

Analysis Revision – Vectors (Total 93 marks)

2017 Analysis Task (19 marks)

Question 1 (13 marks)

A forensic scientist is looking for a murder weapon in a large crime scene.

The weapon is actually located at a point **W** 500 m on a bearing of 030° from her starting point **O**.

Let $\hat{i}$ be a unit vector in the East direction and $\hat{j}$ be a unit vector in the North direction.

She initially moves from the origin in a straight line in the direction of $\vec{v} = 3\hat{i} + 4\hat{j}$.

a) Calculate the unit vector in the direction of her movement $\hat{v}$

b) Find the vector $\vec{w} = \overrightarrow{OW}$ from the starting point **O** to the weapon **W**.

c) Find exact values for the vector projections of $\vec{w}$ in the directions parallel and perpendicular to $\vec{v}$

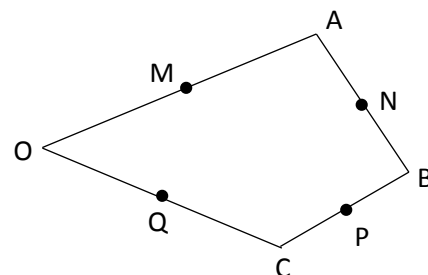
d) Hence show that the shortest distance between her path and the weapon is approximately 59.81 metres.

e) After walking a distance of 500 metres in the direction of $\hat{v}$ to point P she sees the weapon. She immediately stops and walks straight towards it. Calculate the vector $\overrightarrow{PW}$ from the point where she stops to the weapon.

Question 2 (6 marks)

For any quadrilateral OABC let $\overrightarrow{OA} = \underline{a}$, $\overrightarrow{OB} = \underline{b}$ and $\overrightarrow{OC} = \underline{c}$.

- a) Let M, N, P and Q be the midpoints of sides OA, AB, BC and CO. Find $\overrightarrow{OM}$, $\overrightarrow{ON}$, $\overrightarrow{OP}$ and $\overrightarrow{OQ}$ in terms of $\underline{a}$, $\underline{b}$ and $\underline{c}$.



$\overrightarrow{OM} =$	$\overrightarrow{ON} =$	$\overrightarrow{OP} =$	$\overrightarrow{OQ} =$
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- b) Hence prove that MNPQ is a parallelogram

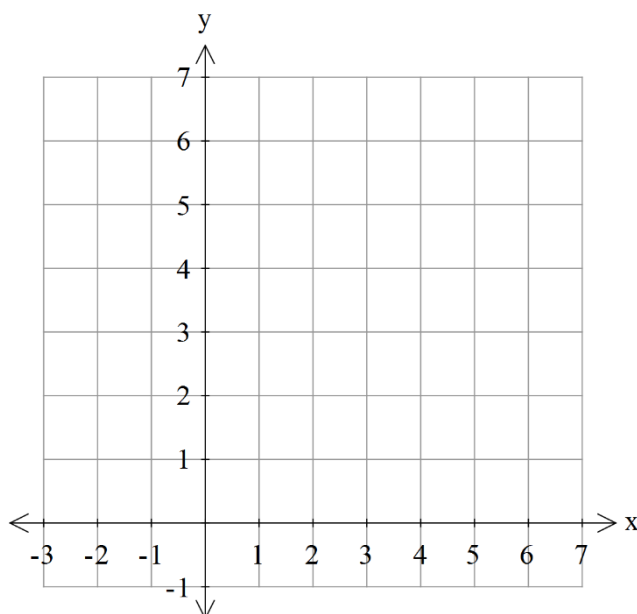
(4 + 2 = 6 marks)

2016 Analysis Task (20 marks)

Question 1

Point A has position vector $\underline{a} = m\underline{i} + 2\underline{j}$, point B has position vector $\underline{b} = \underline{i} + 6\underline{j}$, point C has position vector $\underline{c} = n\underline{i} + 6\underline{j}$ and point D has position vector $\underline{d} = 3\underline{i} + 2\underline{j}$ relative to the origin O, where m and n are integers.

- a) Plot the points B and D on the diagram and label them.



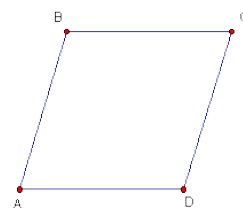
(1 mark)

- b) (i) Find $\overrightarrow{AB}$ in terms of m .

(1 mark)

(ii) Find $\vec{AD}$ in terms of m .

c) Points $ABCD$ form a rhombus. (Note diagram is not to scale)



(1 mark)

(i) Using the fact that $\vec{AB}$ and $\vec{AD}$ have the same magnitude, find the value of m and hence find $\vec{OA}$ and $\vec{AB}$.

$m =$	$\vec{OA} =$	$\vec{AB} =$
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(2 marks)

(ii) Hence find $\vec{OC}$ and hence the value of n .

$\vec{OC} =$	$n =$
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(2 marks)

d) Use vector methods to show that $\vec{AC}$ is perpendicular to $\vec{BD}$.

(2 marks)

e) Find the vector resolute of $\vec{AB}$ parallel to $\vec{AD}$.

(2 marks)

f) Let θ° be the angle between vector $\vec{AB}$ and vector $\vec{AD}$ where $\theta < 90^\circ$. Use a **vector** method to find θ .
(Answer to two decimal places)

(2 marks)

(total 13 marks)

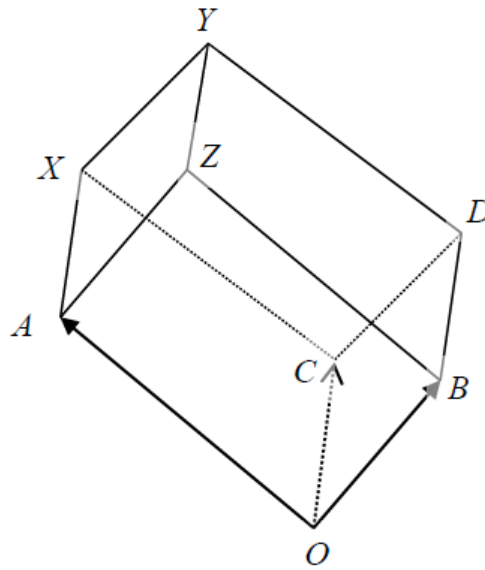
Question 2

A cuboid is positioned on level ground so that it rests on one of its vertices, O .

$$\overrightarrow{OA} = 3\hat{i} - 4\hat{j} - \hat{k}$$

$$\overrightarrow{OB} = \hat{i} + 2w\hat{j} - 5\hat{k}$$

$$\overrightarrow{OC} = x\hat{i} + y\hat{j} + 5\hat{k}$$



NOTE: $\overrightarrow{OA}$, $\overrightarrow{OB}$ and $\overrightarrow{OC}$ are all perpendicular to each other.

- a) i Find $\overrightarrow{OA} \cdot \overrightarrow{OB}$ in terms of w .

(1 mark)

- ii Hence find the value of w .

(1 mark)

- b) i Use the fact that OA is perpendicular to OC to write an equation relating x and y .

(1 mark)

- ii Find another equation relating x and y

(1 mark)

- iii Hence find the values of x and y .

(1 mark)

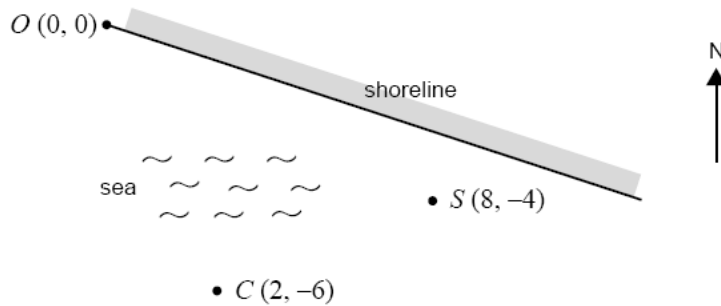
- c) Hence show that the exact volume of this cuboid is 390 units^3 .

(2 marks)
(7 marks total)

2015 Analysis Task (17 marks)

QUESTION 1 – (13 marks)

The diagram shows a straight shoreline with a town at O . Let $\hat{i}$ and $\hat{j}$ be unit vectors in the east and north directions respectively. Displacements are measured in kilometres.



At 12:00 midday, a cargo ship is at $C(2, -6)$ and a sailing ship is at $S(8, -4)$.

- a.** Find $\overrightarrow{CS}$ in terms of $\hat{i}$ and $\hat{j}$ and hence determine the distance between the cargo ship and the sailing ship at 12:00 midday. Give your answer correct to one decimal place.

(2 marks)

Let $P(6m, -2m)$ denote a point on the shoreline, where $m > 0$.

- b. i.** Find $\overrightarrow{PS}$ in terms of m .

(1 mark)

- ii.** Express the scalar product $\overrightarrow{PO} \cdot \overrightarrow{PS}$ in terms of m .

(3 marks)

- iii.** Hence, or otherwise, find the co-ordinates of the closest point on the shoreline to **the sailing ship** at 12:00 midday.

(3 marks)

- iv.** To the nearest tenth of a kilometre, how close is **the sailing ship** to the shore at 12:00 midday?

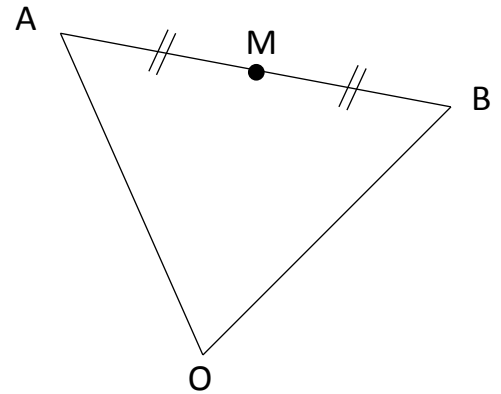
(2 marks)

- b.** Find the distance along the shore (from **O in the direction of P**) to the nearest point to **the cargo ship** at 12:00 midday. Express your answer to 2 decimal places.

(2 marks)

QUESTION 4 – (4 marks)

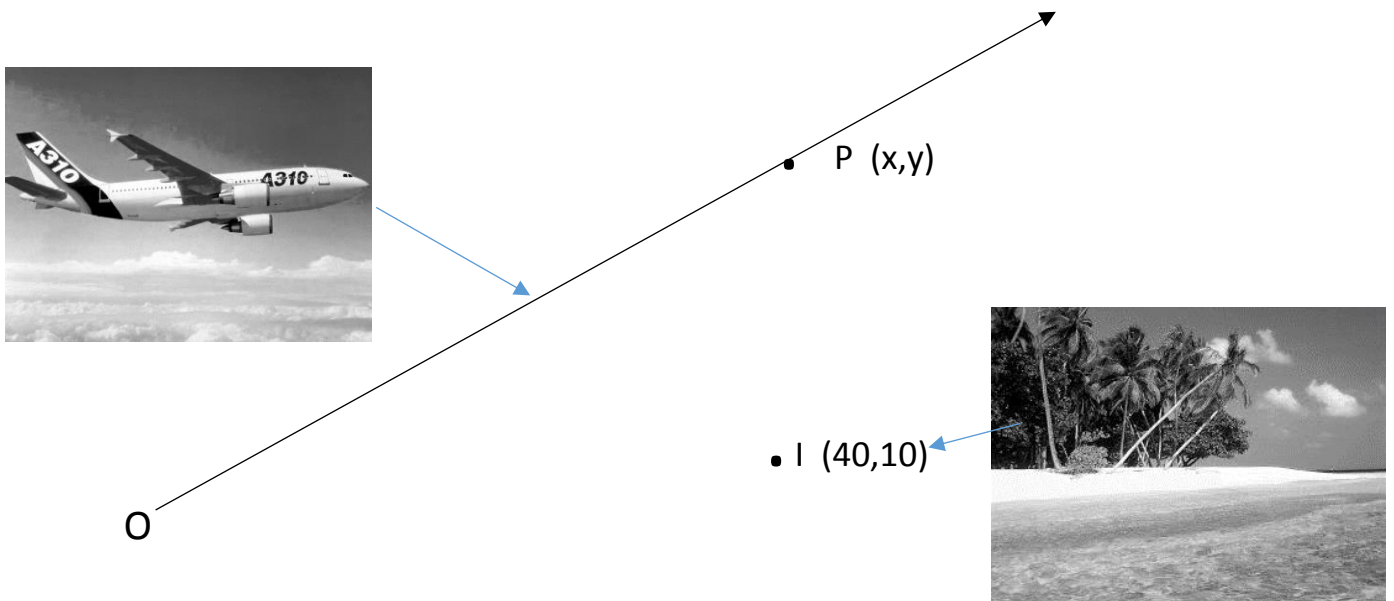
The theorem of Apollonius states that for a triangle OAB , $\overline{OA}^2 + \overline{OB}^2 = 2\overline{AM}^2 + 2\overline{OM}^2$ where M is the midpoint of $\overline{AB}$. Use the dot product of vectors to prove this theorem of Apollonius.



(4 marks)

2014 Analysis Task (12 marks)

1. An airplane is travelling in the direction N 60° E and passes close to an island (I) as shown below:



“O” is taken as the origin. “I” is the position of the island and “P” is a general point somewhere on the path of the airplane.

- (a) Show that the vector $\mathbf{p}$ joining $\overrightarrow{OP}$ is given by $\mathbf{p} = k(\sqrt{3}\mathbf{i} + \mathbf{j})$

2 marks

- (b) Hence find an expression for the vector $\overrightarrow{IP}$ in terms of k .

2 marks

(c) When the plane is at its *closest* to the island, write down the angle $\angle OPI$.

1 mark

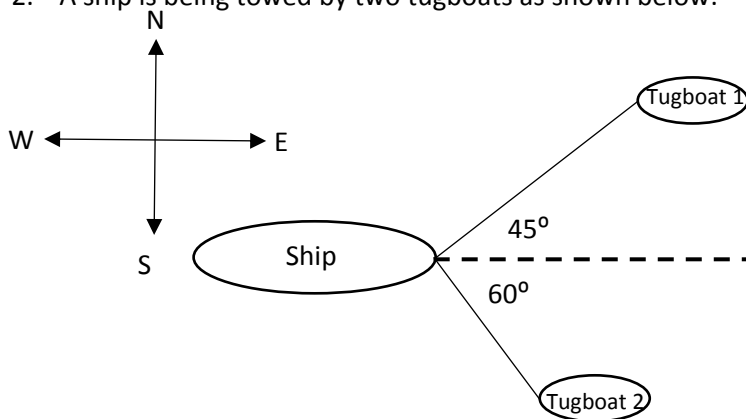
(d) Use the dot product to find the exact value of k when the plane is closest to the island.

2 marks

(e) Hence find the shortest distance (to two decimal places) between the plane's path and the island.

2 marks

2. A ship is being towed by two tugboats as shown below:



The ship is moving at constant speed towards the East. Take $\mathbf{i}$ to be in the direction east and $\mathbf{j}$ to be in the direction north.

Assume that the tugboats apply a force on the ship in the same direction as the ropes attaching them.

The vector sum of the forces applied by tugboat 1, $\vec{F}_1$ and tugboat 2, $\vec{F}_2$ is to the east.

Use vector resolution to find the ratio $\left| \vec{F}_1 \right| : \left| \vec{F}_2 \right|$.

3 marks

2012 Analysis Task (6 marks)

Question 2

A bushwalker heads off from base camp, but after 500m gets injured. He estimates his current position is $300\hat{i} + 400\hat{j}$, relative to the base camp.

A search party sets off from the base camp in the direction parallel to $4\hat{i} + 5\hat{j}$.

- a) How far is the search party from the base camp when it is closest to the injured bushwalker? Express your answer to the nearest metre.

(4 marks)

- b) How far is the search party from the bushwalker at this time? Express your answer to the nearest metre.

(2 marks)

2009 Analysis Task (19 marks)

Question 3

Point A has position vector $\vec{a} = m\hat{i} + 2\hat{j}$, point B has position vector $\vec{b} = \hat{i} + 6\hat{j}$, point C has position vector $\vec{c} = n\hat{i} + 6\hat{j}$ and point D has position vector $\vec{d} = 3\hat{i} + 2\hat{j}$ relative to the origin O, where m and n are real numbers.

- a) Find $\vec{AB}$ and $\vec{AD}$ in terms of m.

(2 marks)

- b) Hence use vector methods to find the values of m and n such that ABCD is a rhombus.

(4 marks)

c) Show that $\vec{AC}$ is perpendicular to $\vec{BD}$.

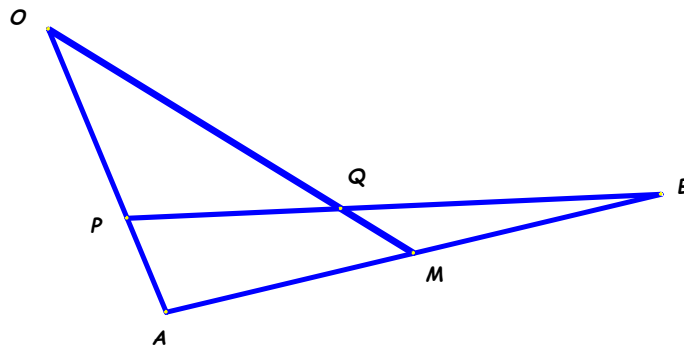
(2 marks)

d) Let θ° be the angle between vector $\vec{AB}$ and vector $\vec{AD}$ where $\theta < 90^\circ$.
Use a vector method to find $\cos \theta^\circ$

(2 marks)

Question 4

A triangle OAB is such that $\vec{OA} = \underline{\underline{a}}$ and $\vec{OB} = \underline{\underline{b}}$. The point M is the mid-point of $\vec{AB}$ and the point P divides $\vec{OA}$ in the ratio 2:1. OM and BP intersect at Q.



a) show that

i) $\vec{OM} = \frac{1}{2}(\underline{\underline{a}} + \underline{\underline{b}})$

ii) $\vec{BP} = \frac{2}{3}\underline{\underline{a}} - \underline{\underline{b}}$

(2 + 2 = 4 marks)

b) Let $\vec{OQ} = \lambda \vec{OM}$ and $\vec{QP} = \ell \vec{BP}$, where $\lambda, \ell \in \mathbb{R}$.

i) Express $\vec{OQ}$ in terms of $\lambda, \mathbf{a}$ and $\mathbf{b}$.

ii) Express $\vec{QP}$ in terms of $\ell, \mathbf{a}$ and $\mathbf{b}$.

iii) Show that $\left(\frac{2\ell}{3} + \frac{\lambda}{2} - \frac{2}{3}\right)\mathbf{a} = \left(\ell - \frac{\lambda}{2}\right)\mathbf{b}$

(1 + 1 + 3 = 5 marks)