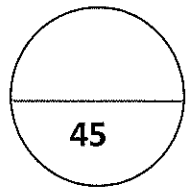




General Maths (Advanced) 2015
Algebra Test
Technology Free Assessment



Name: Solutions

Multiple Choice (10 x 2 = 20 marks). Circle the correct answer.

Question 1

Solve the following equation for x : $\frac{1-2x}{4} + \frac{x}{6} = -\frac{1}{2}$

A. $-\frac{9}{4}$

☒ B. $\frac{9}{4}$

C. $\frac{4}{9}$

D. $-\frac{4}{9}$

E. 0

B

Working space

Question 2

If $P = \frac{2R+5}{6}$ is transposed to make R the subject, then:

A. $R = \frac{5P-6}{2}$

B. $R = 3P - 5$

☒ C. $R = 3P - \frac{5}{2}$

D. $R = 6P - \frac{5}{2}$

E. $R = 2(6P - 5)$

C

Working space

Question 3

Two bottles of Coke and an ice-cream cost \$5, while 1 bottle of Coke and 2 ice-creams cost \$4. Find the cost of a bottle of Coke and an ice-cream.

A. Coke \$1.50, ice-cream \$1.50

B. Coke \$1, ice-cream \$1

C. Coke \$2, ice-cream \$2

D. Coke \$1, ice-cream \$2

☒ E. Coke \$2, ice-cream \$1

E

Working space

Question 4

The point(s) of intersection between the parabola $y = x^2 + 5x - 3$ and the straight line $y = 2x + 1$ is/are:

A. (1, 3)

B. (1, 3), (2, 5)

☒ C. (1, 3), (-4, -7)

D. (2, 5)

E. (5, 2), (2, 5)

C

Working space

Question 5

Working space

James has three lengths of string. One length is x cm, the second y mm and the third z m. The total length of string in centimetres is: A. $x + y + z$ B. $x + 10y + 100z$ C. $x + \frac{y}{10} + \frac{z}{100}$ D. $x + 10y + \frac{z}{100}$ ☒ E. $x + \frac{y}{10} + 100z$ E	
Question 6 A number is multiplied by three, then two thirds is subtracted from the number. The result is three quarters. The number is: A. $\frac{1}{36}$ B. $\frac{3}{21}$ ☒ C. $\frac{17}{36}$ D. $\frac{21}{12}$ E. $\frac{11}{12}$ C	Working space
Question 7 Expressing $\frac{5(x-1)}{3} - \frac{(x+3)}{2x}$ as $\frac{ax^2 + bx + c}{6x}$, then a, b and c, respectively, equal: A. 10, 13, 9 B. 10, -13, 9 C. -10, -13, -9 ☒ D. 10, -13, -9 E. 10, 13, -9 D	Working space
Question 8 The line with equation $y = x + a$ intersects the circle with equation $x^2 + y^2 = 1$ twice if, and only if: A. $a > 0$ ☒ B. $-\sqrt{2} < a < \sqrt{2}$ C. $a^2 > 2$ D. $a \leq 2$ E. $a < 1$ B	Working space

Question 9 A car travels a distance d km at x km/h and then a distance of $2d$ km at y km/h. It takes 1 hour more to travel the $2d$ km than the d km. An equation which describes this is: A. $dx - 2dy = 1$ B. $\frac{2d}{y} - \frac{d}{x} + 1 = 0$ C. $2dy - dx = 1$ D. $\frac{d}{x} - \frac{2d}{y} - 1 = 0$ E. $\frac{2d}{y} - \frac{d}{x} - 1 = 0$ <i>E</i>	Working space
Question 10 The expression $\frac{4}{x+3} - \frac{3}{2x+1}$ can be simplified as: A. $\frac{5x-5}{(x+3)(2x+1)}$ B. $\frac{5x-13}{(x+3)(2x+1)}$ C. $\frac{12}{(x+3)(2x+1)}$ D. $\frac{x+7}{(x+3)(2x+1)}$ E. $\frac{11x-13}{(x+3)(2x+1)}$ <i>A</i>	Working space

Short answer questions

1. Solve the following literal equation for x .

$$\frac{2}{3x} + 11b = \frac{5}{3x} + 2b$$

$$\begin{aligned} \frac{2}{3x} - \frac{5}{3x} &= 2b - 11b \\ \frac{-3}{3x} &= -9b \\ x &= \frac{1}{9b} \end{aligned} \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} \textcircled{1}^M$$

$\textcircled{1}^A$

(2 marks)

2. Express $\frac{2}{x+2} - \frac{3}{(x+2)^2}$ as a single fraction.

$$\frac{2}{x+2} - \frac{3}{(x+2)^2} = \frac{2(x+2) - 3}{(x+2)^2} \quad (1)^M$$

$$= \frac{2x+1}{(x+2)^2} \quad (1)^A$$

(2 marks)

3. Solve the equation $\frac{12}{x} + \frac{12}{x+8} = 4$ for x .

$$\begin{aligned} 12(x+8) + 12x &= 4x(x+8) & \left. \begin{array}{l} \\ \\ \end{array} \right\} (1)^M \\ 12x + 96 + 12x &= 4x^2 + 32x \\ 4x^2 + 8x - 96 &= 0 \\ x^2 + 2x - 24 &= 0 \\ (x+6)(x-4) &= 0 & \left. \begin{array}{l} \\ \\ \end{array} \right\} (1)^M \end{aligned}$$

$$\therefore x = -6 \text{ or } x = +4 \quad (1)^A$$

(3 mark)

4. Express $\frac{6x^2 - 11x + 1}{x^3 - 2x^2 + x}$ as partial fractions

$$\frac{6x^2 - 11x + 1}{x^3 - 2x^2 + x} = \frac{6x^2 - 11x + 1}{x(x-1)^2} \quad \left(\frac{1}{2}\right)^M$$

$$= \frac{A}{x} + \frac{B}{x-1} + \frac{C}{(x-1)^2} \quad (1)^M$$

$$\begin{aligned} 6x^2 - 11x + 1 &= A(x-1)^2 + B(x)(x-1) + Cx & \left(\frac{1}{2}\right)^M \\ &= (A+B)x^2 - (2A+B-C)x + A \end{aligned}$$

Method 1

$$\begin{aligned} A &= 1 \\ A+B &= 6 \\ B &= 5 \\ 2A+B-C &= 11 \\ C &= -4 \end{aligned}$$

Method 2

$$\begin{aligned} \text{Let } x &= 1 & \left(\frac{1}{2}\right)^A & \text{Let } x = -1 \\ -4 &= C & 18 &= 4A + 2B - C \\ \text{Let } x &= 0 & 2B &= 18 + 4 - 4 = 10 \\ 1 &= A & B &= 5 \quad \left(\frac{1}{2}\right)^A \end{aligned}$$

(4 marks)

$$\therefore \frac{6x^2 - 11x + 1}{x^3 - 2x^2 + x} = \frac{1}{x} + \frac{5}{x-1} - \frac{4}{(x-1)^2} \quad \left(\frac{1}{2}\right)^A$$

for rewriting

5. If 5 is subtracted from the numerator of a fraction and 6 from its denominator, the value of the fraction becomes $\frac{1}{2}$. If 3 is added to the numerator and 2 to the denominator, the value becomes $\frac{5}{6}$. What is the fraction?

Let numerator = x and let denominator = y

$$\textcircled{1} \quad \frac{x-5}{y-6} = \frac{1}{2} \quad \text{Therefore } 2x - y = 4 \dots \textcircled{1}$$

$$\textcircled{2} \quad \frac{x+3}{y+2} = \frac{5}{6} \quad \text{Therefore } 6x - 5y = -8 \dots \textcircled{2}$$

Sub $\textcircled{1}$ into $\textcircled{2}$

$$6x - 5(2x - 4) = -8$$

$$6x - 10x + 20 = -8$$

$$-4x = -28$$

$$x = 7$$

$$y = 2x - 4 = 10$$

$\textcircled{1}^M$

$\therefore$ The fraction is $\frac{7}{10}$ $\textcircled{1}^A$

(3 marks)

6. By finding the values of a , b and c , express $5x^2 + 7x + 12$ in the form $a(x+1)^2 + b(x+1) + c$

$$a(x^2 + 2x + 1) + b(x+1) + c = 5x^2 + 7x + 12$$

$$ax^2 + 2ax + a + bx + b + c = 5x^2 + 7x + 12 \quad \textcircled{1}^M$$

$$1) \quad a = 5 \quad \textcircled{2}$$

$$2a + b = 7$$

$$2) \quad b = 7 - 10 = -3 \quad \textcircled{1}$$

$$3) \quad a + b + c = 12$$

$$c = 12 + 3 - 5 = 10 \quad \textcircled{1}$$

$$\therefore 5x^2 + 7x + 12 = 5(x+1)^2 - 3(x+1) + 10 \quad \textcircled{\frac{1}{2}} \text{ for rewriting answer.}$$

(4 marks)

7. A shopkeeper mixed Herb A worth \$2 a kilogram with another one, herb B worth \$3 a kilogram. He sold 20 kilograms of this mixture at \$2.80 per kilogram. Find the weights of the two different herbs he mixed together.

Let herb A = x and herb B = y

$$\begin{aligned} x + y &= 20 & \dots \textcircled{1} & \textcircled{1}^M \\ 2x + 3y &= 20 \times 2.8 & \dots \textcircled{2} & \end{aligned}$$

$$2 \times \textcircled{1} - \textcircled{2}$$

$$\begin{array}{r} 2x + 2y = 40 \\ 2x + 3y = 56 \\ \hline \end{array}$$

$$y = 16$$

Sub $y = 16$ into $\textcircled{1}$

$$\begin{aligned} x &= 20 - 16 \\ &= 4 \end{aligned}$$

$$\therefore \left. \begin{aligned} \text{Herb A} &= 4 \text{ kg} \\ \text{Herb B} &= 16 \text{ kg} \end{aligned} \right\} \textcircled{1}^M$$

(3 marks)

8. Two men 50 km apart move towards one another. One travels at 12 km/h and the other at 4 km/h. After how many minutes do they meet?

$$A \quad \frac{x}{12 \text{ km/hr}} \quad \frac{50-x}{4 \text{ km/hr}} \quad B$$

Time taken equal for both men

$$\frac{x}{12} = \frac{50-x}{4} \quad \textcircled{1}^M$$

$$4x = 600 - 12x$$

$$16x = 600$$

$$x = \frac{75}{2} = 37.5 \text{ km} \quad \textcircled{1}^A$$

$$\begin{aligned} \text{Time taken} &= \frac{37.5}{12} \times 60^5 \quad \textcircled{1}^M \\ &= 187.5 \text{ min.} \end{aligned}$$

The men will meet each other in 187.5 min. $\textcircled{1}^A$
(or $\frac{385}{2}$ min)

OR Relative speed is $12 + 4 = 16 \text{ km/hr}$. $\textcircled{1}$

$$t = \frac{50}{16} \times 60^5 \quad \textcircled{1}$$

(4 marks)

END OF TEST

$$= \frac{750}{4}$$

$$= 187.5 \text{ min.} \quad \textcircled{1}$$

13. The cost \$C\$, of printing a poster consists of two parts:

14. • A fixed charge, \$a\$, for setting up the type
• A charge for printing and paper which increases linearly as the number of copies printed, \$n\$.

If the cost of 150 posters is \$6 and the cost of 900 copies is \$31, find the cost of 1200 copies.

$$C = a + bn$$

$$6 = a + 150b \quad \text{--- (1)} \quad \left(\frac{1}{2}\right)$$

$$31 = a + 900b \quad \text{--- (2)} \quad \left(\frac{1}{2}\right)$$

$$(2) - (1)$$

$$25 = 750b$$

$$\therefore b = \frac{25}{750} = \frac{1}{30} \quad \text{--- (1)}$$

sub into (1)

$$6 = a + 150 \times \frac{1}{30}$$

$$= a + 5$$

$$\therefore a = 1 \quad \text{--- (1)}$$

14. Express $\frac{5x^2 - 6x + 7}{(x^2 + x + 5)(x - 3)}$ in partial fractions.

15.

$$\frac{5x^2 - 6x + 7}{(x^2 + x + 5)(x - 3)} = \frac{Ax + B}{x^2 + x + 5} + \frac{C}{x - 3} \quad \left(\frac{1}{2}\right)$$

$$\therefore 5x^2 - 6x + 7 = (Ax + B)(x - 3) + C(x^2 + x + 5) \quad \left(\frac{1}{2}\right)$$

• Let \$x = 3\$.

$$45 - 18 + 7 = C(9 + 3 + 5)$$

$$34 = 17C$$

$$\therefore C = 2 \quad \text{--- (1)}$$

• Let \$x = 0\$

$$7 = -3B + 5C$$

$$7 = -3B + 10$$

$$\therefore B = 1 \quad \text{--- (1)}$$

• Let \$x = 1\$

$$5 - 6 + 7 = (A + 1)(-2) + 2(7) \quad \left(\frac{1}{2}\right)$$

$$6 - 14 = -2(A + 1)$$

$$A + 1 = 4$$

$$\therefore A = 3 \quad \text{--- (1)}$$

$$\therefore \frac{5x^2 - 6x + 7}{(x^2 + x + 5)(x - 3)} = \frac{3x + 1}{x^2 + x + 5} + \frac{2}{x - 3} \quad \left(\frac{1}{2}\right)$$

$$C(n) = 1 + \frac{1}{30}n$$

$$C(1200) = 1 + \frac{1}{30}(1200) \quad \left(\frac{1}{2}\right)$$

$$= 1 + 40$$

$$= 41$$

cost for 1200 copies is \$41. $\left(\frac{1}{2}\right)$

(4 marks)

$$\begin{aligned} A + C &= 5 & \text{--- (1)} \\ -3A + B + C &= -6 & \text{--- (2)} \\ -3B + 5C &= 7 & \text{--- (3)} \end{aligned}$$

$$\text{From (1)} \quad A = 5 - C$$

$$\text{Using (2)} \quad -3(5 - C) + B + C = -6$$

$$B + 4C = 9 \quad \text{--- (4)}$$

$$3 \times (4) - (3) :$$

$$\begin{array}{r} 3B + 12C = 27 \\ -3B + 5C = 7 \\ \hline 17C = 34 \end{array} \quad +$$

$$17C = 34$$

$$C = 2.$$

$$A = 5 - C = 3$$

$$B = 1$$

(5 marks)

15. Solve the simultaneous literal equations for x and y :

16. $\frac{x}{a} + \frac{y}{b} = 1$ — (1)

$\frac{x}{b} + \frac{y}{a} = 1$ — (2)

(1) $\times ab$

$bx + ay = ab$ — (1') $\left(\frac{1}{2}\right)$

(2) $\times ab$

$ax + by = ab$ — (2') $\left(\frac{1}{2}\right)$

(1') $\times b$

$b^2x + aby = ab^2$ (1'') $\left(\frac{1}{2}\right)$

(2') $\times a$

$a^2x + aby = a^2b$ (2'') $\left(\frac{1}{2}\right)$

(1'') - (2'')

$(b^2 - a^2)x = ab(b - a)$ $\left(\frac{1}{2}\right)$

$x = \frac{ab(b-a)}{b^2 - a^2}$ $\left(\frac{1}{2}\right)$

$= \frac{ab(b-a)}{(b+a)(b-a)}$ $\left(\frac{1}{2}\right)$

$= \frac{ab}{a+b}$ $\left(\frac{1}{2}\right)$

sub into (1')

$\frac{b(ab)}{a+b} + y = ab$ $\left(\frac{1}{2}\right)$

$y = b - \frac{b^2}{a+b}$ $\left(\frac{1}{2}\right)$

$= \frac{ab + b^2 - b^2}{a+b}$ $\left(\frac{1}{2}\right)$

$= \frac{ab}{a+b}$ $\left(\frac{1}{2}\right)$

Alternate by

(1) $\times b$ — (2) $\times a$

$\frac{xb}{a} + y = b$ — (3)

$\frac{xa}{b} + y = a$ — (4)

(3) - (4) $\frac{xb}{a} - \frac{xa}{b} = b - a$

$\frac{xb^2 - a^2x}{ab} = b - a$

$\frac{x(b+a)(b-a)}{ab} = (b-a)$
 $x = \frac{ab}{a+b}$

(6 marks)