

PARTIAL FRACTIONS SOLUTIONS

①

$$\textcircled{1} \quad \frac{-5x^2 + 2x - 6}{(x^2 + 2)(x + 2)} = \frac{Ax + B}{x^2 + 2} + \frac{C}{x + 2}$$

$$= \frac{(Ax + B)(x + 2) + C(x^2 + 2)}{(x^2 + 2)(x + 2)}$$

$$x = -2 : \quad 6C = -20 - 4 - 6 = -30$$

$$C = -5$$

$$x = 0 : \quad 2B + 2C = -6$$

$$2B = -6 + 10 = 4$$

$$B = 2$$

$$x = 1 : \quad 3A + 3B + 3C = -5 + 2 - 6 = -9$$

$$A + 2 - 5 = -3$$

$$A = 0$$

$$\frac{-5x^2 + 2x - 6}{(x^2 + 2)(x + 2)} = \frac{2}{x^2 + 2} - \frac{5}{x + 2}$$

$$\textcircled{2} \quad \frac{-5x^2 + 2x - 26}{(x^2 - 4)(x + 3)} = \frac{A}{x - 2} + \frac{B}{x + 2} + \frac{C}{x + 3}$$

$$= \frac{A(x + 2)(x + 3) + B(x - 2)(x + 3) + C(x - 2)(x + 2)}{(x - 2)(x + 2)(x + 3)}$$

$$x = -2 \quad -4B = -20 - 4 - 26 = -50$$

$$B = 25/2$$

$$x = 2 \quad 20A = -20 + 4 - 26 = -42$$

$$A = -21/10$$

$$x = -3 \quad 5C = -45 - 6 - 26 = -77$$

$$C = -77/5$$

(2)

$$\frac{-5x^2 + 2x - 26}{(x^2 - 4)(x+3)} = \frac{-21}{10(x-2)} + \frac{25}{2(x+2)} - \frac{77}{5(x+3)}$$

$$(3) \quad \frac{-5x+12}{(x+2)(x-2)} = \frac{A}{x+2} + \frac{B}{x-2} = \frac{A(x-2) + B(x+2)}{(x+2)(x-2)}$$

$$x=2: \quad 4B = -10 + 12 = 2$$

$$B = \frac{1}{2}$$

$$x=-2$$

$$-4A = 10 + 12 = 22$$

$$A = -\frac{11}{2}$$

$$\frac{-5x+12}{(x+2)(x-2)} = \frac{-11}{2(x+2)} + \frac{1}{2(x-2)}$$

$$(4) \quad \frac{x^3 - x^2 - 7x + 12}{(x+1)(x-2)} = \frac{x^3 - x^2 - 7x + 12}{x^2 - x - 2}$$

$$\begin{array}{r} x \\ x^2 - x - 2 \overline{) x^3 - x^2 - 7x + 12} \\ \underline{x^3 - x^2 - 2x} \\ -5x + 12 \end{array} = x + \frac{-5x+12}{(x+1)(x-2)}$$

$$\frac{-5x+12}{(x+1)(x-2)} = \frac{A}{x+1} + \frac{B}{x-2} = \frac{A(x-2) + B(x+1)}{(x+1)(x-2)}$$

$$x=2: \quad 3B = -10 + 12 = 2$$

$$B = \frac{2}{3}$$

$$x=-1$$

$$-3A = 5 + 12 = 17$$

$$A = -\frac{17}{3}$$

$$\frac{x^3 - x^2 - 7x + 12}{(x+1)(x-2)} = x - \frac{17}{3(x+1)} + \frac{2}{3(x-2)}$$

(3)

$$(5) \quad \frac{3}{x^3-1} = \frac{A}{x-1} + \frac{Bx+C}{x^2+x+1} = \frac{A(x^2+x+1) + (Bx+C)(x-1)}{x^3-1}$$

$$x=1 : \quad 3 = 3A \quad A = 1$$

$$x=0 \quad 3 = A - C \quad C = -2$$

$$x=-1 \quad A + (C-B)(-2) = 3$$

$$A + 2B - 2C = 3$$

$$B = -1$$

$$\frac{3}{x^3-1} = \frac{1}{x-1} + \frac{-x-2}{x^2+x+1}$$

$$(6) \quad \frac{x^5 + 7x^4 + 18x^3 + 36x^2 + 57x + 57}{(x^2+3)(x+3)^2} = \frac{x^5 + 7x^4 + 18x^3 + 36x^2 + 57x + 57}{x^4 + 6x^3 + 12x^2 + 18x + 27}$$

$$= x + 1 + \frac{6x^2 + 12x + 30}{(x^2+3)(x+3)^2}$$

$$\frac{6x^2 + 12x + 30}{(x^2+3)(x+3)^2} = \frac{Ax+B}{x^2+3} + \frac{C}{(x+3)^2} + \frac{D}{x+3}$$

$$= \frac{(Ax+B)(x+3)^2 + C(x^2+3) + D(x+3)(x^2+3)}{(x^2+3)(x+3)^2}$$

$$x = -3 : \quad 12C = 54 - 36 + 30 = 48 \quad C = 4$$

$$x = 0 : \quad 9B + 3C + 9D = 30 \quad 3B + C + 3D = 10$$

$$B + D = 2 \quad - (1)$$

$$x = 1 : \quad 16A + 16B + 4C + 16D = 48$$

$$A + B + D = 2 \quad - (2)$$

$$x = -1 : \quad -4A + 4B + 4C + 8D = 24$$

$$-A + B + 2D = 2 \quad - (3)$$

(4)

$$\textcircled{1}, \textcircled{2} \text{ \& } \textcircled{3} \text{ INTO CAS} \Rightarrow A=0, B=2, D=0$$

ANS: $x+1 + \frac{2}{x^2+3} + \frac{4}{(x+3)^2}$

$$\textcircled{7} \quad \frac{2ax - bx + 10a - 2b}{(x+5)(x+2)} = \frac{A}{x+5} + \frac{B}{x+2} = \frac{A(x+2) + B(x+5)}{(x+5)(x+2)}$$

$$x = -2: \quad 3B = -4a + 2b + 10a - 2b = 6a \\ B = 2a$$

$$x = -5: \quad -3A = -10a + 5b + 10a - 2b = 3b \\ A = -b$$

$$\frac{2ax - bx + 10a - 2b}{(x+5)(x+2)} = \frac{-b}{x+5} + \frac{2a}{x+2}$$

$$\textcircled{8} \quad \frac{bx - ax - a^2 - b^2}{(x+a)(x-b)} = \frac{A}{x+a} + \frac{B}{x-b} = \frac{B(x+a) + A(x-b)}{(x+a)(x-b)}$$

$$x = -a: \quad A(-a-b) = -ab + \cancel{a^2} - \cancel{a^2} - b^2 = b(-a-b) \\ A = b$$

$$x = b: \quad B(a+b) = \cancel{b^2} - ab - a^2 - \cancel{b^2} = -a(a+b) \\ B = -a$$

$$\frac{bx - ax - a^2 - b^2}{(x+a)(x-b)} = \frac{\cancel{b}^b}{x+a} + \frac{\cancel{a}^a}{x-b}$$