

Answers

TOPIC 2 Algebra and equations

Exercise 2.2 — Substitution

- 1 a 5 d 6 g 30 j 27
2 a -11 d 30 g -125
3 a $\frac{7}{12}$ d $1\frac{1}{3}$
4 a 9 d 18
5 a 9 d 18
6 a 17 d 4 g 46 j 624.6
7 a D b C c B
8 3.9 cm
9 65.45 cm³
10 361 m
11 a -1 — in this case, addition is closed on integers.
b -1 — in this case, subtraction is closed on integers.
c 2 — in this case, multiplication is closed on integers.
d -1 — in this case, division is closed on integers.
e -2 — in this case, subtraction is closed on integers.
f $-\frac{1}{2}$ — in this case, division is not closed on integers.
12 a 10 — in this case, addition is closed on natural numbers.
b -4 — in this case, subtraction is not closed on natural numbers.
c 12 — in this case, multiplication is closed on natural numbers.
d $\frac{4}{3}$ — in this case, division is not closed on natural numbers.
e -2 — in this case, subtraction is not closed on natural numbers.
f 4 — in this case, division is closed on natural numbers.
13 a $(a + 2b) + 4c = a + (2b + 4c)$
b $(x \times 3y) \times 5c = x \times (3y \times 5c)$
c $2p \div q \neq q \div 2p$
d $5d + q = q + 5d$
e $3z + 0 = 0 + 3z = 3z$
f $2x \times \frac{1}{2x} = \frac{1}{2x} \times 2x = 1$
g $(4x \div 3y) \div 5z \neq 4x \div (3y \div 5z)$
h $3d - 4y \neq 4y - 3d$
14 a $s = 71.875$ metres b $t = 4$ seconds
15 $m = \frac{4}{5}$

Challenge 2.1

$$3a^3c^2$$

Exercise 2.3 — Adding and subtracting algebraic fractions

- 1 a $\frac{26}{21}$ or $1\frac{5}{21}$ b $\frac{49}{72}$ c 1
d $\frac{17}{99}$ e $\frac{1}{35}$ f $\frac{6-5x}{30}$
g $\frac{15x-4}{27}$ h $\frac{15-16x}{40}$ i $\frac{15-2x}{3x}$

- 2 a $\frac{5y}{12}$ b $-\frac{3y}{40}$ c $\frac{13x}{12}$
d $\frac{14x}{9}$ e $\frac{3w}{28}$ f $-\frac{y}{5}$
g $\frac{89y}{35}$ h $\frac{32x}{15}$ i $\frac{7x+17}{10}$
j $\frac{7x+30}{12}$ k $\frac{2x-11}{30}$ l $\frac{19x+7}{6}$
3 a $\frac{5}{8x}$ b $\frac{5}{12x}$ c $\frac{38}{21x}$
d $\frac{8}{3x}$ e $\frac{7}{24x}$ f $\frac{9}{20x}$
g $\frac{37}{100x}$ h $\frac{51}{10x}$ i $-\frac{1}{6x}$
4 a $\frac{3x^2+14x-4}{(x+4)(x-2)}$ b $\frac{2x^2+3x+25}{(x+5)(x-1)}$ c $\frac{2x^2+6x-10}{(2x+1)(x-2)}$
d $\frac{4x^2-17x-3}{(x+1)(2x-7)}$ e $\frac{7x^2+x}{(x+7)(x-5)}$ f $\frac{2x^2+6x+7}{(x+1)(x+4)}$
g $\frac{-x^2+7x+15}{(x+1)(x+2)}$ h $\frac{x-7}{(x+3)(x-2)}$ i $\frac{x^2+3x+9}{(x+2)(3x-1)}$
j $\frac{5-5x}{(x-1)(1-x)} = \frac{5}{x-1}$
k $\frac{3x+7}{(x+1)^2}$ l $\frac{3x-4}{(x-1)^2}$
5 a The student transcribed the denominator incorrectly and wrote $(x+2)$ instead of $(x-2)$ in line 2. Also, the student forgot that multiplying a negative number by a negative number gives a positive number. Line 3 should have +3 in the numerator, not -1. They didn't multiply.
b $\frac{x^2-5x+3}{(x-1)(x-2)}$
6 a $\frac{4x^2+17x+17}{(x+2)(x+1)(x+3)}$ b $\frac{7x^2-20x+4}{(x-1)(x+2)(x-4)}$
c $\frac{4x^2+17x+19}{(x+1)(x+3)(x+2)}$ d $\frac{2(2x^2-9x+25)}{(x-4)(x-1)(x+3)}$
7 The lowest common denominator may not always be the product of the denominators. Each fraction must be multiplied by the correct multiple.
8 $a = 4$
9 $\frac{4(x-1)}{(x+3)(x+4)(x-2)}$
10 $\frac{2(x-1)}{(x-7)(x-4)}$

Exercise 2.4 — Multiplying and dividing algebraic fractions

- 1 a $\frac{4x}{y}$ b $\frac{3x}{y}$ c $\frac{4y}{x}$
d $\frac{9x}{4y}$ e $-\frac{5x}{4y}$ f $\frac{3w}{2x}$

- g** $\frac{6z}{7x}$ **h** $\frac{2z}{7x}$ **i** $\frac{-3x}{2y}$ **7 a** $a = 4$ **b** $b = 6$ **c** $i = 3$
- j** $\frac{5}{24}$ **k** $\frac{12z}{x}$ **l** $\frac{-x}{6w}$ **d** $f = 9$ **e** $q = 1\frac{1}{8}$ **f** $r = 5\frac{2}{5}$
- 2 a** $\frac{2}{3x-2}$ **b** $\frac{5}{x-3}$ **c** $\frac{9}{2(x-6)}$ **g** $s = 4\frac{5}{6}$ **h** $t = 9\frac{4}{5}$ **i** $a = -7\frac{1}{2}$
- d** $\frac{1}{x+3}$ **e** $\frac{2x}{(x+1)^2}$ **f** $\frac{x+1}{2(2x-3)}$ **8 a** $f = 40$ **b** $g = 30$ **c** $r = -10$
- g** $\frac{a}{10(a+3)}$ **h** $\frac{35d}{8(d-3)}$ **i** $\frac{9}{32x^2(x-2)}$ **d** $m = 18$ **e** $n = 28$ **f** $p = 62.4$
- j** $\frac{3x}{10(x-1)}$ **3 a** $\frac{3}{5}$ **b** $\frac{2}{9}$ **c** $\frac{1}{3}$ **g** $x = 1\frac{1}{3}$ **h** $y = 9$ **c** $m = 4\frac{2}{5}$
- d** 3 **e** $\frac{1}{25}$ **f** $\frac{35}{6}$ or $5\frac{5}{6}$ **d** $k = 1\frac{1}{2}$ **e** $n = 5\frac{2}{3}$ **f** $c = 1\frac{1}{3}$
- g** $\frac{4y^2}{7}$ **h** $\frac{2y^2}{25}$ **i** $\frac{8y^2}{9}$ **g** $x = 10$ **h** $x = 4$ **i** $x = -7\frac{1}{5}$
- j** $\frac{32xy}{15}$ **k** $\frac{2}{3}$ **l** y^2 **10 a** $k = 25$ **b** $m = 16$ **c** $p = -11\frac{3}{7}$
- 4 a** $\frac{9}{(3x-7)(x+3)}$ **b** $\frac{1}{(x+2)(x-9)}$ **d** $u = -4\frac{1}{8}$ **e** $x = \frac{8}{11}$ **f** $v = 3$
- c** $\frac{21(x-3)}{x+5}$ **d** $\frac{13}{9(x-4)(x+1)}$ **11 a** $x = 26$ **b** $m = -5$ **c** $w = \frac{25}{3}$

5 Yes, because all of the fractions have the same denominator and therefore can be added together.

- 6 a** -1
b $4 - x$ considered to be the same as $x - 4$
- 7** 1
8 $\frac{(x+1)^2}{x^2+1}$

Challenge 2.2

$$\frac{x(x+2)}{(x-1)(x+1)} \text{ or } \frac{x(x+2)}{x^2-1}$$

Exercise 2.5 — Solving simple equations

- 1 a** $a = 24$ **b** $k = 121$ **c** $g = 2.9$
d $r = 3$ **e** $h = 0.26$ **f** $i = -2$
- g** $t = 5$ **h** $q = \frac{1}{6}$ **i** $x = 0$
2 a $f = 12$ **b** $i = -60$ **c** $z = -7$
- d** $v = 7$ **e** $w = -5\frac{1}{3}$ **f** $k = 10$
g $a = 0.425$ **h** $m = 16\frac{5}{8}$ **i** $y = 21\frac{1}{2}$
- 3 a** $t = 100$ **b** $y = \pm 17$ **c** $q = 6.25$
d $f = \pm 1.2$ **e** $h = \frac{16}{49}$ **f** $p = \pm \frac{3}{8}$
- g** $g = \frac{225}{484}$ **h** $j = \pm \frac{14}{31}$ **i** $a = \pm 1\frac{2}{3}$
4 a $t = 25$ **b** $x = \pm 6$ **c** $m = 16$
d $t = \pm 3$ **e** $t = \pm 10$ **f** $m = 25$
- 5 a** $x = 8$ **b** $x = -3$ **c** $m = \frac{1}{8}$
d $x = \frac{3}{4}$ **e** $m = 0.008$ **f** $w = 2\frac{1}{2}$
- 6 a** $x = -1$ **b** $x = -2$ **c** $m = 1$
d $w = -512$ **e** $t = 125$ **f** $x = 2$

- 7 a** $a = 4$ **b** $b = 6$ **c** $i = 3$
d $f = 9$ **e** $q = 1\frac{1}{8}$ **f** $r = 5\frac{2}{5}$
- g** $s = 4\frac{5}{6}$ **h** $t = 9\frac{4}{5}$ **i** $a = -7\frac{1}{2}$
8 a $f = 40$ **b** $g = 30$ **c** $r = -10$
d $m = 18$ **e** $n = 28$ **f** $p = 62.4$
- 9 a** $x = 1\frac{1}{3}$ **b** $y = 9$ **c** $m = 4\frac{2}{5}$
d $k = 1\frac{1}{2}$ **e** $n = 5\frac{2}{3}$ **f** $c = 1\frac{1}{3}$
g $x = 10$ **h** $x = 4$ **i** $x = -7\frac{1}{5}$
- 10 a** $k = 25$ **b** $m = 16$ **c** $p = -11\frac{3}{7}$
d $u = -4\frac{1}{8}$ **e** $x = \frac{8}{11}$ **f** $v = 3$
- 11 a** $x = 26$ **b** $m = -5$ **c** $w = \frac{25}{3}$
d $t = 5$ **e** $x = 9$ **f** $n = -\frac{19}{3}$
- 12 a** B **b** E **c** C
13 a $x = -5$ **b** $d = -1$ **c** $p = 7$
d $x = -11$ **e** $h = -2$ **f** $t = 5$
g $v = -20$ **h** $r = -3$ **i** $g = -0.8$
- 14 a** $x = -1$ **b** $v = 1$ **c** $l = 2$
d $g = -2$ **e** $t = 3$ **f** $e = -23\frac{1}{3}$
- g** $j = -3\frac{3}{8}$ **h** $k = -36$ **i** $f = -12\frac{1}{4}$
15 a $x = 2$ **b** $b = 5$ **c** $w = 2$
d $f = 7$ **e** $t = 3$ **f** $r = 2\frac{1}{3}$
g $g = -1\frac{1}{3}$ **h** $h = -2\frac{1}{5}$ **i** $a = 0$
- 16 a** $x = -1$ **b** $c = 2$ **c** $r = 2\frac{2}{3}$
d $k = 1$ **e** $y = -1\frac{1}{8}$ **f** $g = 7$
g $w = 1$ **h** $m = \frac{1}{5}$ **i** $p = 1\frac{2}{3}$
- 17 a** $x = -15$ **b** $y = -4\frac{4}{5}$ **c** $t = 21$
d $u = -2\frac{5}{7}$ **e** $f = 12\frac{1}{2}$ **f** $r = 7\frac{1}{2}$
g $d = -6$ **h** $h = -12$ **i** $x = 1$
- 18 a** A **b** D **c** B
19 a 6 cm **b** 1.26 cm
20 a 2.5 cm **b** 41 cm
21 a 5 m **b** 2.8 s
22 a 314 cm **b** 6.3 cm
23 1.8 cm
24 6.2 cm
25 Dimensions are 10 m by 6 m.

- 26** $\frac{10\sqrt{3}}{3}$ cm

Exercise 2.6 — Solving multi-step equations

- 1 a** $x = \frac{20}{31}$ **b** $x = 3\frac{5}{8}$ **c** $x = \frac{29}{36}$
d $x = -7$ **e** $x = -2\frac{8}{11}$ **f** $x = \frac{10}{43}$
- 2 a** $x = 3$ **b** $x = 12$ **c** $x = -2$
d $x = \frac{3}{2}$ **e** $x = -\frac{11}{3}$ or $x = -3\frac{2}{3}$
f $x = \frac{2}{13}$ **g** $x = 3$ **h** $x = \frac{5}{7}$

- 3 a $x = \frac{5}{17}$ b $x = 15$ c $x = -6\frac{2}{9}$
 d $x = -\frac{10}{19}$ e $x = -1\frac{1}{2}$ f $x = -192$
 g $x = \frac{4}{7}$ h $x = 12$ i $x = 3\frac{1}{4}$
 j $x = 3$ k $x = 52$ l $x = 1\frac{5}{8}$
 4 a $x = \frac{5}{19}$ b $x = 1\frac{31}{58}$ c $x = 4\frac{11}{14}$
 d $x = -3\frac{15}{17}$ e $x = 5\frac{20}{43}$ f $x = -1\frac{10}{13}$
 g $x = 1\frac{2}{61}$ h $x = -4\frac{9}{26}$ i $x = 1.5$
 j $x = -4\frac{1}{3}$ k $x = 3$ l $x = 1$
 5 \$180 6 60 hours 7 \$12 000
 8 \$60
 9 a 4, 9, 25, 49 b 16, 81 c 64
 10 a $6x - 450 = 1000$

b $241\frac{1}{3}$ tickets. This means they need to sell 242 tickets to qualify, as the number of tickets must be a whole number.

11 Teacher to check

12 4

13 $a = 3, b = 5$

Exercise 2.7 — Literal equations

- 1 a $x = \frac{bcd}{a}$ b $x = a(d + bc)$ c $x = (m - n)^2$
 d $x = \pm\sqrt{\frac{w}{ac}}$ e $x = \frac{ay}{b}$ f $x = nw - m$
 g $x = \frac{c}{ab} - b$ h $x = \frac{ac}{b + mc}$ i $x = \frac{ay}{m + b}$
 j $x = \frac{mc + amd}{d}$
 2 a $l = \frac{V}{bh}$ b $b = \frac{P - 2l}{2}$ c $h = \frac{2A}{b}$
 d $a = \pm\sqrt{c^2 - b^2}$ e $C = \frac{5}{9}(F - 32)$ f $r = \pm\sqrt{\frac{A}{\pi}}$
 g $a = \frac{v - u}{t}$ h $N = \frac{100I}{PR}$ i $m = \frac{2E}{v^2}$
 j $v = \pm\sqrt{\frac{2E}{m}}$ k $a = \frac{v^2 - u^2}{2s}$
 l $u = \pm\sqrt{v^2 - 2as}$ m $a = \frac{xb}{b - x}$
 n $x_1 = \frac{x(m + n) - mx_2}{n}$
 3 a $a = \pm 12$ b $h = 7$ c $C = 30$ d $a = 9.8$
 e $s = \pm 13$ f $u = \pm 11$ g $a = 25$ h $x_2 = 13$

- 4 a i No restrictions on x ii $x = \pm\sqrt{y - 4}$ iii $y \geq 4$
 b i $x \neq 3$ ii $x = \frac{2}{y} + 3$ iii $y \neq 0$
 c i No restrictions ii $t = \frac{v - u}{a}$ iii $a \neq 0$
 d i $c \geq 0$ ii $b = \pm\sqrt{c^2 - a^2}$ iii $|c| \geq |a|$
 e i $r \neq 1$ ii $r = \frac{s - a}{s}$ iii $s \neq 0$
 f i $p \neq -q$ ii $b = \frac{m(p + q) - qa}{p}$ iii $p \neq 0$
 g i $a \neq 0, b^2 \geq 4ac$
 ii $c = \frac{b^2 - (2ax + b)^2}{4a}$ or $c = -ax^2 - bx$
 iii No new restrictions
 h i $p \neq -q$ ii $p = \frac{q(a - m)}{m - b}$ iii $m \neq b$

5 a No restrictions, all values must be positive for a cylinder to exist.

b $r = \sqrt{\frac{V}{\pi h}}$ c $h \neq 0$, no new restrictions

6 a T and l must be greater than zero.

b $l = \frac{T^2 g}{4\pi^2}$

c The restrictions still hold.

d 2.2 m

7 a $C = \frac{5}{9}(F - 32)$

b -40°

8 Distance Jing has ridden is $\frac{ja}{j - p}$ kilometres.

Investigation — Rich task

- 1 a $9(c - d)$
 b Yes, this is a multiple of 9 as the number that multiplies the brackets is 9.
 2 $90(b - c)$; 90 is a multiple of 9 so the difference between the correct and incorrect one is a multiple of 9.
 3 $900(a - b)$; again 900 is a multiple of 9.
 4 If two adjacent digits are transposed, the difference between the correct number and the transposed number is a multiple of 9.

Code puzzle

Australia