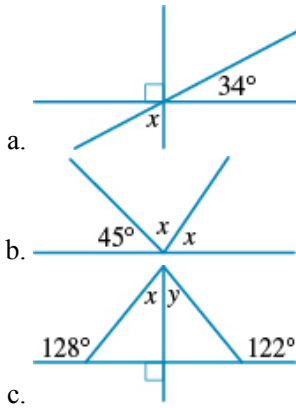


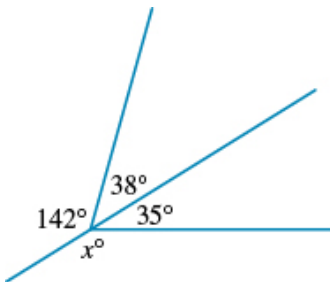
EXERCISE 4.2 Review of basic geometry

PRACTISE

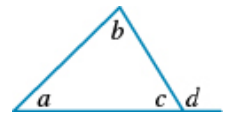
1. **WE1** Find the values of the pronumerals in the following figures.



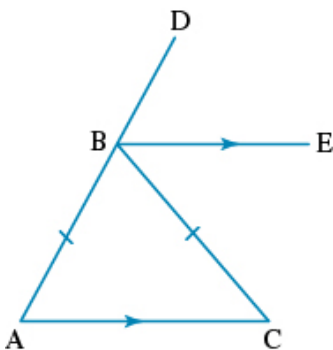
2. Find the value of x .



3. **WE2** Prove the following theorem: 'The value of the exterior angle of a triangle equals the sum of the two interior opposite angles'. In other words, in referring to the figure at right,
 $d = a + b$ $d = a + b$.



4. $\triangle ABC$ $\triangle ABC$ shown below is isosceles. AB is produced to D , and BE is drawn through B , parallel to the base AC . Prove that BE bisects $\angle CBD$.



CONSOLIDATE

5. Angles that add up to 180 degrees are called:

- A. parallel
- B. isosceles
- C. complementary

D. supplementary

E. right-angled

6. Which of the following statements is false?

A. An angle is always formed when two lines meet.

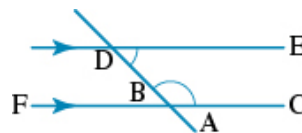
B. The sum of angles in a right-angled triangle is 180 degrees 180 degrees.

C. A line segment extended to infinity in both directions is called a ray.

D. Parallel lines never meet.

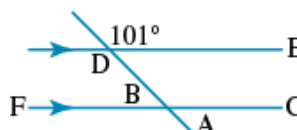
E. Reflex angles are bigger than acute angles.

7. Prove that the co-interior (or allied) angles marked with arcs in the figure at right are supplementary. In other words, prove that $\angle EDB + \angle DBC = 180^\circ$ $\angle EDB + \angle DBC = 180^\circ$.

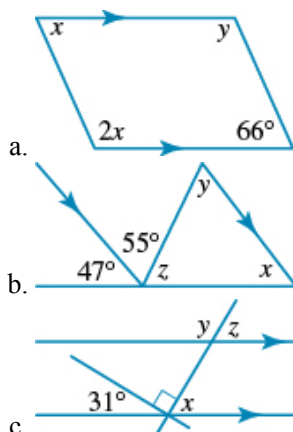


8. From the result in Worked example 2 and question 7, show that there are only two different angle values in that figure: all angles must take on one or the other of these values.

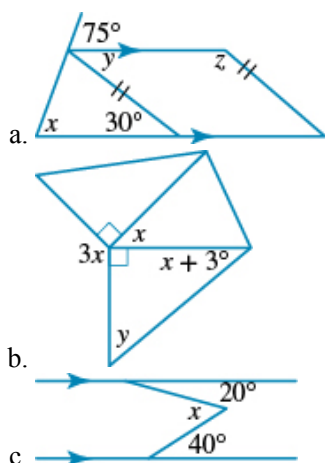
Furthermore, show that if you know only one of these angles, then all other angles can be determined. Demonstrate this using the value in the figure at right.



9. Find the values of the unknown angles in the following figures.



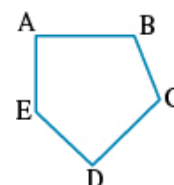
10. Find the values of the unknown angles in the following figures.

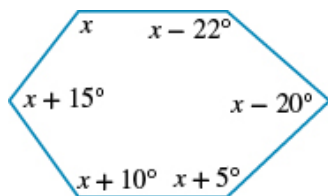


11. Show that the sum of the interior angles of the pentagon at right equals 540 degrees 540 degrees. In other words, show that:

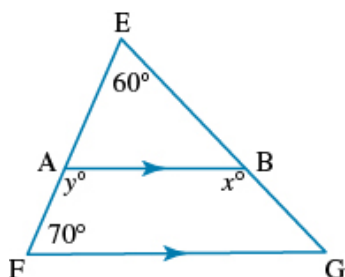
$$\angle AED + \angle EDC + \angle DCB + \angle CBA + \angle BAE = 540^\circ \quad \angle AED + \angle EDC + \angle DCB + \angle CBA + \angle BAE = 540^\circ.$$

12. Find the value of xx in the figure below.

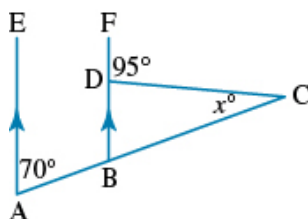




13. Find the values of the pronumerals in the figure at right.

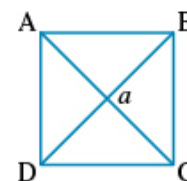


14. Using the figure below, find the value of x , giving reasons for your answer.



MASTER

15. Prove that the diagonals of a square form a right angle; that is, that angle a in the figure at right equals 90 degrees.



16.

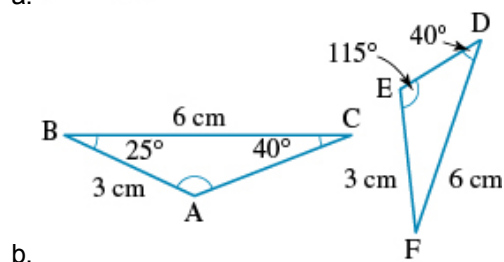
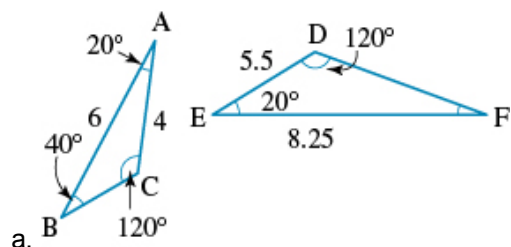
- Copy and complete the following table relating the sum of the interior angles to the number of sides of a polygon.
- Can you establish a general formula for the sum in terms of n , the number of sides?

Number of sides (n)	3	4	5	6	7	8	9	10	20
Sum of interior angles	180°	360°	540°	720°					

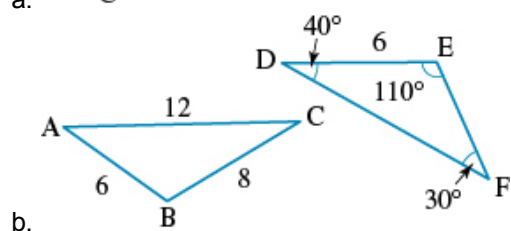
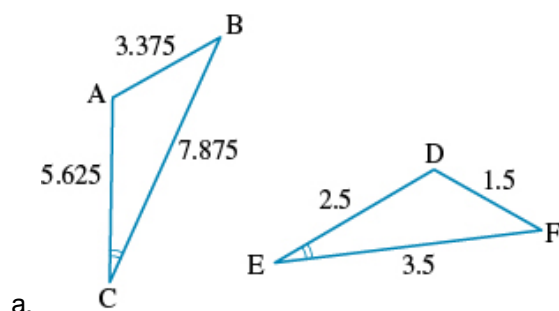
EXERCISE 4.3 Congruence and similarity

PRACTISE

1. **WE3** For each of the following, compare $\triangle ABC$ and $\triangle EFD$ and state whether they are similar, congruent, or there is not enough information given for a decision to be made. Justify your answers.

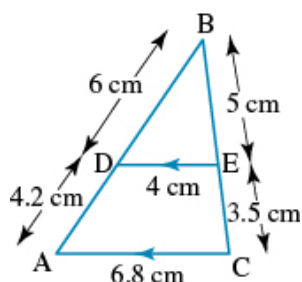


2. For each of the following, compare $\triangle ABC$ and $\triangle EFD$ and state whether they are similar, congruent, or there is not enough information given for a decision to be made. Justify your answers.



3. **WE4** Show that $\triangle ABC$ is similar to $\triangle DBE$ because:

- the corresponding angles are equal
- the corresponding side ratios are equal (to a value other than 1).



4. Show that $\triangle ABC$ is similar to $\triangle DBE$ because:

- the corresponding angles are equal

The diagram shows two triangles. The top triangle has side lengths of 37 cm and 31 cm, and a third side labeled y . The bottom triangle has side lengths of 9.6 cm and 6.2 cm, and a third side labeled x . Both triangles have a black dot in the top-left interior angle and an asterisk in the bottom-right interior angle, indicating that these angles are equal. The triangles are oriented such that the sides of length 37 cm and 9.6 cm are on the left, and the sides of length 31 cm and 6.2 cm are on the right.

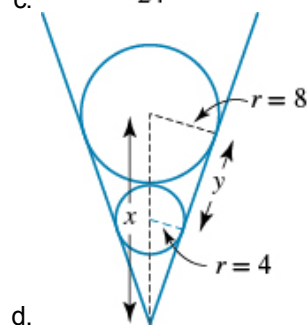
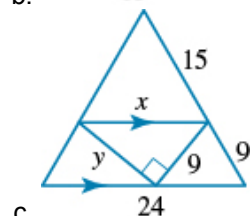
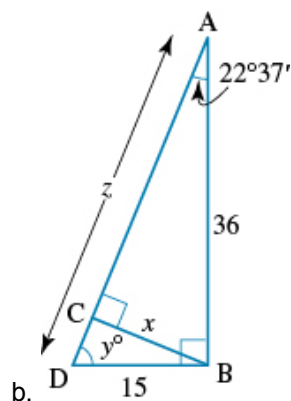
The first triangle has a top side of 2.4, a left side of x , and a right side of y . It has two interior angles marked with dots at the top vertices.

The second triangle has a left side of 9.6, a right side of 12.4, and an unlabeled bottom side. It has two interior angles marked with dots at the top-left and bottom-left vertices.

Diagram showing two triangles, $\triangle ABC$ and $\triangle EDC$, sharing vertex C . The segments AC and EC are collinear, with $AC = 15$ and $EC = 5$. The segments BC and DC are also collinear, with $BC = x$ and $DC = y$. Angle $A = 6^\circ$ and angle $E = 4^\circ$.

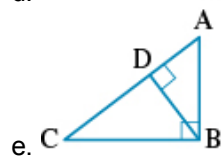
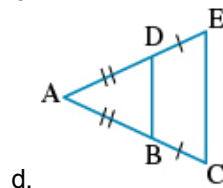
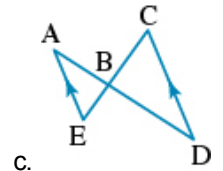
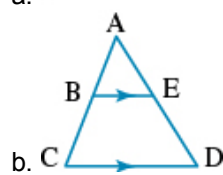
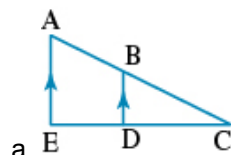
The diagram shows a right-angled triangle ACE with the right angle at E . The height AE is 3.4 m . A line segment BD is drawn from B on AC to D on CE , such that $BD \perp CE$. The segment DE is 4 m , and the segment CD is labeled x .

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CONSOLIDATE

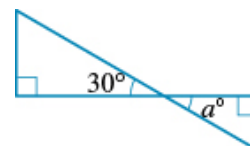
7. In each of the following diagrams, find and re-draw two triangles that are similar. Give reasons for your answer.



8. The magnitude of angle a° is:

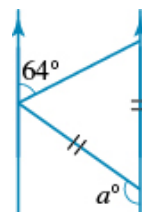
A. 30°

- B. $180^\circ - 30^\circ$ $180^\circ - 30^\circ$
 C. 60° 60°
 D. $180^\circ - 60^\circ$ $180^\circ - 60^\circ$
 E. 70° 70°



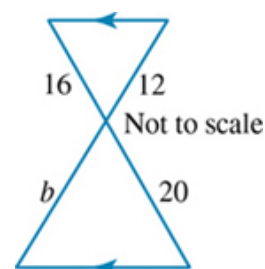
9. The magnitude of angle a° is:

- A. 64°
 B. 128
 C. 52°
 D. 154°
 E. 104°



10. In the given diagram, the length of side b is closest to:

- A. 24
 B. 22
 C. 16
 D. 15
 E. 9.6

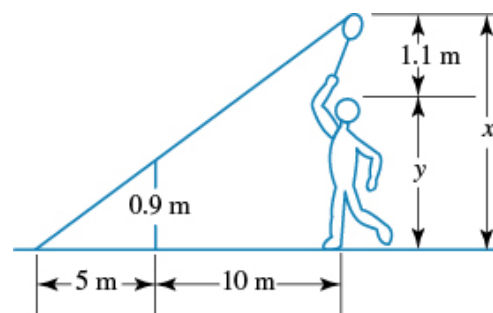


Questions 11 and 12 refer to the following information.

A young tennis player's serve is shown in the diagram at right. Assume the ball travels in a straight line.

11. The height of the ball just as it is hit, x , is closest to:

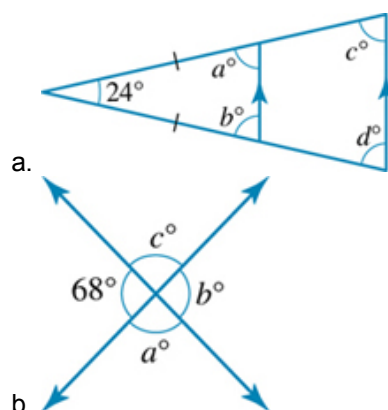
- A. 3.6 m
 B. 2.7 m
 C. 2.5 m
 D. 1.8 m
 E. 1.6 m

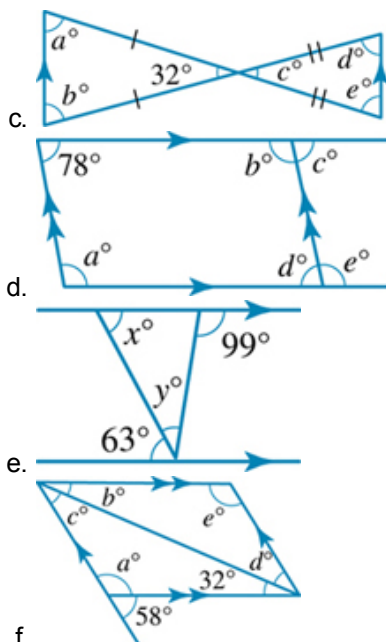


12. The height of the player, y , is closest to:

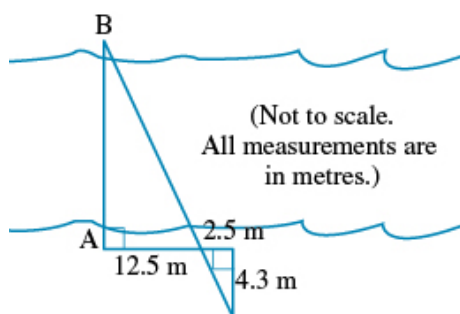
- A. 190 cm
 B. 180 cm
 C. 170 cm
 D. 160 cm
 E. 150 cm

13. Calculate the size of the missing angles. Justify your answer.





14. Find the length of the bridge, $\overline{AB}$, needed to span the river, using similar triangles as shown (to 1 decimal point).



15. A triangle with sides 12 cm, 24 cm and 32 cm is similar to a smaller triangle that has a longest side measurement of 8 cm. Draw a diagram to represent this situation and then calculate the perimeter of the smaller triangle.
16. It just happens that you always carry a wooden rod 1.5 m in length and a tape measure. For each of the following situations, draw a diagram showing the two similar triangles and calculate the height of the vertical object.
- You have always wanted to know how high the basketball players had to jump to touch the top of the ring. Your wooden rod held next to the basketball pole created a shadow half the length of the basketball pole's 1 m shadow. If the ring is 45 cm from the top of the board, how far up from the ground do the players have to reach to touch the ring?
 - The measuring tape for the pole vault height is missing. How high did Australian National Champion Elizabeth Parnov pole vault if the pole vault creates a shadow of 2.4 m on the ground, while your wooden rod creates a shadow of 80 cm?
 - Does the fire brigade need to bring an extension ladder longer than 15 m to rescue people from the top of a high-rise building if the shadow created by the building is 24 m and the shadow created by your pole is 2 m?
 - The clearance sign is missing from a low bridge over a road. The sun has created an image of the bridge and its opening on the ground. The image of the opening is 1.44 m in length. The semi-trailer you are driving is 2.5 m high. You step out of the truck and place your trusty rod on the ground, producing a 90 cm shadow. Will you need to find an alternative route or can you proceed?



MASTER

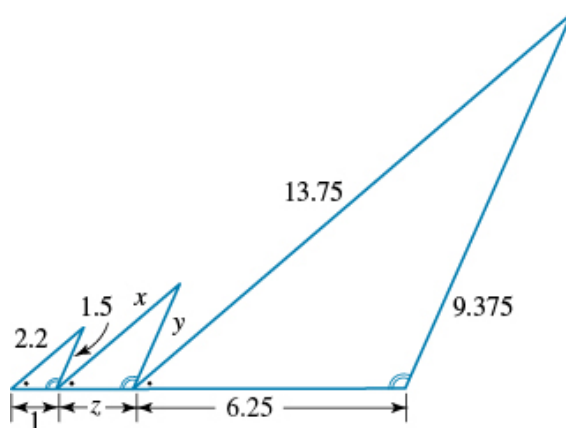
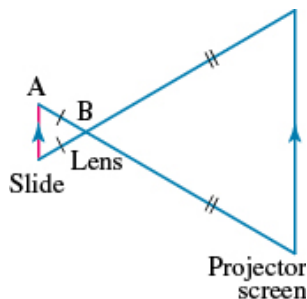
17. A curved section of a fun-ride is to be supported by similar triangles.

- a. If the dimensions of the first and last triangle are as shown, calculate the dimensions of the triangle in between.
- b. The construction people decide to make the track even steeper by extending it with one more similar triangle. What are the dimensions of the next triangle?

18. At your 18th birthday party you want to show slides of you and your friends growing up. The diagram at right shows the set-up of the projector lens, slide and projector screen.

How far horizontally from the screen do you need to place the projector's lens so the image just covers the entire screen if:

- a slide is $5\text{ cm} \times 5\text{ cm}$
- the distance from AA to BB is 10 cm
- the projector screen is $1.5\text{ m} \times 1.5\text{ m}$



EXERCISE 4.2

1.

a. 56° 56°

b. 67.5° 67.5°

c. $x = 38^\circ$ $x=38^\circ$, $y = 32^\circ$ $y=32^\circ$

2. $x = 145^\circ$ $x=145^\circ$

3. Check with your teacher.

4. Check with your teacher.

5. DD

6. CC

7. Check with your teacher.

8. Check with your teacher.

9.

a. $x = 60^\circ$ $x=60^\circ$, $y = 114^\circ$ $y=114^\circ$

b. $x = 47^\circ$ $x=47^\circ$, $y = 55^\circ$ $y=55^\circ$, $z = 78^\circ$ $z=78^\circ$

c. $x = 59^\circ$ $x=59^\circ$, $y = 121^\circ$ $y=121^\circ$, $z = 59^\circ$ $z=59^\circ$

10.

a. $x = 75^\circ$ $x=75^\circ$, $y = 30^\circ$ $y=30^\circ$, $z = 150^\circ$ $z=150^\circ$

b. $x = 45^\circ$ $x=45^\circ$, $y = 42^\circ$ $y=42^\circ$

c. 60° 60°

11. Check with your teacher.

12. 122° 122°

13. $x = 130^\circ$ $x=130^\circ$, $y = 110^\circ$ $y=110^\circ$

14. $x = 25^\circ$ $x=25^\circ$

15. $AB = BC$ $AB=BC$, therefore $\triangle ABC$ is isosceles.

$\angle BAC = \angle BCA = 45^\circ$ $\angle BAC=\angle BCA=45^\circ$ (property of isosceles triangles).

In triangle OAB , $\angle OAB + \angle ABO + \angle BOA = 180^\circ$ $\angle OAB+\angle ABO+\angle BOA=180^\circ$ (sum of angles in a triangle).

$\angle OAB = \angle ABO = 45^\circ$ $\angle OAB=\angle ABO=45^\circ$, so $\angle BOA = 90^\circ$ $\angle BOA=90^\circ$.

16.

a.

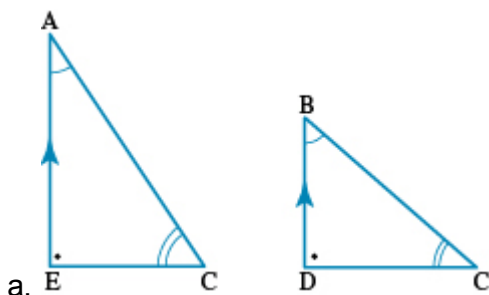
Number of sides	Sum of interior angles
3	180°
4	360°
5	540°
6	720°

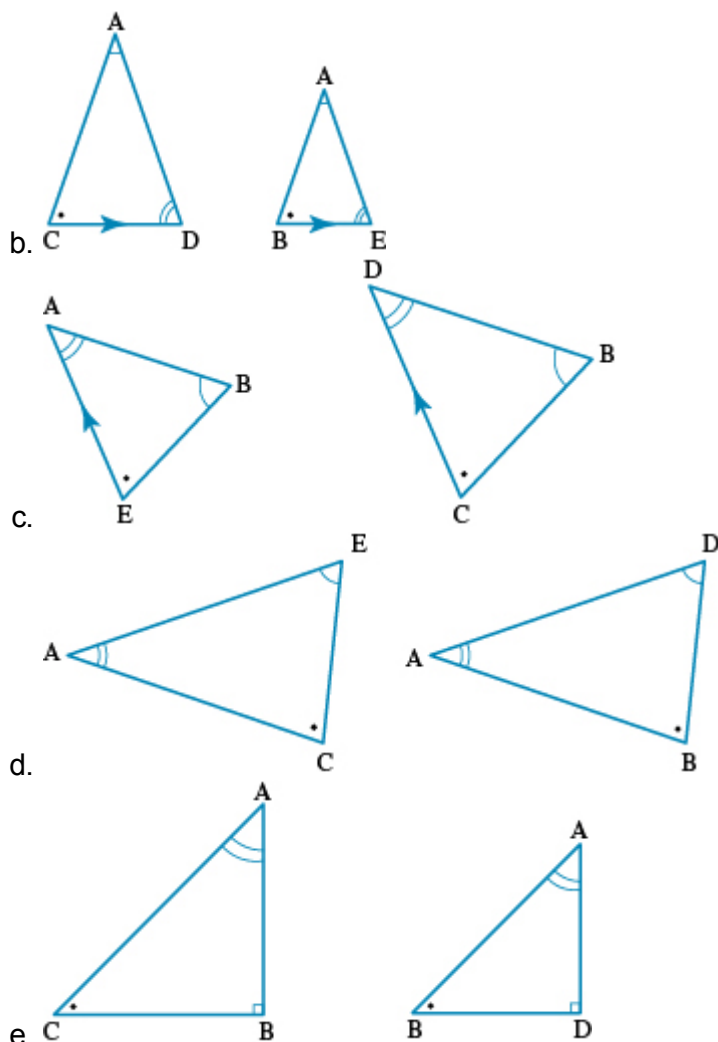
77	900° 900°
88	1080° 1080°
99	1260° 1260°
1010	1440° 1440°
20	3240° 3240°

b. General formula: $180^\circ(n - 2)$ $180^\circ(n-2)$

EXERCISE 4.3

1.
 - a. Similar
 - b. Congruent
2.
 - a. Similar
 - b. Not enough information (unless cosine rule is known)
3. Check with your teacher.
4. Check with your teacher.
5.
 - a. $x = 7.4$ cm $x=7.4$ cm, $y = 48$ cm $y=48$ cm
 - b. $x = 3.1$ cm $x=3.1$ cm, $y = 3.1$ cm $y=3.1$ cm
 - c. $x = 12.5$ cm $x=12.5$ cm, $y = 19.5$ cm $y=19.5$ cm
 - d. $x = 7.5$ cm $x=7.5$ cm, $y = 10$ cm $y=10$ cm
 - e. $x = 4$ $x=4$
6.
 - a. $x = 75^\circ$ $x=75^\circ$, $y = 75^\circ$ $y=75^\circ$, $z = 4$ cm $z=4$ cm
 - b. $x = 13.8$ $x=13.8$, $y = 67^\circ 23'$ $y=67^\circ 23'$, $z = 39$ $z=39$
 - c. $x = 15$ $x=15$, $y = 12$ $y=12$
 - d. $x = 24$ $x=24$, $y = 11.3$ $y=11.3$
- 7.





8. AA

9. BB

10. DD

11. BB

12. DD

13.

a. $a = b = c = d = 78^\circ$ $a=b=c=d=78^\circ$

b. $a = c = 112^\circ$ $a=c=112^\circ$, $b = 68^\circ$ $b=68^\circ$

c. $a = b = d = e = 74^\circ$ $a=b=d=e=74^\circ$, $c = 32^\circ$ $c=32^\circ$

d. $a = b = e = 102^\circ$ $a=b=e=102^\circ$, $c = d = 78^\circ$ $c=d=78^\circ$

e. $x = 63^\circ$ $x=63^\circ$, $y = 36^\circ$ $y=36^\circ$

f. $a = e = 122^\circ$ $a=e=122^\circ$, $b = 32^\circ$ $b=32^\circ$, $c = d = 26^\circ$ $c=d=26^\circ$

14. 21.5 metres 21.5 metres

15. 17 cm 17 cm

16.

a. 2.55 metres 2.55 metres

b. 4.5 metres 4.5 metres

c. 18 metres 18 metres. **Need a longer ladder.**

d. 2.4 metres 2.4 metres. **Need to find an alternative route.**

17.

a. $x = 5.5$ $x=5.5$, $y = 3.75$ $y=3.75$, $z = 2.5$ $z=2.5$

b. 34.38 34.38, 23.44 23.44, 15.63 15.63

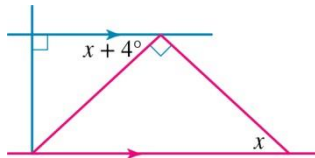
18. 2.9 metres

Review — Topic 4: Geometry in the plane

- 1 The sum of angles in an 11-sided polygon is:

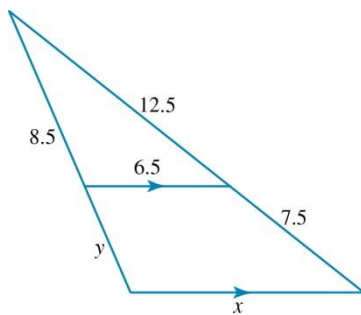
A 360°
 B 1620°
 C 1980°
 D 3240°
 E 3960°

- 2 The value of x in the figure below is:



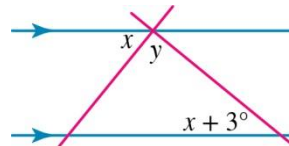
A 42°
 B 43°
 C 45°
 D 47°
 E unable to be determined from the given information

- 3 In the diagram below the lengths of x and y are:



A $x = 5.1, y = 10.4$
 B $x = 12.3, y = 6.2$
 C $x = 6.2, y = 12.3$
 D $x = 5.1, y = 5.1$
 E $x = 10.4, y = 5.1$

- 4 a If $x = 40^\circ$, find y .
 b Express the angle y in terms of x .



- 5 A tunnel 500 m long has a semicircular cross-section. It requires temporary perpendicular supports, AB and BC, as shown at right. Point B is vertically above D.

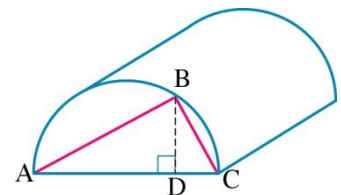
a What is the relationship between $\triangle ABC$ and $\triangle DBC$?

b If $AB = 24$ m and $BC = 7$ m, find:

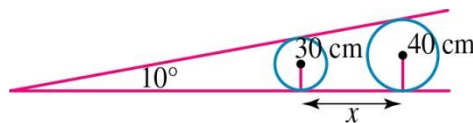
- the diameter of the tunnel
- the height of B above the floor of the tunnel
- the distance of D from the wall at C
- the distance of D from the wall at A.

c While the temporary supports are in place, what should drivers of large trucks entering the tunnel take into account?

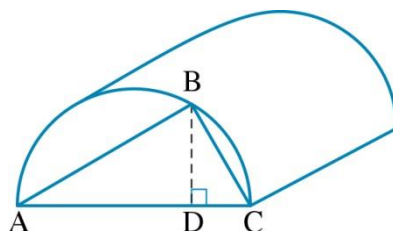
d What is the maximum height of a 3 m wide truck so that it can pass under the supports?



- e Determine whether a truck that is 4 m wide and 5.2 m high will pass through the tunnel safely.
- 6 A plank rests on two cylindrical rollers, as shown. The radii of the rollers are 30 cm and 40 cm. The plank makes an angle of 10° with the horizontal ground. Find the distance, x cm, between the rollers.



- 7 A tunnel 300 m long has a semicircular cross-section. It requires temporary supports, AB and BC, as shown.



Point B is vertically above D. If $AB = 12$ m and $BC = 5$ m, find:

- the diameter of the tunnel
- the height of B above the floor of the tunnel
- the distance D is from the wall at C
- the maximum height a 3 m wide truck could be so that it can pass under the supports.

Answers:

- B 2 B 3 E
- a 97°
b $y = 177^\circ - 2x$
- a $\triangle ABC$ and $\triangle DBC$ are similar triangles as they both contain a right angle and share $\angle C$. Therefore the third angle is the same.
b i 25 m
ii 6.72 m
iii 1.96 m
iv 23.04 m
c Drivers must take into account that the height and width of the tunnel space will have decreased and therefore larger trucks may not be able to pass through safely.
d 5.91 m
e Yes, the maximum height is 5.64 m.
- The distance between the rollers is $x = 114.3$ cm.
- a The diameter of the tunnel is 13 m.
b B is 4.62 m above the floor of the tunnel.
c D is 1.92 m from C.
d The maximum height of the truck is 3.55 m.