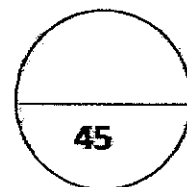




Year 11 Specialist Maths - 2017
Complex Numbers Test
Technology Free Assessment – 45 minutes writing time



Name: Solutions

Multiple Choice (10 x 2 = 20 marks). Circle the correct answer.

Question 1 The solution to the equation $2iz + 5 = \frac{4-i}{i}$ is A. $z = 2 + 3i$ B. $z = 2 - 3i$ ☒ C. $z = -2 + 3i$ D. $z = -2 - 3i$ E. $z = 3 - 2i$	Working space $2iz + 5i = 4 - i$ $-2z = 4 - i - 5i$ $= 4 - 6i$ $z = -2 + 3i$
Question 2 If $(x + iy)^2 = -32i$ for real values of x and y, then: A. $x = 4, y = 4$ B. $x = -4, y = 4$ C. $x = 4, y = -4$ ☒ D. $x = 4, y = -4$ or $x = -4, y = 4$ E. $x = 4, y = 4$ or $x = -4, y = -4$	Working space $x^2 + 2xyi - y^2 = -32i$ $x^2 - y^2 = 0$ $xy = -32$
Question 3 If $u = 4 \operatorname{cis}\left(\frac{\pi}{4}\right)$ and $v = \operatorname{cis}\left(\frac{-\pi}{6}\right)$, then $\frac{u}{v}$ equals: A. $3 \operatorname{cis}\left(\frac{\pi}{12}\right)$ B. $3 \operatorname{cis}\left(\frac{5\pi}{12}\right)$ C. $4 \operatorname{cis}\left(\frac{-3}{2}\right)$ D. $4 \operatorname{cis}\left(\frac{\pi}{12}\right)$ ☒ E. $4 \operatorname{cis}\left(\frac{5\pi}{12}\right)$	Working space $\frac{u}{v} = 4 \operatorname{cis}\left(\frac{\pi}{4} + \frac{\pi}{6}\right)$ $= 4 \operatorname{cis}\left(\frac{3\pi + 2\pi}{12}\right)$

Question 4

Let $u = \sqrt{3} - i$ and $v = i - 1$. Then $|uv|$ is equal to:

- A. $\sqrt{3}$
- ☒ B. $2\sqrt{2}$
- C. 2
- D. 0
- E. 1

Working space

$$|u| = 2$$

$$|v| = \sqrt{2}$$

$$|uv| = 2\sqrt{2}$$

Question 5

If $|z| = 2$, then $|\frac{1}{z}|$ is equal to:

- A. $\frac{1}{\sqrt{2}}$
- B. $\frac{-1}{\sqrt{2}}$
- ☒ C. $\frac{1}{2}$
- D. $\frac{-1}{2}$
- E. $\sqrt{2}$

Working space

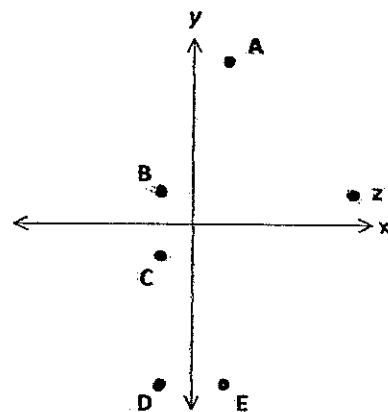
$$\left| \frac{1}{z} \right| = \frac{|1|}{|z|} = \frac{1}{2}$$

Question 6

The complex number, z , is shown on the Argand diagram to the right. The position of $-iz$ is:

- A. A
- B. B
- C. C
- D. D
- ☒ E. E

Working space

**Question 7**

If $z = \sqrt{2} \operatorname{cis}\left(-\frac{7\pi}{12}\right)$, z^4 is:

- A. $-2 + 2\sqrt{3}i$
- ☒ B. $2 - 2\sqrt{3}i$
- C. $2\sqrt{3} - 2i$
- D. $-2\sqrt{3} + 2i$
- E. $-2 - 2\sqrt{3}i$

Working space

$$z^4 = 2^2 \operatorname{cis}\left(4 \times -\frac{7\pi}{12}\right)$$

$$= 4 \operatorname{cis}\left(-\frac{7\pi}{3}\right)$$

$$= 4 \operatorname{cis}\left(-\frac{\pi}{3}\right)$$

$$= 4 \cos\left(-\frac{\pi}{3}\right) + 4i \sin\left(-\frac{\pi}{3}\right)$$

$$= 2 - 2\sqrt{3}i$$

Question 8

Which of the following equations show how the inverse z^{-1} and the conjugate $\bar{z}$ of the complex number $z = r \operatorname{cis} \theta$ are related?

- A. $\bar{z} = z^{-1}$
 B. $\bar{z} \times z^{-1} = 1$
 C. $\frac{\bar{z}}{z^{-1}} = r^2$
 D. $\frac{z^{-1}}{\bar{z}} = r^2$
 E. $\bar{z} \times z^{-1} = r^2$

Working space

$$\begin{aligned} \text{Let } z &= a+bi \\ \frac{1}{z} &= \frac{a-bi}{a^2+b^2} \\ \bar{z} &= a-bi \\ \frac{1}{z} &= \frac{1}{r^2} \times \bar{z} \\ r^2 &= \frac{\bar{z}}{z^{-1}} \end{aligned}$$

Question 9

If $z = 2\operatorname{cis}\left(\frac{\pi}{4}\right)$ and $w = 2\operatorname{cis}(\pi)$, then $\operatorname{Arg}(zw) =$

- A. $\frac{5\pi}{4}$
 B. $-\frac{3\pi}{4}$
 C. $-\frac{5\pi}{4}$
 D. $\frac{\pi^2}{4}$
 E. $\frac{5\pi}{2}$

Working space

$$\begin{aligned} \operatorname{Arg}(zw) &= \left(\frac{\pi}{4} + \pi\right) \\ &= \frac{5\pi}{4} \\ &= -\frac{3\pi}{4} \end{aligned}$$

Question 10

If k is a real constant and the imaginary part of $\frac{6+ki}{2k+3i}$ equal to zero, then

- A. $k = \pm 1$
 B. $k = 0$
 C. $k = \pm 3$
 D. $k = 9$
 E. $k = -6$

Working space

$$\begin{aligned} \frac{6+ki}{2k+3i} &= \frac{6+ki}{2k+3i} \times \frac{2k-3i}{2k-3i} \\ &= \frac{12k - 18i + 2k^2i + 3k}{(2k)^2 + 3^2} \\ 2k^2 - 18 &= 0 \\ k^2 &= 9 \\ k &= \pm 3 \end{aligned}$$

Short answer questions (25 marks)

1. Find a and b if $(3-i)(a+bi) = 5+5i$.

$$3a + 3bi - ia + b = 5 + 5i \quad (1)^M$$

$$3a + b = 5 \quad \text{--- (1)}$$

$$-a + 3b = 5 \quad \text{--- (2)}$$

$$\left. \begin{array}{l} 3 \times (1) - (2) \text{ gives } 9a + 3b = 15 \\ -a + 3b = 5 \end{array} \right\} (1)^M$$

$$\hline 10a = 10$$

$$a = 1$$

$$\begin{aligned} b &= 5 - 3a \\ &= 5 - 3 \\ &= 2 \end{aligned}$$

$$\therefore a = 1 \quad (1)^A$$

$$b = 2 \quad (1)^A$$

(3 marks)

2. If $z = 2 + i$, find $(z - \bar{z})^2$

$$\bar{z} = 2 - i \quad (1)$$

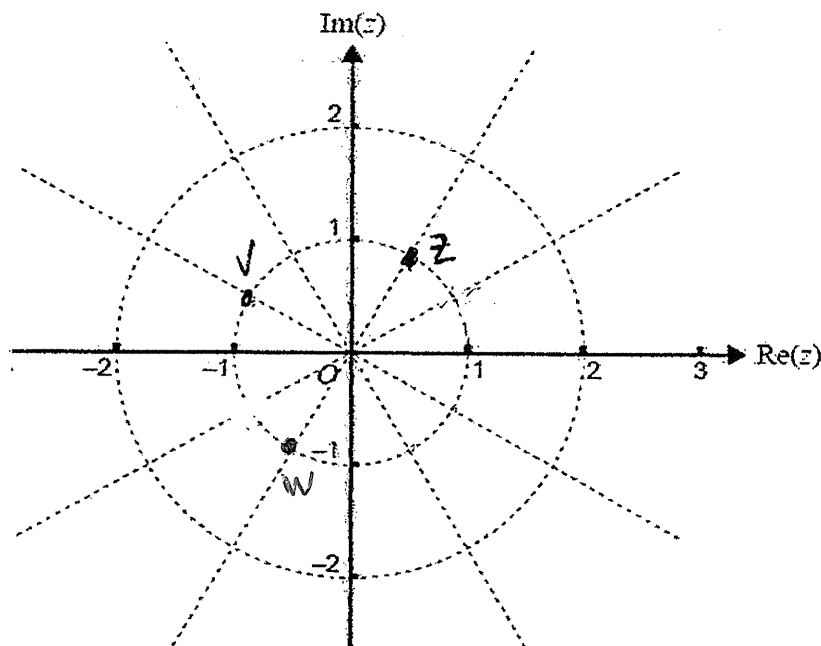
$$\begin{aligned} z - \bar{z} &= 2 + i - 2 + i \\ &= 2i \quad (1) \end{aligned}$$

$$\begin{aligned} (z - \bar{z})^2 &= (2i)^2 \\ &= -4 \quad (1) \end{aligned}$$

(3 marks)

3. a) On the Argand plane below, plot and clearly label the complex number $z = \text{cis}\left(\frac{\pi}{3}\right)$.

deduct 1-mark
if not
labelled.



(1) point z

(1) point w

(1) point v

(1)^M for getting
to w and v .

b) On the Argand plane above also plot and clearly label $w = (\bar{z})^2$ and $v = -i^3 z$

$$\bar{z} = \text{cis}\left(-\frac{\pi}{3}\right)$$

$$(\bar{z})^2 = \text{cis}\left(-\frac{2\pi}{3}\right)$$

$$v = \text{cis}\left(\frac{5\pi}{6}\right)$$

(1+3=4 marks)

4. The complex numbers z_1 and z_2 are given by: $z_1 = \frac{1+i}{1-i}$ and $z_2 = \frac{1}{2} + \frac{\sqrt{3}}{2}i$

a) Show that $z_1 = i$.

$$z_1 = \frac{1+i}{1-i} \times \frac{1+i}{1+i} \quad (1)$$

$$= \frac{1+2i+i^2}{2} \quad (2)$$

$$= \frac{2i}{2} \quad (3)$$

$$= i \quad \text{as required.}$$

b) Show that $|z_1| = |z_2|$.

$$|z_1| = \sqrt{1^2} = 1 \quad (1)$$

$$|z_2| = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} \quad (2)$$

$$= \sqrt{\frac{1}{4} + \frac{3}{4}}$$

$$= \sqrt{1} \quad (3)$$

$$= 1$$

$\therefore |z_1| = |z_2|$ as required.

c) Express both z_1 and z_2 in polar form.

$$z_1 = i = 1 \operatorname{cis} \frac{\pi}{2}$$

$$z_2 = \frac{1}{2} + \frac{\sqrt{3}}{2}i$$

$$\tan \theta = \frac{\sqrt{3}}{1}$$

$$\theta = \frac{\pi}{3} \quad (1)$$

$$\therefore z_1 = 1 \operatorname{cis} \frac{\pi}{2} \quad (1)$$

$$z_2 = 1 \operatorname{cis} \frac{\pi}{3} \quad (1)$$

d) Hence, find $z_1 z_2$ in polar form.

$$z_1 z_2 = \left(\operatorname{cis} \frac{\pi}{2} \right) \times \left(\operatorname{cis} \frac{\pi}{3} \right)$$

$$= \operatorname{cis} \left(\frac{\pi}{2} + \frac{\pi}{3} \right) \quad (1)$$

$$= \operatorname{cis} \left(\frac{5\pi}{6} \right) \quad (1)$$

(2 + 2 + 3 + 2 = 9 marks)

5. Solve $z^4 - 9 = 0$. Express your answers in Cartesian form.

$$(z^2)^2 - 3^2 = (z^2 - 3)(z^2 + 3) = 0 \quad \textcircled{1}$$

$$(z - \sqrt{3})(z + \sqrt{3})(z + \sqrt{3}i)(z - \sqrt{3}i) = 0 \quad \textcircled{1}$$

$$z = \sqrt{3}, -\sqrt{3}, -\sqrt{3}i, \sqrt{3}i \quad \textcircled{1}$$

Alternate method can be used.

(3 marks)

6. Factorise $z^2 - 2z + 17$ over the set of complex numbers, $\mathbb{C}$.

$$z^2 - 2z + 17 = z^2 - 2z + 1^2 + 17 - 1^2 \quad \textcircled{1}$$

$$= (z - 1)^2 + 16 \quad \textcircled{1}$$

$$= (z - 1 - 4i)(z - 1 + 4i) \quad \textcircled{1}$$

Deduct 1 mark if they solved not factorised.

(3 marks)