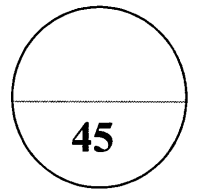




Year 11 Specialist Maths 2016
Statistics and Probability Test

Technology Allowed Assessment



Name: SOLUTIONS

Multiple Choice (10 x 2 marks = 20 marks)

1. A random sample of 20 people is taken from a population of 500. The people in the sample have an average of 1.2 pets. The numbers 20 and 1.2 are the:
- A Population size and sample mean
 - B Sample size and sample proportion
 - ☒ C Sample size and sample mean
 - D Population size and population mean
 - E Sample size and population proportion

2. A random sample must:

- A Be selected using a random number generator.
- B Include all subgroups of the population.
- C Be conducted anonymously.
- D Consider the opinions of all minority groups.
- ☒ E Have an equal chance of every member of the population being included.

3. Using the table below, find $\Pr(\hat{P} \geq 0.6)$.

$\hat{p}$	0	0.2	0.4	0.6	0.8	1
$\Pr(\hat{P} = \hat{p})$	0.12	0.09	0.17	0.22	0.21	0.19

- A 0.40
 - B 0.60
 - C 0.21
 - ☒ D 0.62
 - E 0.81
4. In Australia the probability of having blue eyes is 0.22. In a random sample of 10 people the probability of exactly three people having blue eyes is:
- A $(0.22)^3(0.78)^7$
 - ☒ B ${}^{10}C_3(0.22)^3(0.78)^7$
 - C ${}^{10}C_3(0.22)^7(0.78)^3$
 - D ${}^7C_3(0.22)^3(0.78)^7$
 - E ${}^{10}C_3(0.22)^3(0.78)^{10}$

5. A box of squash balls has 10 slow and 15 fast balls. Five balls are selected **with** replacement. The probability of selecting 2 slow and 3 fast balls is approximately:
- A 0.3854
 - B 0.2304
 - ☒ C 0.3456
 - D 0.2372
 - E 0.3957
6. Mangos from a farm have a 0.08 probability of being rotten. A buyer takes a random sample of 20 mangos and will reject the batch if two or more mangos are rotten. The probability of rejecting the batch is closest to:
- A 0.5169
 - B 0.8113
 - C 0.1887
 - D 0.3859
 - ☒ E 0.4831
7. A class contains 12 boys and 10 girls. A team of five students is selected without replacement. The probability that the proportion of girls is greater than 0.5 is approximately:
- ☒ A 0.4060
 - B 0.5940
 - C 0.7820
 - D 0.0549
 - E 0.4152
8. It is acceptable to use a binomial distribution to calculate probabilities for a random sample if:
- A The sample size is large.
 - B The population size is small.
 - C The sample size is less than the population size.
 - ☒ D The population size is much bigger than the sample size.
 - E It is never acceptable.
9. In a batch of 50 cars, 30% are manual and 70% are automatic. Eight cars are sold in one week. The probability that the proportion of manuals is less than 30% is closest to:
- A 0.5
 - ☒ B 0.5492
 - C 0.4508
 - D 0.5518
 - E 0.4482

10. When considering the distribution of the sample mean, the effect of increasing the sample size is:

- A To reduce the mean of the sample distribution.
- B To increase the mean of the sample distribution.
- C To make the sampling distribution more asymmetric.
- D To increase the variation of the sampling distribution.
- ☒ E To reduce the variation of the sampling distribution.

Short Answer Questions (25 Marks)

1. In the table below, x is the number of colds a child gets per year.

x	1	2	3	4	5
$\Pr(X=x)$	n	$3n$	n^2	$2n$	0.0144

a) Find the value of n .

$$n^2 + 6n + 0.0144 = 1 \quad (1/2)$$

$$n = -6.16, 0.16 \quad (1/2)$$

$$\text{SINCE } n \geq 0 \Rightarrow n = 0.16 \quad (1/2)$$

b) Calculate the probability that a child gets three or more colds per year.

$$\Pr(X \geq 3) = 0.16^2 + 2 \times 0.16 + 0.0144 \quad (1)$$

$$= 0.36 \quad (1)$$

(4 marks)

2. A school has 1284 students of which 693 are female. A random sample of 25 students is taken and 12 are female.

a) What is the value of the population proportion p ? (Answer to four decimal places)

$$\frac{693}{1284} \approx 0.5397 \quad (1/2)$$

b) What is the value of the sample proportion $\hat{p}$? (Exact value)

$$\frac{12}{25} \quad \text{OR} \quad 0.48 \quad (1)$$

c) Using your CAS and assuming a **hypergeometric** distribution calculate the probability of 12 students in the sample being female. (Answer to four decimal places)

$$0.1331 \quad (2)$$

d) Now using your CAS and assuming a **binomial** distribution calculate the probability of 12 students in the sample being female. (Answer to four decimal places)

$$0.1322 \quad (2)$$

e) Comment on the results in b) and c) in relation to the sizes of the population and sample.

• VERY SIMILAR (1)

• LARGE POPULATION SIZE COMPARED TO SAMPLE SIZE (1)

(1 + 1 + 2 + 2 + 2 = 8 marks)

3. A school has 150 lockers of which 10 are broken. A class of 20 are randomly allocated lockers.

Give all answers in this question to four decimal places.

a) Complete the table below to show the hypergeometric distribution of the proportion of broken lockers given to the class ($\hat{p}$).

(-1M EACH INCORRECT ANSWER)

$\hat{p}$	0	$\frac{1}{20}$	$\frac{1}{10}$	$\frac{3}{20}$	$\frac{1}{5}$	$\frac{1}{4}$	$\geq \frac{3}{10}$
$\Pr(\hat{p} = \hat{p})$	0.2277	0.3765	0.2639	0.1030	0.0247	0.0038	0.0004

Use the table to calculate:

$$\text{b) } \Pr(\hat{p} \leq 0.1) = 0.2277 + 0.3765 + 0.2639 \quad (1/2)$$

$$= 0.8681 \quad (1/2)$$

$$\text{c) } \Pr(\hat{p} \leq 0.2) = 0.8681 + 0.1030 + 0.0247 \quad (1/2)$$

$$= 0.9958 \quad (1/2)$$

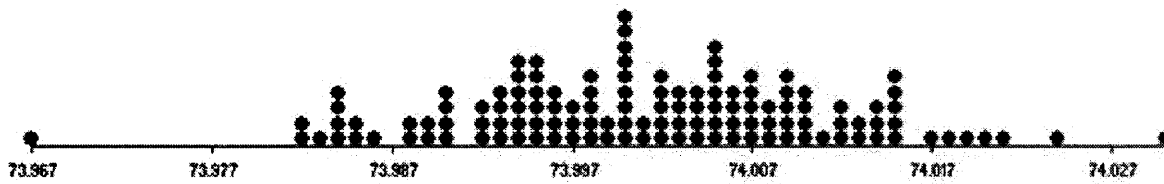
$$\text{d) } \Pr(\hat{p} \leq 0.1 | \hat{p} \leq 0.2) = \frac{0.8681}{0.9958} = 0.8718$$

(1/2) (1/2)

(5 + 1 + 1 + 1 = 8 marks)

4. The dot plot below shows the distribution of the sample mean for samples of ten pistons.

Piston Diameter Dot Plot



a) Use the dot plot to estimate the population mean (Do not do any calculations, estimate visually)

74.001 (ACCEPT 73.998 → 74.004)

b) Use the dot plot to estimate the probability that the sample mean is greater than 74.016 given that it is known the sample mean is greater than 74.007

$$Pr(X > 74.016 \mid Pr(X > 74.007)) = \frac{Pr(X > 74.016)}{Pr(X > 74.007)} = \frac{7}{33}$$

(1) (1)

c) Explain what effect there would be on the sample distribution if the sample size was increased to 50.

• MEAN REMAINS APPROXIMATELY THE SAME (1)

• LESS VARIATION (1)

(1 + 2 + 2 = 5 marks)