C $5x - 2y - 20 = 0$
 D $-5x + 2y - 20 = 0$ E $-5x - 2y + 20 = 0$
- 13 The straight-line distance between the x - and y -intercepts of the line with equation $2x - 3y - 12 = 0$ is:
 A $2\sqrt{15}$ B $2\sqrt{13}$ C $4\sqrt{15}$ D $4\sqrt{13}$ E $2\sqrt{5}$
- 14 The point that divides the line segment AB in the ratio $1:2$ closer to A is:
 A $(-2, 6)$ B $(0, 5)$ C $(-3, 6\frac{1}{2})$ D $(-1, 5\frac{1}{2})$ E $(1, 4\frac{1}{2})$

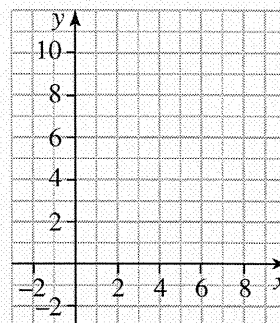
Short answer

- Find the equation of the perpendicular bisector of the line joining the points $(-5, 2)$ and $(1, 6)$ and hence find its axis intercepts.
- The coordinates of three points are $A(1, 9)$, $B(-3, -5)$ and $C(4, 2)$. Find the three positions of the point D that allow the points to be joined to form a parallelogram.
- Find the equation of the line whose points are all equidistant from the points $(2, 4)$ and $(8, -2)$.
- A triangle ABC has vertices with coordinates $A(4, 8)$, $B(-8, 3)$ and $C(-1, -4)$.
 - Show that the triangle ABC is isosceles.
 - Find the midpoint M of $\overline{BC}$ and show that the lines $\overline{BC}$ and $\overline{AM}$ are perpendicular.
- Find the gradient of the line L_1 with equation $3y - 2x + 7 = 0$.
 - Find the equation of the straight line that passes through the point $(1, 1)$ and is:
 - parallel to L_1
 - perpendicular to L_1
- Find the value of k so that:
 - the line with equation $2x + ky - 10 = 0$ passes through the point $(3, 1)$
 - the line with equation $kx - 5y - 11 = 0$ has a gradient of $\frac{3}{10}$
 - the line with equation $2x + 3y - k = 0$ has a y -intercept of -4
- Find the acute angle between the line with equations $x - 2y + 2 = 0$ and $x + y + 3 = 0$.

- 1 The equations of two lines L_1 and L_2 are $kx + 2y + 3 = 0$ and $(k + 1)x - 6y - 1 = 0$ respectively.
- Find the gradient of both lines in terms of k .
 - Find the value of k so that $L_1 \parallel L_2$.
 - Find the value of k so that $L_1 \perp L_2$.

- 2 The points $A(2, 8)$, $B(-1, 5)$, $C(3, 2)$ and $D(a, b)$ are the vertices of a parallelogram.

- Plot the points A , B and C on the set of axes and sketch in two possible parallelograms using the point D .
- Find the two different coordinate sets of the point D for the two parallelograms sketched in part a.
- The sketch in part a should show three triangles inside a trapezium. Find the area of the trapezium in exact form and so find the area of each triangle.



- 3 $A(-1, -2)$ and $B(11, 16)$ are the endpoints of the line segment $\overline{AB}$. Find the coordinates of the point P that divides $\overline{AB}$:

- internally in the ratio: i 1:2 ii 5:1 iii 1:5
- externally in the ratio: i 1:2 ii 5:1 iii 1:5

- 4 Find the equation of a straight line so that all the points on the line are equidistant from $(-3, 2)$ and (a, b) .

- 5 The line with equation $3x - 7y + 27 = 0$ is the perpendicular bisector of $\overline{AB}$. If A is $(-5, 10)$, find the coordinates of B .

- 6 Find the equation of the straight line that passes through the origin and is:

- parallel to the straight line with equation $y = ax + b$
- perpendicular to the straight line with equation $y = ax + b$
- parallel to the straight line with equation $\frac{x}{a} + \frac{y}{b} = 1$
- perpendicular to the straight line with equation $\frac{x}{a} + \frac{y}{b} = 1$

- 7 Find the equation of the straight line passing through the point $(ab^2, 2ab)$ and with gradient:

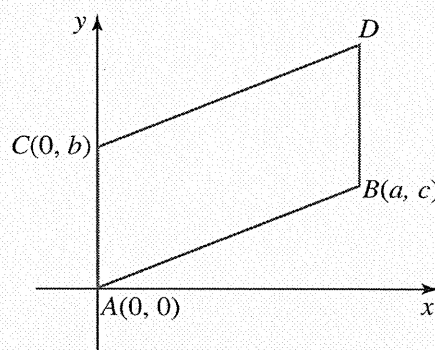
- $\frac{1}{a}$
- $\frac{1}{b}$
- $\frac{1}{ab}$

- 8 Find the equation of the straight line passing through the points:

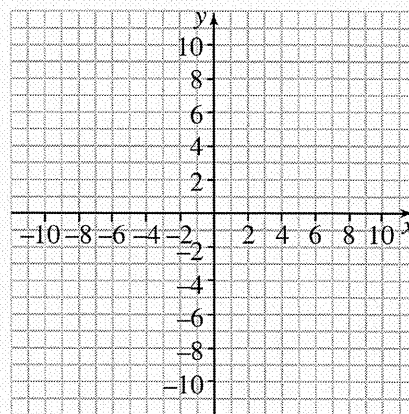
- $(2a, b)$ and $(-a, 2b)$
- $(a, 2b)$ and $(2a, -b)$
- $\left(\frac{a}{b}, \frac{b}{a}\right)$ and $\left(-\frac{a}{b}, \frac{2b}{a}\right)$
- $(a^2, 2a)$ and $(b^2, 2b)$
- $(ab^2, 2ab)$ and $(ac^2, 2ac)$
- $\left(a, \frac{1}{a}\right)$ and $\left(b, \frac{1}{b}\right)$

- 9 Find the equation of the straight line whose x - and y -intercepts are b and c respectively.
- 10 Find the value of b , and hence the equation of the straight line through $(1, b)$ and $(3, -b)$:
- a with angle of inclination 135° b with a y -intercept of 2

- 11 The figure shown on this set of axes is a parallelogram.
- a Find the coordinates of point D .
- b Find the equation of the diagonals and find the intersection of the diagonals.
- c Show that the intersection of the diagonals bisects each diagonal.

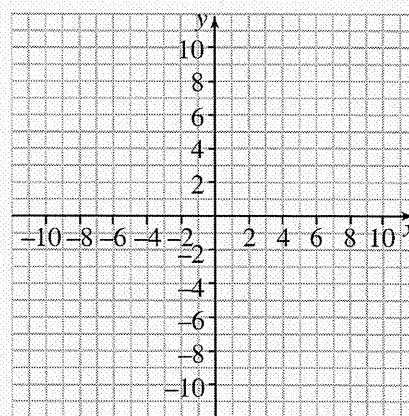


- 12 ABC is a triangle with $A(0, 1)$, $B(2, -2)$ and $C(-3, -3)$. $\overline{AP}$, $\overline{BQ}$ and $\overline{CR}$ are perpendicular to $\overline{BC}$, $\overline{AC}$ and $\overline{AB}$. Find the point of intersection of the altitudes (the orthocentre), hence show that the altitudes are concurrent.



- 13 ABC is a triangle with vertices $(4, 6)$, $(-8, 2)$ and $(0, -6)$ respectively.

- a Plot the points onto this set of axes.
- b Find the equations of the perpendicular bisectors of the sides $\overline{AB}$, $\overline{AC}$ and $\overline{BC}$.
- c Show that these are concurrent by finding their point of intersection (the circumcentre P).
- d Find the lengths $d\overline{AP}$, $d\overline{BP}$ and $d\overline{CP}$ and so show that the circumcentre is equidistant from A , B and C .
- e Draw the circle that includes the three vertices A , B and C .



- 14 $A(0, 0)$, $B(6, 2)$ and $C(4, 6)$ are the vertices of a triangle.

- a Find:
- $d\overline{AB}$, $d\overline{AC}$, $d\overline{BC}$
 - the coordinates of the midpoint D of $\overline{BC}$
 - the length of the median $\overline{AD}$
- b Hence show that the theorem of Apollonius (shown below) applies to the triangle:

$$(d\overline{AB})^2 + (d\overline{AC})^2 = 2\left(\frac{1}{2}d\overline{BC}\right)^2 + 2(d\overline{AD})^2$$

Revision Questions

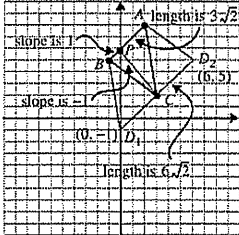
Multiple choice

- ~~1 A~~ 2 D 3 D 4 C
 5 A 6 D 7 C 8 C
 9 C 10 D 11 A 12 C
 13 B ~~14 B~~

Short answer

- 1 $2y + 3x = 2, \left(\frac{2}{3}, 0\right), (0, 1)$
 2 $(0, -12), (-6, 2), (8, 16)$
 3 $y = x - 4$
 4 a $d\overline{AB} = d\overline{AC} = \sqrt{(144 + 25)}$ so the triangle is isosceles.
 b i M is $\left(\frac{-9}{2}, \frac{-1}{2}\right)$. The slope of $\overline{BC}$ is -1 and the slope of $\overline{AM} = 1$ so the lines are perpendicular.
 5 a $\frac{2}{3}$
 b i $y = \frac{2x}{3} + \frac{1}{3}$ ii $y = \frac{-3x}{2} + \frac{5}{2}$
 6 a $k = 4$ b $k = \frac{3}{2}$ c $k = -12$
 7 71.6°

Challenging Tasks

- 1 a $\frac{-k}{2}$ and $\frac{k+1}{6}$ b $k = \frac{-1}{4}$
 c $k = -4$ or 3
 2 a 
 b $(6, 5)$ and $(0, -1)$ c $\frac{9\sqrt{37}}{2}$ and $\frac{3\sqrt{37}}{2}$
~~3 a i (3, 4) ii (9, 13) iii (1, 1)~~
~~b i (-13, -20) ii (14, 20.5) iii (-4, 6.5)~~
 4 $2(b-2)y = -2(a+3)x + a^2 + b^2 + 13$
 5 $(1, -4)$
 6 a $y = ax$ b $ay = -x$
 c $ay + bx = 0$ d $by = -ax$
 7 a $ay - x - ab(2a - b) = 0$
 b $by - x - ab^2 = 0$
 c $aby - x - ab^2(2b - 1) = 0$
 8 a $3ay + bx - 5ab = 0$
 b $ay + 3bx - 5ab = 0$
 c $2a^2y - b^2x - ab = 0$
 d $(a+b)y - 2x - 2ab = 0$
 e $(c+b)y - 2x - 2ac = 0$
 f $aby + x - a - b = 0$

9 $\frac{x}{b} + \frac{y}{c} = 1$

10 a $b = 1, y = -x + 2$ b $b = 1, y = -x + 2$

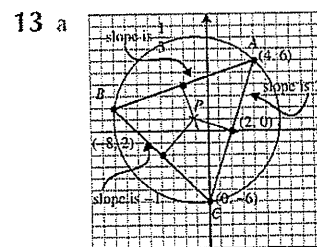
11 b $\left(\frac{a}{2}, \frac{c+b}{2}\right)$ c $\left(\frac{a}{2}, \frac{c+b}{2}\right)$

12 $\overline{AP} y = -5x + 1$

$\overline{BQ} 4y = -3x - 2$

$\overline{CR} 3y = 2x - 3$

The orthocentre is the point $\left(\frac{6}{17}, -\frac{13}{17}\right)$ through which these three altitudes pass.



b $AB y = -3x - 2$

$BC y = x + 2$

$CA 3y = -x - 2$

c $P, (-1, -1)$

d $\sqrt{50}$

e The circle centre P with radius $\sqrt{50}$ has the three points on its circumference.

14 a i $d\overline{AB} = \sqrt{40}, d\overline{AC} = \sqrt{52}$ and $d\overline{BC} = \sqrt{20}$

ii $(5, 4)$

iii $\sqrt{41}$

b $(d\overline{AB})^2 + (d\overline{AC})^2 = 92$

$2\left(d\frac{1}{2}\overline{BC}\right)^2 + 2(d\overline{AD})^2 = 92$

$(d\overline{AB})^2 + (d\overline{AC})^2 = 2\left(d\frac{1}{2}\overline{BC}\right)^2 + 2(d\overline{AD})^2$