



Glen Waverley Secondary College 2017

Specialist Mathematics Unit 2 Technology Access Examination



Student Name: _____

Student ID Code: _____ Form: _____

Teacher (circle): **CAL** **PLG** **GMN**

QUESTION AND ANSWER BOOK

Reading Time: 15 minutes
Writing Time: 120 minutes

Structure of Book

Section	Number of questions	Number of questions to be answered	Number of marks
MULTIPLE CHOICE	20	20	40
SHORT ANSWER	9	9	60

- Students are permitted to bring into the room: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring into the exam room: blank sheets of paper and/or white out liquid/tape.
- A CAS calculator, a scientific calculators and Bound Reference are allowed in this exam

Materials supplied

- Question and Answer booklet.
- Formula sheet (attached).

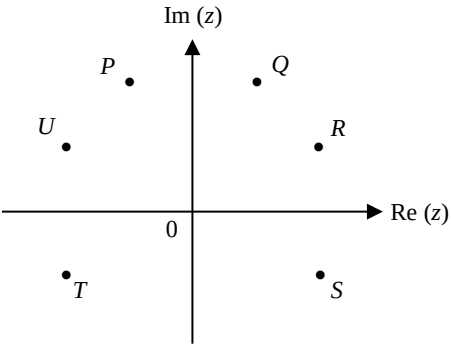
Instructions

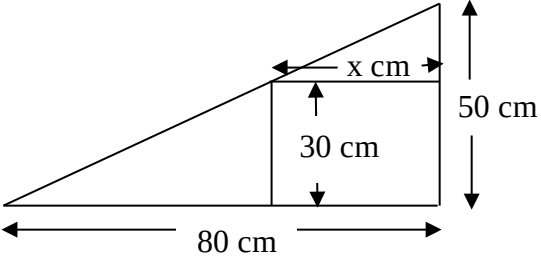
- Write your **student name and student ID code** in the space provided above on this page.
- All written responses must be in English.
- Write your answers to the **MULTIPLE CHOICE questions** in the table provided on page 7.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Answer multiple choice questions on the table at the end of this section.

Question 1	
$\frac{-x^2 + 10}{(x + 1)(x - 2)^2}$ is equal to: A. $\frac{1}{x + 1} - \frac{2}{x - 2} + \frac{2}{(x - 2)^2}$ B. $\frac{-x^2}{(x - 2)^2} + 10$ C. $\frac{-x^2}{x + 1} - \frac{10}{(x - 2)^2}$ D. $\frac{1}{x + 1} - \frac{1}{(x - 2)^2}$ E. $\frac{1}{x + 1} - \frac{x}{(x - 2)^2}$	
Question 2	
If $t_1 = 10$, $t_{n+1} = t_n - 3$, then t_5 is: A. 7 B. -5 C. -2 D. -8 E. -1	
Question 3	
$-\cos x + \sin x$ in the form $r \sin(x + \alpha)$, where $r > 0$, is: A. $\sqrt{2} \sin\left(x + \frac{\pi}{4}\right)$ B. $-\sin\left(x + \frac{\pi}{4}\right)$ C. $\sqrt{2} \sin\left(x - \frac{3\pi}{4}\right)$ D. $\sqrt{2} \sin\left(x - \frac{\pi}{4}\right)$ E. $\sqrt{2} \sin\left(x + \frac{3\pi}{4}\right)$	

Question 4	
The complex number u is represented on the Argand diagram by the point U. The conjugate of u ($\bar{u}$) would be represented by the point: A. P B. Q C. R D. S E. T	
Question 5	
The reciprocal of the function $y = x^2 + 4x + k$ has no vertical asymptotes if: A. $k < 4$ B. $k > 4$ C. $k = 4$ D. $k \leq 4$ E. $k \geq 4$	
Question 6	
Let $A = \begin{bmatrix} 2 & 0 \\ -1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 \\ -1 & -3 \end{bmatrix}$. Find $A^2B - 2AB$: A. $\begin{bmatrix} -10 & 10 \\ 19 & -5 \end{bmatrix}$ B. $\begin{bmatrix} 4 & -4 \\ -16 & -19 \end{bmatrix}$ C. $\begin{bmatrix} -6 & 6 \\ 9 & -9 \end{bmatrix}$ D. $\begin{bmatrix} 0 & 0 \\ -6 & -15 \end{bmatrix}$ E. $\begin{bmatrix} 6 & -6 \\ -9 & 9 \end{bmatrix}$	
Question 7	
The matrix $\begin{bmatrix} 0 & 2 \\ 1 & 0 \end{bmatrix}$ corresponds to: A. a dilation by factor of 2 from the y-axis followed by a reflection in the line $y = x$ B. a dilation by factor of 2 from the x-axis followed by a reflection in the line $y = x$ C. a dilation by factor of 2 from the y-axis followed by a reflection in the x-axis D. a dilation by factor of 2 from the y-axis followed by a reflection in the y-axis E. a dilation by factor of 2 from the x-axis followed by a reflection in the y-axis	

Question 8	
For what values of k is $A = \begin{bmatrix} 2k & 1 \\ 3 & k \end{bmatrix}$ singular? A. $k \neq \pm\sqrt{\frac{3}{2}}$ B. $k = \pm\sqrt{\frac{3}{2}}$ C. $k < -\sqrt{\frac{3}{2}}$ D. $k > -\sqrt{\frac{3}{2}}$ E. $k \in \mathbb{R}$	
Question 9	
Which of the following side lengths would form a right angled triangle: A. 5 cm, 7cm, 12 cm B. 2.4 cm, 3.0 cm, 3.6 cm C. 1.2 cm, 1.6 cm, 2.0 cm D. 9.6 cm, 12 cm, 20 cm E. 4.5 cm, 4.5 cm, 9 cm	
Question 10	
The value of x in the diagram on the right is: A. 32 B. 48 C. 12.5 D. 20 E. 24	
Question 11	
Two similar cylinders have heights of 12 m and 20 m. The ratio of their volumes is: A. 3:5 B. 9:25 C. 27:125 D. 64:100 E. 144:400	

Question 12	
If a vector $\vec{v}$ of magnitude 3 is parallel to the vector $-3\vec{i} + 6\vec{j}$ then $\vec{v}$ is represented by the vector: A. $-\vec{i} + 2\vec{j}$ B. $\frac{1}{\sqrt{5}}(-\vec{i} + 2\vec{j})$ C. $\frac{3}{\sqrt{5}}(-\vec{i} + 2\vec{j})$ D. $\frac{5}{\sqrt{5}}(-\vec{i} + 2\vec{j})$ E. $\frac{1}{3}(-\vec{i} + 2\vec{j})$	
Question 13	
Let $\vec{a} = (n-4)\vec{i} - 3\vec{j}$ and $\vec{b} = (n-4)\vec{i} + 3\vec{j}$ If $\vec{a}$ is perpendicular to $\vec{b}$ then n is: A. 4 B. 13 or -5 C. -1 or -7 D. 1 or 7 E. 4 or 9	
Question 14	
A vector $\vec{a}$ has a magnitude of 13 and its scalar resolute in the direction of $3\vec{i} + 4\vec{j}$ is -6.6. $\vec{a}$ could be: A. $5\vec{i} + 12\vec{j}$ B. $-13\vec{i}$ C. $-13\vec{j}$ D. $5\vec{i} - 12\vec{j}$ E. $-5\vec{i} + 12\vec{j}$	

Question 15	
A particle has a velocity given by $v(t) = t^2 - 4t$ Its acceleration at time $t = 4$ is: A. 12 m/s^2 B. 4 m/s^2 C. 0 m/s^2 D. 8 m/s^2 E. $-\frac{32}{3} \text{ m/s}^2$	
Question 16	
An object has a position given by: $x(t) = -2t^3 + 21t^2 + 30t \quad 0 \leq t \leq 10$ Its maximum velocity is closest to: A. 3.5 m/s B. 563.08 m/s C. 7.65 m/s D. 25.63 m/s E. 103.5 m/s	
Question 17	
A car accelerates uniformly from a speed of 4 m/s to 12 m/s over a distance of 160 metres. Its acceleration is: A. 0.4 m/s^2 B. 0.05 m/s^2 C. 0.45 m/s^2 D. 0.16 m/s^2 E. 0.32 m/s^2	
Question 18	
A sample of 4 people is selected randomly from a group of 10 men and 12 women. The probability of selecting at least two women is closest to: A. 0.7744 B. 0.4060 C. 0.3684 D. 0.6316 E. 0.4135	

Question 19															
In Australia the probability of a person having red hair is 0.15. In a random selection of 15 people the probability that exactly two have red hair is: A. 0.9126 B. 0.2312 C. 0.6814 D. 0.6042 E. 0.2856															
Question 20															
The following table represents the sampling distribution for a random variable. $Pr\left(\hat{p} > 0.4 \mid \hat{p} < 0.8\right)$ is: <table><tr><td>$\hat{p}$</td><td>0</td><td>0.2</td><td>0.4</td><td>0.6</td><td>0.8</td><td>1</td></tr><tr><td>$Pr\left(\hat{P} = \hat{p}\right)$</td><td>0.1</td><td>0.3</td><td>0.2</td><td>0.2</td><td>0.1</td><td>0.1</td></tr></table> A. $\frac{3}{8}$ B. $\frac{1}{4}$ C. $\frac{5}{9}$ D. $\frac{2}{9}$ E. $\frac{1}{2}$	$\hat{p}$	0	0.2	0.4	0.6	0.8	1	$Pr\left(\hat{P} = \hat{p}\right)$	0.1	0.3	0.2	0.2	0.1	0.1	
$\hat{p}$	0	0.2	0.4	0.6	0.8	1									
$Pr\left(\hat{P} = \hat{p}\right)$	0.1	0.3	0.2	0.2	0.1	0.1									

Multiple Choice Answer Table									
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20

Question 1

a) Find the common ratio of the jumps.

b) Assuming that every jump follows this pattern, calculate the distance of the 12th jump.

c) If the rabbit continued to jump forever, how far would it travel in total?

(1 + 2 + 2 = 5 marks)

Question 2

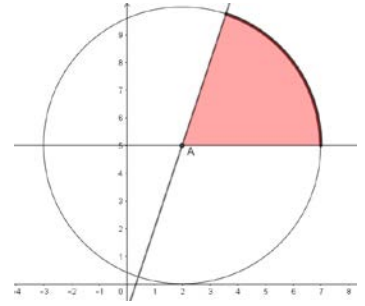
Let $z = \frac{2k-3i}{1+ki}$, where $k \in \mathbb{R}$, and $\text{Im}(z) = -\frac{5}{2}$. Find all possible values of $\text{Re}(z)$.

(4 marks)

Question 3

A circle is defined by the locus of points $P(x, y)$ whose distance from $A = (2, 5)$ is 5.

- a) A minor arc is bounded by the lines $y = 5$ and $y = 3x - 1$. Find the length of this arc (shown in bold on the diagram) to 2 decimal places.



- b) Calculate the area of the minor sector bounded by the same lines (shaded on the diagram) to 2 decimal places.

- c) Write the equation of the circle in Cartesian form.

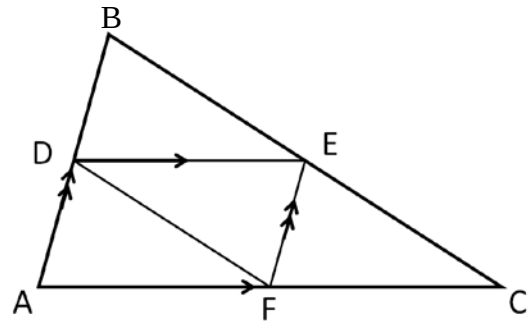
- d) The circle is transformed by the matrix $\begin{bmatrix} 3 & 0 \\ 0 & \frac{1}{4} \end{bmatrix}$. Write the Cartesian equation of the resulting ellipse.

- e) Find the area of the transformed sector from part b), to 2 decimal places.

(3 + 2 + 1 + 3 + 2 = 11 marks)

Question 4

a) Show that $\triangle ADF \cong \triangle FED$



b) Hence, show that $\triangle ABC \sim \triangle FED$

c) If $\angle BAC = 50^\circ$, $AB = 5$, and $AC = 8$, find the total area of triangles $\triangle ADF$, $\triangle FEC$ and $\triangle DBE$.
(Assume point D is the midpoint of AB) (Answer to two decimal places)

(2 + 3 + 3 = 8 marks)

Question 5

The rule for a transformation is $(x, y) \rightarrow (-x + 2y, 4x - y)$.

a) Find the image of the point $(2, -4)$.

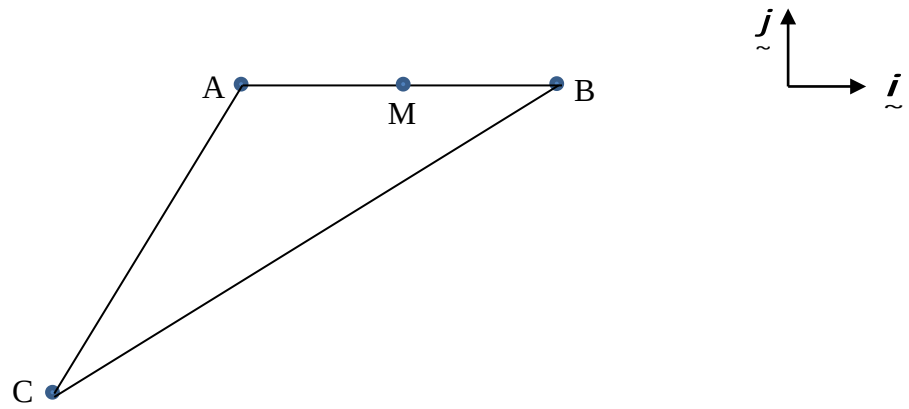
b) Find the matrix of this transformation.

c) Find the matrix of the inverse transformation.

d) Find the point (x, y) that is mapped to the point $(-7, 14)$.

(1 + 1 + 1 + 2 = 5 marks)

Question 6



In the diagram above A and B are two goal posts, M is the midpoint of the two goal posts and C is a footballer trying to kick a ball between the posts.

$$\overrightarrow{CA} = (10\hat{i} + 30\hat{j}) \text{ m and } \overrightarrow{AB} = (8\hat{i}) \text{ m}$$

a) Find vector $\overrightarrow{CM}$.

b) Hence find the straight line distance from C to M. (Answer to two decimal places)

A wind is blowing with a velocity $\vec{w} = (10\hat{i} + \lambda\hat{j})$ m/s. The footballer finds that if he kicks the ball with a velocity $\vec{v}$ which has a magnitude of 10 and at an angle 30° measured clockwise from $\hat{j}$ the ball moves directly towards the goal post A. Assume the total velocity of the ball will be $\vec{v} + \vec{w}$.

Let $\hat{q}$ be a unit vector in the direction of $\overrightarrow{CA}$.

c) Find $\hat{q}$.

d) If the ball travels directly from C to A, then $\vec{v} + \vec{w} = k\hat{q}$. Use this information to determine k and λ

e) Hence find $\vec{w}$.

(2 + 2 + 2 + 2 + 1 = 9 marks)

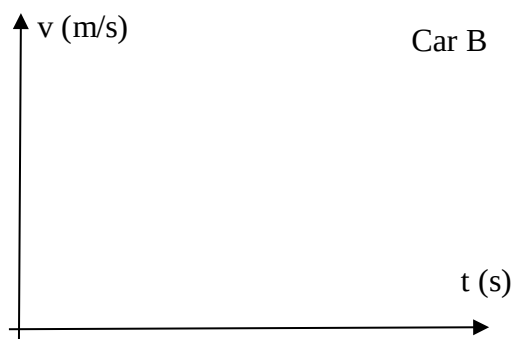
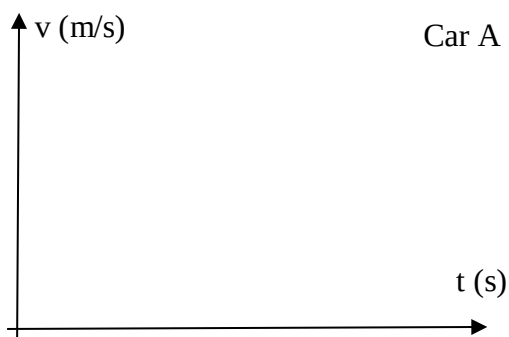
Question 7

Two traffic lights are 400 metres apart.

Car A starts from one traffic light and accelerates from rest uniformly at 2.5 m/s^2 up to a maximum speed of 20 m/s and then maintains a constant speed.

Car B starts from the other traffic light in the opposite direction. It accelerates from rest uniformly at 1.5 m/s^2 up to a maximum speed of 15 m/s and then maintains a constant speed.

a) Sketch velocity – time graphs on the axes below for both cars.



b) Hence find an expression for the distance covered by each car $d_A(t)$ and $d_B(t)$.

c) At what time will they pass each other?

- d) At this time how far is Car A from its starting point? (Answer to three decimal places)
- e) When Car A is 40 metres from the other traffic light it brakes uniformly to a stop at the lights. How long does it take to stop?

(2 + 2 + 2 + 1 + 2 = 9 marks)

Question 8

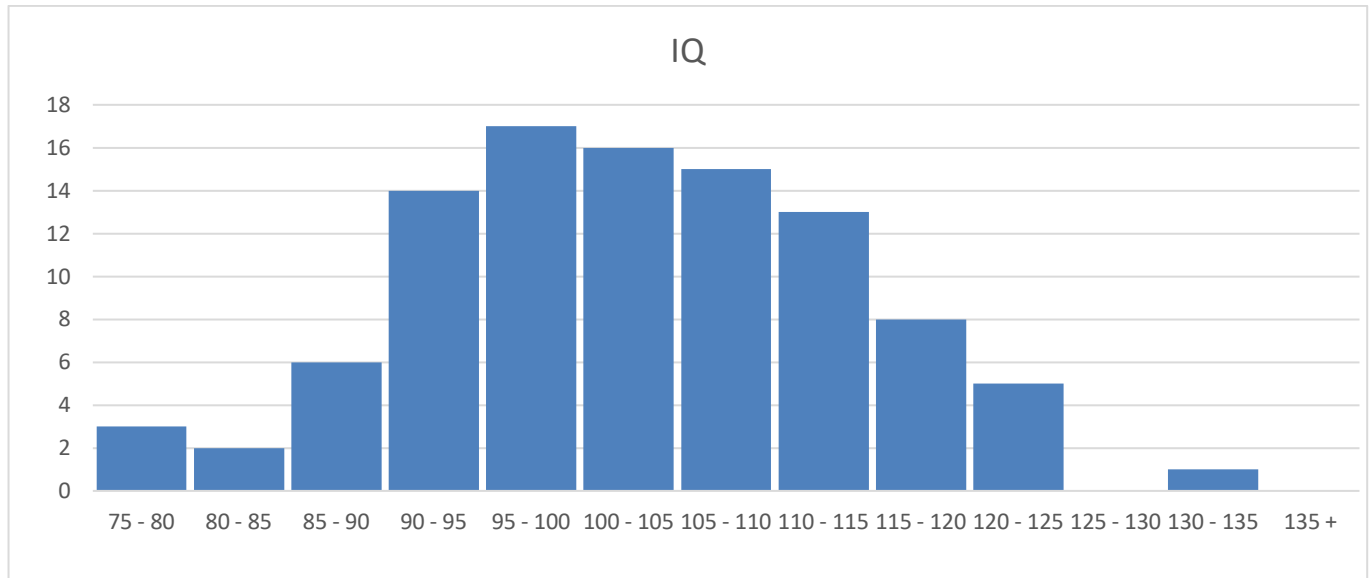
A school has 1600 students of whom 850 are female. A random selection of six students is selected.

- a) Using a hypergeometric distribution calculate the probability that the sample has equal numbers of boys and girls. (Answer to four decimal places)
- b) Now use a binomial approximation to estimate the probability that the sample has equal numbers of boys and girls. (Answer to four decimal places)
- c) Comment on the suitability of using a binomial approximation in this case.

(2 + 2 + 1 = 5 marks)

Question 9

The histogram below shows the distribution of the sample mean of IQ for 100 random samples of 20 people.



- a) State the proportion of the sample means which are greater than 120.
- b) Using the table, find the probability that the sample mean will be greater than 120 given that it is greater than 100.
- c) How would you expect the distribution to be different if a sample size of 100 was used?

(1 + 2 + 1 = 4 marks)

END OF EXAM

Formula Sheet

Coordinate Geometry

angle to the x-axis: $m = \tan \theta$

angle between two lines: $\tan \theta = \pm \frac{m_2 - m_1}{1 + m_1 m_2}$

distance between two points:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

midpoint: $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$

Vectors

$$\vec{r} = x\vec{i} + y\vec{j} \quad |\vec{r}| = \sqrt{x^2 + y^2}$$

$$\vec{r}_1 \cdot \vec{r}_2 = |\vec{r}_1| |\vec{r}_2| \cos \theta = x_1 x_2 + y_1 y_2$$

$$\vec{V}_{a \text{ rel } b} = \vec{V}_a - \vec{V}_b$$

Complex Numbers

$$z = x + yi = r(\cos \theta + i \sin \theta) = r \operatorname{cis} \theta$$

where $r = \sqrt{x^2 + y^2}$ and $\theta = \tan^{-1} \frac{y}{x}$

$$-\pi < \operatorname{Arg} z < \pi$$

$$z_1 z_2 = r_1 r_2 \operatorname{cis}(\theta_1 + \theta_2) \text{ and } \frac{z_1}{z_2} = \frac{r_1}{r_2} \operatorname{cis}(\theta_1 - \theta_2)$$

$$z^n = r^n \operatorname{cis}(n\theta) \text{ (de Moivre's Theorem)}$$

Trigonometry

$$\text{sine rule: } \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\text{cosine rule: } c^2 = a^2 + b^2 - 2ab \cos C$$

$$\text{area of a triangle: } A = \frac{1}{2}bh = \frac{1}{2}ab \sin C$$

$$\text{Heron's formula: } A = \sqrt{s(s-a)(s-b)(s-c)}$$

$$\text{where } s = \frac{1}{2}(a+b+c)$$

Reciprocal trigonometric functions:

$$\sec \theta = \frac{1}{\cos \theta} \quad \operatorname{cosec} \theta = \frac{1}{\sin \theta} \quad \cot \theta = \frac{1}{\tan \theta}$$

Fundamental trigonometric identities:

$$\cos^2 x + \sin^2 x = 1$$

$$1 + \tan^2 x = \sec^2 x$$

$$\cot^2 x + 1 = \operatorname{cosec}^2 x$$

Compound angle (Addition) formulae:

$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$$

$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$

$$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$$

Double angle formulae:

$$\sin 2x = 2 \sin x \cos x$$

$$\cos 2x = 2 \cos^2 x - 1$$

$$= 1 - 2 \sin^2 x$$

$$= \cos^2 x - \sin^2 x$$

$$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$$

$a \sin \theta \pm b \cos \theta = R \sin(\theta \pm \alpha)$ where:

$$R = \sqrt{a^2 + b^2} \text{ and } \tan \alpha = \frac{b}{a}$$

$$\sin \alpha = \frac{a}{\sqrt{a^2 + b^2}}, \cos \alpha = \frac{b}{\sqrt{a^2 + b^2}}$$

$a \sin \theta \pm b \cos \theta = R \cos(\theta \mp \alpha)$ where:

$$R = \sqrt{a^2 + b^2} \text{ and } \tan \alpha = \frac{b}{a}$$

$$\sin \alpha = \frac{a}{\sqrt{a^2 + b^2}}, \cos \alpha = \frac{b}{\sqrt{a^2 + b^2}}$$

Circle Mensuration

Arc length: $r\theta$

Chord length: $2r \sin\left(\frac{\theta}{2}\right)$

Sector area: $\frac{1}{2}r^2\theta$

Segment area: $\frac{1}{2}r^2(\theta - \sin \theta)$

Matrices

Determinant of the 2×2 matrix:

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$\det A = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

Inverse of a 2×2 matrix:

$$A^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \text{ where}$$

$$\det(A) \neq 0$$

Translation Matrix: $T_{(a,b)} = \begin{bmatrix} a \\ b \end{bmatrix}$

Rotation Matrix

(anticlockwise about (0,0)):

$$R_\theta = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$$

Matrix for Reflection in the X axis:

$$M_x = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

Matrix for Reflection in the Y axis:

$$M_y = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

Matrix for Reflection in the line $y = x$:

$$M_{y=x} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

Matrix for dilation parallel to X axis by a factor

of m : $D_x = \begin{bmatrix} m & 0 \\ 0 & 1 \end{bmatrix}$

Matrix for dilation parallel to Y axis by a factor

of n : $D_y = \begin{bmatrix} 1 & 0 \\ 0 & n \end{bmatrix}$

Matrix for dilation about the origin by a factor of

$$k: D_k = \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$$

Kinematics

$$v = u + at$$

$$v^2 = u^2 + 2as$$

$$s = ut + \frac{1}{2}at^2$$

$$s = \frac{1}{2}(u + v)t$$

Statistics & Probability

$$\mu_{\bar{x}} = \mu$$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

Hypergeometric:

$$Pr(X = x) = \frac{{}^D C_x \times {}^{N-D} C_{n-x}}{{}^N C_n}$$

Binomial:

$$Pr(X = x) = {}^n C_x \times p^x \times (1-p)^{n-x}$$

Sequences and Series

Arithmetic:

$$t_n = a + (n-1)d$$

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

Geometric:

$$t_n = ar^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$S_\infty = \frac{a}{1-r}$$