



2018 Year 11 Specialist Maths Unit 1  
Algebra Test – Technology Free Assessment

Name: Solutions

Time allowed: 40 minutes

Total: / 35 marks

Multiple Choice Questions. Circle the correct answer (5 x 2 = 10 marks)

|   |                                                                                                                                                                                                                                                                                        |               |
|---|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------|
| 1 | If $(x+a)^2 + b \equiv x^2 - 4x + 10$ , then the values of $a$ and $b$ respectively are:<br>A. 2, 6<br>B. 6, -2<br><input checked="" type="radio"/> C. -2, 6<br>D. 2, -6<br>E. -2, -6                                                                                                  | Working Space |
| 2 | If $\frac{x+\frac{a}{2}}{a} = x$ , then:<br>A. $x = \frac{-a}{2(a+1)}$<br>B. $x = \frac{-a}{2(a-1)}$<br>C. $x = \frac{a}{2(1-a)}$<br><input checked="" type="radio"/> D. $x = \frac{a}{2(a-1)}$<br>E. $x = \frac{a}{2(a+1)}$                                                           | Working Space |
| 3 | Expressed in the form $a(2x-1)^2 + b(2x-1) + c$ , $24x^2 - 32x - 2$ is<br>A. $24(2x-1)^2 - 32(2x-1) - 2$ ✗<br>B. $12(2x-1)^2 - 16(2x-1) - 2$ ✗<br>C. $6(2x-1)^2 - 16(2x-1) - 2$ ✓<br><input checked="" type="radio"/> D. $6(2x-1)^2 - 4(2x-1) - 12$<br>E. $6(2x-1)^2 - 4(2x-1) + 10$ ✗ | Working Space |
| 4 | The solution of the simultaneous equations $3x - 2y = 0$ and $x + 6y = 20$ is<br>A. $x = -2, y = 3$<br><input checked="" type="radio"/> B. $x = 2, y = 3$<br>C. $x = 2, y = -3$<br>D. $x = -2, y = -3$<br>E. $x = 3, y = 2$                                                            | Working Space |

|   |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |               |
|---|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------|
| 5 | <p>An error in solving the simultaneous equations</p> $ay - x = c$ $y + bx = d$ <p>occurs in which line?</p> <p>Line 1 <math>x = ay - c</math></p> <p>Line 2 <math>\therefore y + b(ay - c) = d</math></p> <p>Line 3 <math>\therefore y + aby - cb = d</math></p> <p>Line 4 <math>\therefore y(1 + ab) = d - cb</math></p> <p>Line 5 <math>\therefore y = \frac{d - cb}{1 + ab}</math></p> <p>A. Line 1</p> <p>B. Line 2</p> <p>C. Line 3</p> <p><u>D.</u> Line 4</p> <p>E. Line 5</p> | Working Space |
|---|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------|

Note award any other correct alternative method for all questions.

### Short Answer Questions (25 marks)

6. Find the value of  $x$  that satisfies the equation  $\frac{1-2x}{3} + \frac{x}{4} = 2$

$$4(1-2x) + 3x = 24$$

$$4 - 8x + 3x = 24$$

$$-5x = 20$$

$$x = -4$$

①<sup>M</sup>

} ①<sup>M</sup>

①<sup>A</sup>

3 marks

7. Express  $x$  in terms of  $a$  where  $\frac{1}{1 + \frac{1}{x+1}} = a$

$$\frac{1}{\frac{x+1+1}{x+1}} = a$$

$$\frac{x+1}{x+2} = a$$

$$ax + 2a = x + 1$$

$$x(a-1) = 1-2a$$

$$x = \frac{1-2a}{a-1}$$

①<sup>M</sup>

} ①<sup>M</sup>

①<sup>A</sup>

3 marks

8. Express  $\frac{2}{x+3} - \frac{4}{x-1}$  as a single fraction

$$\begin{aligned} \textcircled{1}^M \quad \frac{2x-2-4x-12}{(x+3)(x-1)} &= \frac{-2x-14}{(x+3)(x-1)} \quad \textcircled{1}^M \\ &= \frac{-2(x+7)}{(x+3)(x-1)} \quad \textcircled{1}^A \end{aligned}$$

accept expanded form

3 marks

9. Transpose the equation  $A = 2\sqrt{\frac{r-h}{m}}$  to make  $h$  the subject.

$$\begin{aligned} \textcircled{1}^M \quad A^2 &= 4 \left( \frac{r-h}{m} \right) \\ &= \frac{4r-4h}{m} \end{aligned} \quad \boxed{\therefore h = -\frac{1}{4}mA^2 + r} \quad \textcircled{1}^A$$

$$\textcircled{1}^M \quad mA^2 - 4r = -4h$$

3 marks

10. The line with equation  $y = -x$  intersects the hyperbola with equation  $y = \frac{1}{x-a} + a$ , where  $a > 1$ . Find the possible values of  $x$  in terms of  $a$  that satisfies the simultaneous equations.

$$\begin{aligned} -x &= \frac{1}{x-a} + a \quad \textcircled{1}^M \\ -x &= \frac{1+a(x-a)}{x-a} \quad \textcircled{\frac{1}{2}}^M \\ -x^2 + ax &= 1 + ax - a^2 \quad \textcircled{\frac{1}{2}}^M \\ x^2 &= a^2 - 1 \\ x &= \pm \sqrt{a^2 - 1}, \quad a > 1 \quad \textcircled{1}^A \end{aligned}$$

3 marks

11. Solve the following simultaneous equations. Express your answer in terms of  $a$  and  $b$ .

$$x + y = 2(a+b-1)$$

$$y = x - 2(a+1)$$

$$x + x - 2a - 2 = 2a + 2b - 2$$

$$2x = 4a + 2b$$

$$x = 2a + b$$

$$y = 2a + b - 2a - 2$$

$$= b - 2$$

$$\boxed{\begin{aligned} \therefore x &= 2a + b \\ y &= b - 2 \end{aligned}}$$

$\textcircled{1}^A$

3 marks

12. Express  $\frac{5x^2 - 6x + 7}{(x^2 + x + 5)(x - 3)}$  in partial fractions

$$\frac{5x^2 - 6x + 7}{(x^2 + x + 5)(x - 3)} = \frac{Ax + B}{x^2 + x + 5} + \frac{C}{x - 3}$$

$$5x^2 - 6x + 7 = (Ax + B)(x - 3) + C(x^2 + x + 5)$$

(1)<sup>M</sup>

Let  $x = 3$  :  $34 = 9C + 3x + 5C$   
 $34 = 17C$   
 $C = 2$

(1)<sup>M</sup>

Let  $x = 0$  :  $7 = -3B + 5C$   
 $3B = 10 - 7 = 3$   
 $B = 1$

Let  $x = 1$  :  $6 = -2(A + B) + C$   
 $6 = -2A - 2 + 4$   
 $A = 3$

$$\therefore \frac{5x^2 - 6x + 7}{(x^2 + x + 5)(x - 3)} = \frac{3x + 1}{x^2 + x + 5} + \frac{2}{x - 3}$$

(1/2)<sup>M</sup>

(1)<sup>A</sup>

4 marks

13. Resolve the expression into partial fractions  $\frac{2ab}{(ax - b)(ax + b)}$

$$\frac{2ab}{(ax - b)(ax + b)} = \frac{A}{ax - b} + \frac{B}{ax + b}$$

$$2ab = A(ax + b) + B(ax - b)$$

$$2ab = Aax + Ab + aBx - Bb$$

(1/2)<sup>M</sup>

Equate  
Coefficients

$$\Rightarrow 2ab = Ab - Bb$$

$$2ab = b(A - B)$$

$$A - B = 2a \quad \dots \dots \dots (1)$$

(1/2)<sup>M</sup>

$$\Rightarrow Aa + aB = 0$$

$$a(A + B) = 0$$

$$a = 0 \text{ or } A + B = 0$$

$$A = -B \quad \dots \dots (2)$$

(1/2)<sup>M</sup>

3 marks

Sub (2) into (1) gives  $-2B = 2a$

$$B = -a$$

$$A = a$$

(1/2)<sup>M</sup>

$$\therefore \frac{a}{ax - b} - \frac{a}{ax + b}$$

(1)<sup>A</sup>