

SPECIALIST MATHEMATICS

Multiple-choice questions

- 1 The image of the point $(2, -3)$ under the transformation $(x, y) \rightarrow (2x + 3y, x - y)$ is
- A** $(-5, 5)$
- B** $(5, 5)$
- C** $(-5, -1)$
- D** $(5, -1)$
- E** $(5, -5)$
- 2 For a given linear transformation, it is known that $(1, 0) \rightarrow (2, 3)$ and $(0, 1) \rightarrow (-1, 2)$. Under this transformation, the image of the point $(-2, 3)$ is
- A** $(-7, 12)$
- B** $(-7, -12)$
- C** $(-7, 0)$
- D** $(-1, 0)$
- E** $(1, 12)$
- 3 The matrix that corresponds to a reflection in the y -axis is
- A** $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$
- B** $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$
- C** $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$
- D** $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
- E** $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$

SPECIALIST MATHEMATICS

- 4 The matrix that corresponds to a shear by a factor of 2 parallel to the y-axis is

A $\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$

B $\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$

C $\begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}$

D $\begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}$

E $\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$

- 5 The matrix that corresponds will rotate the plane by 90° clockwise is

A $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

B $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

C $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

D $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

E $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

- 6 The matrix $\begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$ will rotate the plane by

A 45° clockwise

B 45° anticlockwise

C 135° clockwise

D 135° anticlockwise

E 90° anticlockwise

SPECIALIST MATHEMATICS

- 7 The matrix $\begin{bmatrix} 0 & 2 \\ 1 & 0 \end{bmatrix}$ corresponds to
- A** a dilation by factor of 2 from the y -axis followed by a reflection in the line $y = x$
 - B** a dilation by factor of 2 from the x -axis followed by a reflection in the line $y = x$
 - C** a dilation by factor of 2 from the y -axis followed by a reflection in the x -axis.
 - D** a dilation by factor of 2 from the y -axis followed by a reflection in the y -axis.
 - E** a dilation by factor of 2 from the x -axis followed by a reflection in the y -axis
- 8 The transformation that will reflect the point (x, y) in the line $x = 2$ is
- A** $(x, y) \rightarrow (2 + x, y)$
 - B** $(x, y) \rightarrow (2 - x, y)$
 - C** $(x, y) \rightarrow (4 - x, y)$
 - D** $(x, y) \rightarrow (2 - x, 2 + y)$
 - E** $(x, y) \rightarrow (4 + x, 4 - y)$
- 9 The matrix that corresponds to dilation by a factor of 2 from the x -axis followed by a reflection in the y -axis
- A** $\begin{bmatrix} 0 & 2 \\ 1 & 0 \end{bmatrix}$
 - B** $\begin{bmatrix} 0 & 2 \\ -1 & 0 \end{bmatrix}$
 - C** $\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$
 - D** $\begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix}$
 - E** $\begin{bmatrix} 1 & 0 \\ 0 & -2 \end{bmatrix}$

SPECIALIST MATHEMATICS

- 10** The matrix that corresponds to a shear by a factor of 2 parallel to the x-axis is

A $\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$

B $\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$

C $\begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}$

D $\begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}$

E $\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$

- 11** The rule for the transformation that is a translation by vector $\begin{bmatrix} 2 \\ -4 \end{bmatrix}$ followed by a

rotation by 90° clockwise is

A $(x, y) \rightarrow (y - 2, -x - 4)$

B $(x, y) \rightarrow (-y + 2, x + 4)$

C $(x, y) \rightarrow (x - 2, -y - 4)$

D $(x, y) \rightarrow (y - 4, -x - 2)$

E $(x, y) \rightarrow (x + 2, -y + 4)$

- 12** A transformation has matrix $\begin{bmatrix} 0 & -1 \\ 2 & 0 \end{bmatrix}$. The image of the line $x + 2y = 3$ under this

transformation has equation

A $4x - y = 6$

B $y - 4x = 6$

C $2x - 2y = 3$

D $2x + 2y = 3$

E $2y - 2x = 3$

SPECIALIST MATHEMATICS

- 13 A transformation has matrix $\begin{bmatrix} a & -1 \\ 3 & a \end{bmatrix}$ and maps the unit square to a parallelogram of area 6 units. Therefore, a has values
- A ± 2
 - B $\pm\sqrt{3}$
 - C $\pm\sqrt{5}$
 - D $\pm\sqrt{6}$
 - E $\pm\sqrt{7}$
- 14 The graph of $2x + 3y = 6$ is rotated by 90° anticlockwise then dilated by a factor of 2 from the y -axis. The axial intercepts of its image are
- A $(-2, 0)$ and $(0, 3)$
 - B $(2, 0)$ and $(0, 3)$
 - C $(-2, 0)$ and $(0, 6)$
 - D $(-4, 0)$ and $(0, 3)$
 - E $(-4, 0)$ and $(0, 6)$

Short-answer questions (technology-free)

- 1 The rule for a transformation is $(x, y) \rightarrow (x - y, -x + 2y)$.
- a Find the image of the point $(-2, 3)$.
 - b Find the matrix of this transformation.
 - c Find the rule for the inverse transformation.
 - d Find the point (x, y) that is mapped to the point $(2, -1)$.
- 2 Find the matrix corresponding to the following linear transformations:
- a a reflection in the x -axis
 - b a dilation by a factor of 5 from the y -axis
 - c a shear by a factor of 3 parallel to the y -axis
 - d a projection onto the y -axis
 - e a rotation by 60° anticlockwise
 - f a reflection in the line $y = -x$

SPECIALIST MATHEMATICS

- 3** The matrix $\begin{bmatrix} a & b \\ b & a \end{bmatrix}$ maps the point $(3, 4)$ to the point $(11, 10)$. Find the values of a and b .
- 4** Find the matrix that will reflect the point (x, y) in the line $y = 5x$.
- 5** Find the transformation matrix that will correspond to
- a** a reflection in the x -axis then a dilation by a factor of 2 from the y -axis.
 - b** a rotation about the origin by 90° anticlockwise, then a shear by a factor of 3 parallel to the x -axis.
- 6** The straight line with equation $2x - 3y = 6$ is transformed by the rule $(x, y) \rightarrow (x + 3y, 2y)$. Find the equation of its image.
- 7**
- a** Find the matrix of a linear transformation that maps the line $y = x + 1$ to the line $3x + 2y = 6$.
 - b** By considering the origin $(0, 0)$, explain why there is no matrix that maps the line $y = x$ to the line $3x + 2y = 6$.
- 8**
- a** Find the rule for the transformation that will reflect the point (x, y) in the x -axis and then translate the result by vector $\begin{bmatrix} 2 \\ -3 \end{bmatrix}$.
 - b** Find the rule for this transformation if the translation takes place before the reflection.
- 9** The matrix $\begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}$ maps the unit square to a parallelogram. Sketch the parallelogram, and find its area.
- 10** For what value(s) of a does the matrix $\begin{bmatrix} 3a & -2 \\ 1 & 2 \end{bmatrix}$ map the unit square to a parallelogram of area 5 square units?

SPECIALIST MATHEMATICS

Extended-response questions

- 1** **a** Find the rule for the transformation that will reflect the plane in the line $y = x + 2$. Hint: translation line down by two units, reflect in the line $y = x$, then translate the line up by two units.
- b** Find the image of the point (1,1) under this transformation.
- c** Sketch the image of the unit square under this transformation.
- 2** Consider the transformation defined by the matrix
- $$\begin{bmatrix} a & 3 \\ a & a \end{bmatrix}.$$
- a** Sketch the image of the unit square under this transformation when $a = 2$, and find its area.
- b** For what value(s) of a does the above matrix map the unit square to a parallelogram of area 2 square units?
- c** Suppose the transformation maps the unit square to a parallelogram of area c square units. For what values of c will there always be four corresponding values of a ?
- 3** **a** Write down the matrix that will rotate the plane by 45° anticlockwise.
- b** Find the image of the points (1, 1) and $(-1, 1)$ under this transformation.
- c** Sketch the graph of $y = x^2$ under this transformation. Label the axial intercepts.
- d** Find the equation of the curve found in part **c** under this transformation.

SPECIALIST MATHEMATICS

Answers to Chapter 19 test 1**Answers to multiple-choice questions**

- 1 A
- 2 C
- 3 A
- 4 B
- 5 C
- 6 B
- 7 B
- 8 C
- 9 D
- 10 A
- 11 D
- 12 B
- 13 B
- 14 D

Answers to short-answer questions (technology-free)

- 1
 - a $(-5, 8)$
 - b $\begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$
 - c $(x, y) \rightarrow (2x + y, x + y)$
 - d $(3, 1)$
- 2
 - a $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$
 - b $\begin{bmatrix} 5 & 0 \\ 0 & 1 \end{bmatrix}$
 - c $\begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$
 - d $\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$
 - e $\begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$
 - f $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$
- 3 $a = 1$ and $b = 2$
- 4 $\begin{bmatrix} -\frac{12}{13} & \frac{5}{13} \\ \frac{5}{13} & \frac{12}{13} \end{bmatrix}$

SPECIALIST MATHEMATICS

5 **a** $\begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix}$

b $\begin{bmatrix} 3 & -1 \\ 1 & 0 \end{bmatrix}$

6 $4x - 9y = 12$

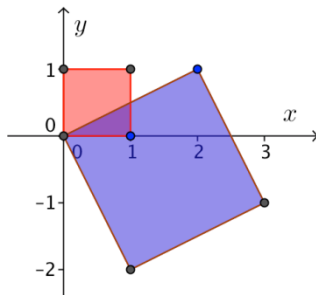
7 **a** $\begin{bmatrix} -2 & 0 \\ 0 & 3 \end{bmatrix}$

b The origin will always be mapped to the origin. Since the line $3x + 2y = 6$ does not contain the origin, no such transformation will exist.

8 **a** $(x, y) \rightarrow (x + 2, -y - 3)$

b $(x, y) \rightarrow (x + 2, 3 - y)$

9 Area = 5



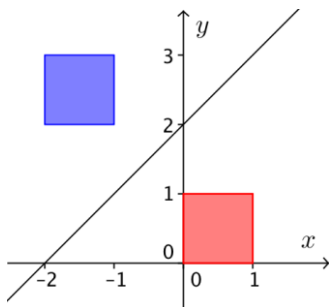
10 $a = -\frac{7}{6}, \frac{1}{2}$

Answers to extended-response questions

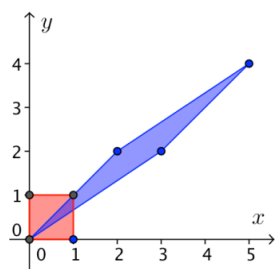
1 **a** $(x, y) \rightarrow (y - 2, x + 2)$

b $(-1, 3)$

c



2 **a** Area = 2



SPECIALIST MATHEMATICS

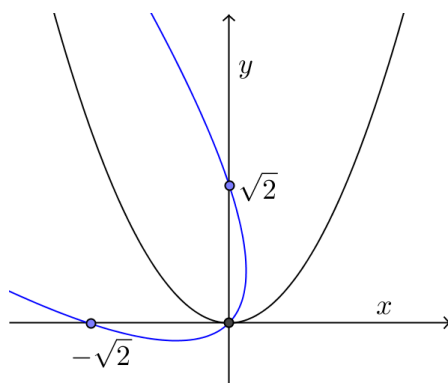
b $a = 1, 2, \frac{1}{2}(3 - \sqrt{17}), \frac{1}{2}(3 + \sqrt{17})$

c $0 < c < \frac{9}{4}$

3 a $\begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$

b $(0, \sqrt{2}), (-\sqrt{2}, 0)$

c



d $x^2 + 2xy + y^2 = \sqrt{2}(y - x)$