

2017 Analysis Task (19 marks)

Question 1 (13 marks)

A forensic scientist is looking for a murder weapon in a large crime scene.

The weapon is actually located at a point **W** 500 m on a bearing of  $030^\circ$  from her starting point **O**.

Let  $\underline{i}$  be a unit vector in the East direction and  $\underline{j}$  be a unit vector in the North direction.

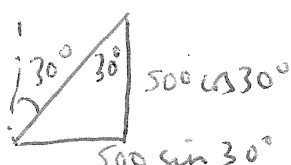
She initially moves from the origin in a straight line in the direction of  $\underline{v} = 3\underline{i} + 4\underline{j}$ .

a) ~~2016~~ Calculate the unit vector in the direction of her movement  $\hat{v}$

$$|\underline{v}| = \sqrt{3^2 + 4^2} = 5$$

$$\hat{v} = \frac{1}{5}(3\underline{i} + 4\underline{j}) = \frac{3}{5}\underline{i} + \frac{4}{5}\underline{j}$$

b) Find the vector  $\underline{w} = \overrightarrow{OW}$  from the starting point **O** to the weapon **W**.



$$\underline{w} = 500 \sin 30^\circ \underline{i} + 500 \cos 30^\circ \underline{j}$$

$$= 250 \underline{i} + 250\sqrt{3} \underline{j}$$

c) Find exact values for the vector projections of  $\underline{w}$  in the directions parallel and perpendicular to  $\underline{v}$

PARALLEL  $(\underline{w} \cdot \hat{v}) \hat{v} = (250 \times \frac{3}{5} + 250\sqrt{3} \times \frac{4}{5}) (\frac{3}{5}\underline{i} + \frac{4}{5}\underline{j})$

$$= (120\sqrt{3} + 90) \underline{i} + (160\sqrt{3} + 120) \underline{j}$$

PERPENDICULAR  $\underline{w} - (\underline{w} \cdot \hat{v}) \hat{v} = 250 \underline{i} + 250\sqrt{3} \underline{j} - [(120\sqrt{3} + 90) \underline{i} + (160\sqrt{3} + 120) \underline{j}]$

$$= (160 - 120\sqrt{3}) \underline{i} + (90\sqrt{3} - 120) \underline{j}$$

d) Hence show that the shortest distance between her path and the weapon is approximately 59.81 metres.

$$|\underline{w}_\perp| = \sqrt{(160 - 120\sqrt{3})^2 + (90\sqrt{3} - 120)^2}$$

$$= 50 \sqrt{43 - 24\sqrt{3}}$$

$$\approx 59.81$$

e) After walking a distance of 500 metres in the direction of  $\hat{v}$  to point **P** she sees the weapon. She immediately stops and walks straight towards it. Calculate the vector  $\overrightarrow{PW}$  from the point where she stops to the weapon.

$$\underline{p} = 500 \hat{v} = 300 \underline{i} + 400 \underline{j}$$

$$\overrightarrow{PW} = -\underline{p} + \underline{w} = -300 \underline{i} - 400 \underline{j} + 250 \underline{i} + 250\sqrt{3} \underline{j}$$

$$= -50 \underline{i} + (250\sqrt{3} - 400) \underline{j}$$

### Question 2 (6 marks)

For any quadrilateral OABC let  $\vec{OA} = \underline{a}$ ,  $\vec{OB} = \underline{b}$  and  $\vec{OC} = \underline{c}$ .

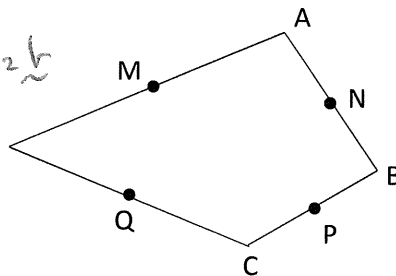
- a) **2016)** Let M, N, P and Q be the midpoints of sides OA, AB, BC and CO. Find  $\vec{OM}$ ,  $\vec{ON}$ ,  $\vec{OP}$  and  $\vec{OQ}$  in terms of  $\underline{a}$ ,  $\underline{b}$  and  $\underline{c}$ .

$$\vec{OM} = \frac{1}{2} \vec{OA} = \frac{1}{2} \underline{a}$$

$$\vec{ON} = \vec{OA} + \frac{1}{2} \vec{AB} = \underline{a} + \frac{1}{2} (-\underline{a} + \underline{b}) = \frac{1}{2} \underline{a} + \frac{1}{2} \underline{b}$$

$$\vec{OP} = \vec{OB} + \frac{1}{2} \vec{BC} = \underline{b} + \frac{1}{2} (-\underline{b} + \underline{c}) = \frac{1}{2} \underline{b} + \frac{1}{2} \underline{c}$$

$$\vec{OQ} = \frac{1}{2} \vec{OC} = \frac{1}{2} \underline{c}$$



|                                        |                                                                    |                                                                    |                                        |
|----------------------------------------|--------------------------------------------------------------------|--------------------------------------------------------------------|----------------------------------------|
| $\vec{OM} = \frac{1}{2} \underline{a}$ | $\vec{ON} = \frac{1}{2} \underline{a} + \frac{1}{2} \underline{b}$ | $\vec{OP} = \frac{1}{2} \underline{b} + \frac{1}{2} \underline{c}$ | $\vec{OQ} = \frac{1}{2} \underline{c}$ |
|----------------------------------------|--------------------------------------------------------------------|--------------------------------------------------------------------|----------------------------------------|

- b) Hence prove that MNPQ is a parallelogram

$$\vec{MN} = \vec{ON} - \vec{OM} = \frac{1}{2} \underline{b}$$

$$\vec{QP} = \vec{OP} - \vec{OQ} = \frac{1}{2} \underline{b}$$

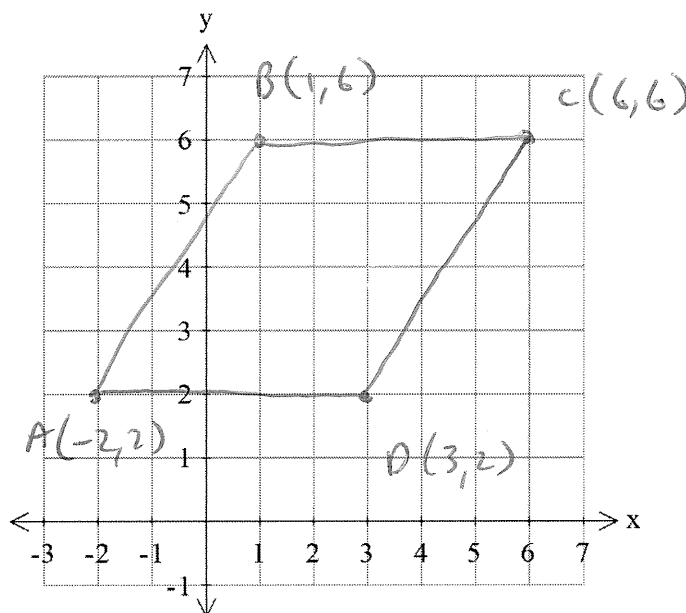
$$\therefore \vec{MN} = \vec{QP} \Rightarrow \text{MNPQ IS A PARALLELOGRAM (4 + 2 = 6 marks)}$$

**2016 Analysis Task (20 marks)**

### Question 1

Point A has position vector  $\underline{a} = m\underline{i} + 2\underline{j}$ , point B has position vector  $\underline{b} = \underline{i} + 6\underline{j}$ , point C has position vector  $\underline{c} = n\underline{i} + 6\underline{j}$  and point D has position vector  $\underline{d} = 3\underline{i} + 2\underline{j}$  relative to the origin O, where m and n are integers.

- a) Plot the points B and D on the diagram and label them.



(1 mark)

- b) (i) Find  $\vec{AB}$  in terms of m.

$$\begin{aligned} \vec{AB} &= -\vec{OA} + \vec{OB} = \underline{b} - \underline{a} = \underline{i} + 6\underline{j} - (m\underline{i} + 2\underline{j}) \\ &= (1-m)\underline{i} + 4\underline{j} \end{aligned}$$

(1 mark)

(ii) Find  $\vec{AD}$  in terms of  $m$ .

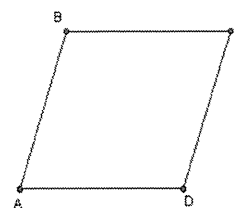
$$\vec{AD} = -\vec{OA} + \vec{OD} = \underline{\underline{2}} - \underline{\underline{5}} = 3\underline{\underline{i}} + 2\underline{\underline{j}} - (m\underline{\underline{i}} + 2\underline{\underline{j}}) \\ = (3-m)\underline{\underline{i}}$$

(1 mark)

c) Points  $ABCD$  form a rhombus. (Note diagram is not to scale)

$$|\vec{AB}| = |\vec{AD}| \Rightarrow |\vec{AB}|^2 = |\vec{AD}|^2$$

$$(1-m)^2 + 4^2 = (3-m)^2 \Rightarrow 1 - 2m + m^2 + 16 = 9 - 6m + m^2 \Rightarrow m = -2$$



(i) Using the fact that  $\vec{AB}$  and  $\vec{AD}$  have the same magnitude, find the value of  $m$  and hence find  $\vec{OA}$  and  $\vec{AB}$ .

$$\therefore \vec{OA} = 2\underline{\underline{i}} + 2\underline{\underline{j}}$$

$$\vec{AB} = 3\underline{\underline{i}} + 4\underline{\underline{j}}$$

|       |                                                                      |                                                                      |
|-------|----------------------------------------------------------------------|----------------------------------------------------------------------|
| $m =$ | $\vec{OA} = 2\underline{\underline{i}} + 2\underline{\underline{j}}$ | $\vec{AB} = 3\underline{\underline{i}} + 4\underline{\underline{j}}$ |
|-------|----------------------------------------------------------------------|----------------------------------------------------------------------|

(2 marks)

(ii) Hence find  $\vec{OC}$  and hence the value of  $n$ .

$$\vec{DC} = \vec{AB} = 3\underline{\underline{i}} + 4\underline{\underline{j}}$$

$$\vec{OC} = \vec{OD} + \vec{DC} = 3\underline{\underline{i}} + 2\underline{\underline{j}} + 3\underline{\underline{i}} + 4\underline{\underline{j}} = 6\underline{\underline{i}} + 6\underline{\underline{j}}$$

$$\vec{OC} = n\underline{\underline{i}} + 6\underline{\underline{j}} \Rightarrow n = 6$$

|                                                                      |         |
|----------------------------------------------------------------------|---------|
| $\vec{OC} = 6\underline{\underline{i}} + 6\underline{\underline{j}}$ | $n = 6$ |
|----------------------------------------------------------------------|---------|

(2 marks)

d) Use vector methods to show that  $\vec{AC}$  is perpendicular to  $\vec{BD}$ .

$$\vec{AC} = \underline{\underline{c}} - \underline{\underline{a}} = (6\underline{\underline{i}} + 6\underline{\underline{j}}) - (-2\underline{\underline{i}} + 2\underline{\underline{j}}) = 8\underline{\underline{i}} + 4\underline{\underline{j}}$$

$$\vec{BD} = \underline{\underline{d}} - \underline{\underline{b}} = (3\underline{\underline{i}} + 2\underline{\underline{j}}) - (\underline{\underline{i}} + 6\underline{\underline{j}}) = 2\underline{\underline{i}} - 4\underline{\underline{j}}$$

$$\vec{AC} \cdot \vec{BD} = 8 \times 2 + 4 \times -4 = 0 \Rightarrow \vec{AC} \perp \vec{BD}$$

(2 marks)

e) Find the vector resolute of  $\vec{AB}$  parallel to  $\vec{AD}$ .

$$(\vec{AB} \cdot \vec{AD}) \frac{\vec{AD}}{|\vec{AD}|^2}$$

$$\vec{AD} = (3-m)\underline{\underline{i}} = 5\underline{\underline{i}} \quad \vec{AD} = \underline{\underline{i}}$$

$$= (3\underline{\underline{i}} + 4\underline{\underline{j}}) \cdot (\underline{\underline{i}}) \underline{\underline{i}}$$

$$= 3\underline{\underline{i}}$$

(2 marks)

f) Let  $\theta^\circ$  be the angle between vector  $\vec{AB}$  and vector  $\vec{AD}$  where  $\theta < 90^\circ$ . Use a **vector** method to find  $\theta$ .

(Answer to two decimal places)

$$|\vec{AB}| = \sqrt{3^2 + 4^2} = 5 \quad |\vec{AD}| = 5$$

$$\vec{AB} \cdot \vec{AD} = (3\underline{\underline{i}} + 4\underline{\underline{j}}) \cdot 5\underline{\underline{i}} = 15$$

$$\cos \theta = \frac{\vec{AB} \cdot \vec{AD}}{|\vec{AB}| |\vec{AD}|} = \frac{15}{5 \times 5} = \frac{3}{5}$$

$$\theta = 53.13^\circ$$

(2 marks)

(total 13 marks)

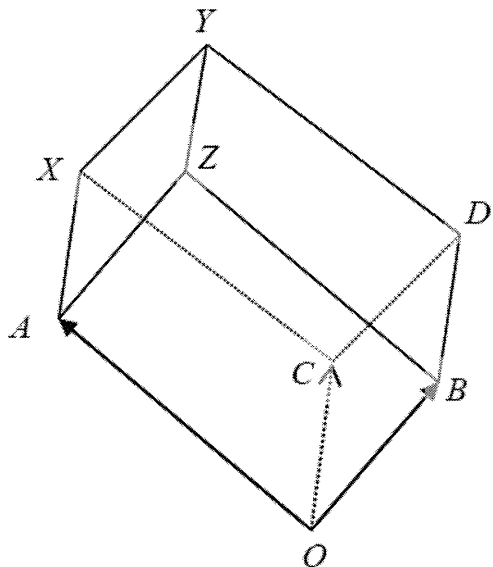
## Question 2

A cuboid is positioned on level ground so that it rests on one of its vertices,  $O$ .

$$\overrightarrow{OA} = 3\mathbf{i} - 4\mathbf{j} - \mathbf{k}$$

$$\overrightarrow{OB} = \mathbf{i} + 2w\mathbf{j} - 5\mathbf{k}$$

$$\overrightarrow{OC} = x\mathbf{i} + y\mathbf{j} + 5\mathbf{k}$$



NOTE:  $\overrightarrow{OA}$ ,  $\overrightarrow{OB}$  and  $\overrightarrow{OC}$  are all perpendicular to each other.

a) i Find  $\overrightarrow{OA} \cdot \overrightarrow{OB}$  in terms of  $w$ .  $\overrightarrow{OA} \cdot \overrightarrow{OB} = (3\mathbf{i} - 4\mathbf{j} - \mathbf{k}) \cdot (\mathbf{i} + 2w\mathbf{j} - 5\mathbf{k})$  (1 mark)

$$= 3 - 8w + 5 = 8 - 8w$$

ii Hence find the value of  $w$ .

$$\overrightarrow{OA} \cdot \overrightarrow{OB} = 0 \quad (\text{since } \overrightarrow{OA} \perp \overrightarrow{OB})$$

$$\therefore 8 - 8w = 0 \Rightarrow w = 1$$

(1 mark)

b) i Use the fact that  $OA$  is perpendicular to  $OC$  to write an equation relating  $x$  and  $y$ .

$$\overrightarrow{OA} \cdot \overrightarrow{OC} = (3\mathbf{i} - 4\mathbf{j} - \mathbf{k}) \cdot (x\mathbf{i} + y\mathbf{j} + 5\mathbf{k}) = 0$$

(1 mark)

ii Find another equation relating  $x$  and  $y$

$$3x - 4y - 5 = 0$$

$$\overrightarrow{OB} \cdot \overrightarrow{OC} = (\mathbf{i} + 2\mathbf{j} - 5\mathbf{k}) \cdot (x\mathbf{i} + y\mathbf{j} + 5\mathbf{k}) = 0$$

(1 mark)

iii Hence find the values of  $x$  and  $y$ .

$$x + 2y - 25 = 0$$

$$\text{Solving: } x = 11, y = 7$$

(1 mark)

c) Hence show that the exact volume of this cuboid is  $390 \text{ units}^3$ .

$$|\overrightarrow{OA}| = \sqrt{3^2 + 4^2 + 1^2} = \sqrt{26}$$

$$|\overrightarrow{OB}| = \sqrt{1^2 + 2^2 + 5^2} = \sqrt{30}$$

$$|\overrightarrow{OC}| = \sqrt{11^2 + 7^2 + 5^2} = \sqrt{195}$$

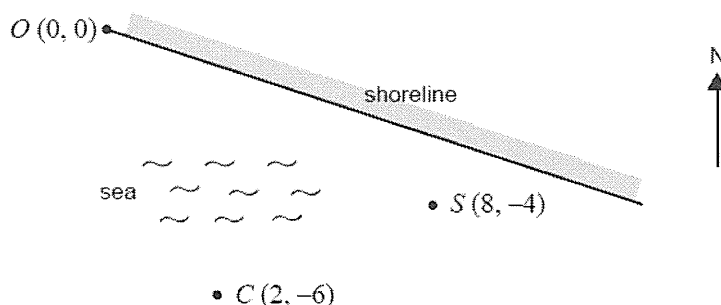
$$V = \sqrt{26} \times \sqrt{30} \times \sqrt{195} = 390 \text{ unit}^3 \quad \text{As req'd}$$

(2 marks)

(7 marks total)

**QUESTION 1 – (13 marks)**

The diagram shows a straight shoreline with a town at O. Let  $\hat{i}$  and  $\hat{j}$  be unit vectors in the east and north directions respectively. Displacements are measured in kilometres.



At 12:00 midday, a cargo ship is at C (2, -6) and a sailing ship is at S(8, -4).

- a. Find  $\overrightarrow{CS}$  in terms of  $\hat{i}$  and  $\hat{j}$  and hence determine the distance between the cargo ship and the sailing ship at 12:00 midday. Give your answer correct to one decimal place.

$$\overrightarrow{CS} = -\overrightarrow{OC} + \overrightarrow{OS} = -(2\hat{i} - 6\hat{j}) + (8\hat{i} + 4\hat{j}) = 6\hat{i} + 2\hat{j}$$

$$|\overrightarrow{CS}| = \sqrt{36 + 4} = \sqrt{40} \approx 6.3 \text{ km}$$

(2 marks)

Let P(6m, -2m) denote a point on the shoreline, where  $m > 0$ .

- b. i. Find  $\overrightarrow{PS}$  in terms of  $m$ .

$$\overrightarrow{PS} = -\overrightarrow{OP} + \overrightarrow{OS} = -(6m\hat{i} - 2m\hat{j}) + (8\hat{i} + 4\hat{j}) \quad (1 \text{ mark})$$

- ii. Express the scalar product  $\overrightarrow{PO} \cdot \overrightarrow{PS}$  in terms of  $m$ .  $= (8 - 6m)\hat{i} + (2m - 4)\hat{j}$

$$\begin{aligned} \overrightarrow{PO} \cdot \overrightarrow{PS} &= (-6m\hat{i} + 2m\hat{j}) \cdot [(8 - 6m)\hat{i} + (2m - 4)\hat{j}] \\ &= -6m(8 - 6m) + 2m(2m - 4) \\ &= 40m^2 - 56m \end{aligned}$$

(3 marks)

- iii. Hence, or otherwise, find the co-ordinates of the closest point on the shoreline to the sailing ship at 12:00 midday. Closest when  $\overrightarrow{PO} \perp \overrightarrow{PS} \Rightarrow \overrightarrow{PO} \cdot \overrightarrow{PS} = 0$

$$m(40m - 56) = 0 \Rightarrow m = 0, 7/5$$

$$m > 0 \Rightarrow m = 7/5 \quad P = (6m, -2m) = (8.4, -2.8) \quad (3 \text{ marks})$$

- iv. To the nearest tenth of a kilometre, how close is the sailing ship to the shore at 12:00 midday?

$$\overrightarrow{PS} = (8 - 8.4)\hat{i} + (2.8 - 4)\hat{j} = -0.4\hat{i} - 1.2\hat{j}$$

$$|\overrightarrow{PS}| = \sqrt{0.4^2 + 1.2^2} \approx 1.3 \text{ km} \quad (2 \text{ marks})$$

- b. Find the distance along the shore (from O in the direction of P) to the nearest point to the cargo ship at 12:00 midday. Express your answer to 2 decimal places.

$$|\overrightarrow{OP}| = \frac{14\sqrt{10}}{5} \quad \overrightarrow{OC} \cdot \overrightarrow{OP} = (2\hat{i} - 6\hat{j}) \cdot (8.4\hat{i} - 2.8\hat{j}) \approx 3.79 \text{ km} \quad (2 \text{ marks})$$

$$\sqrt{8.4^2 + 2.8^2}$$

#### QUESTION 4 – (4 marks)

The theorem of Apollonius states that for a triangle  $OAB$ ,  $\overline{OA}^2 + \overline{OB}^2 = 2\overline{AM}^2 + 2\overline{OM}^2$  where  $M$  is the midpoint of  $\overline{AB}$ . Use the dot product of vectors to prove this theorem of Apollonius.

Let  $\vec{OA} = \underline{a}$  &  $\vec{OB} = \underline{b}$

LHS =  $|\underline{a}|^2 + |\underline{b}|^2 = \underline{a} \cdot \underline{a} + \underline{b} \cdot \underline{b}$

$\vec{AM} = \frac{1}{2} \vec{AB} = \frac{1}{2} (\underline{b} - \underline{a})$

$\vec{OM} = \vec{OA} + \vec{AM} = \underline{a} + \frac{1}{2} (\underline{b} - \underline{a}) = \frac{1}{2} (\underline{a} + \underline{b})$

RHS =  $2 \vec{AM} \cdot \vec{AM} + 2 \vec{OM} \cdot \vec{OM}$

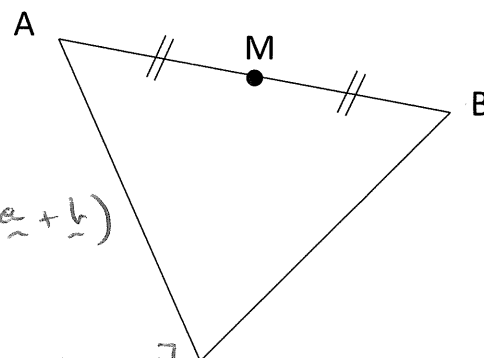
$= 2 \left[ \frac{1}{4} (\underline{b} - \underline{a}) \cdot (\underline{b} - \underline{a}) + \frac{1}{4} (\underline{a} + \underline{b}) \cdot (\underline{a} + \underline{b}) \right]$

$= \frac{1}{2} [ \underline{b} \cdot \underline{b} - 2\underline{a} \cdot \underline{b} + \underline{a} \cdot \underline{a} + \underline{a} \cdot \underline{a} + 2\underline{a} \cdot \underline{b} + \underline{b} \cdot \underline{b} ]$

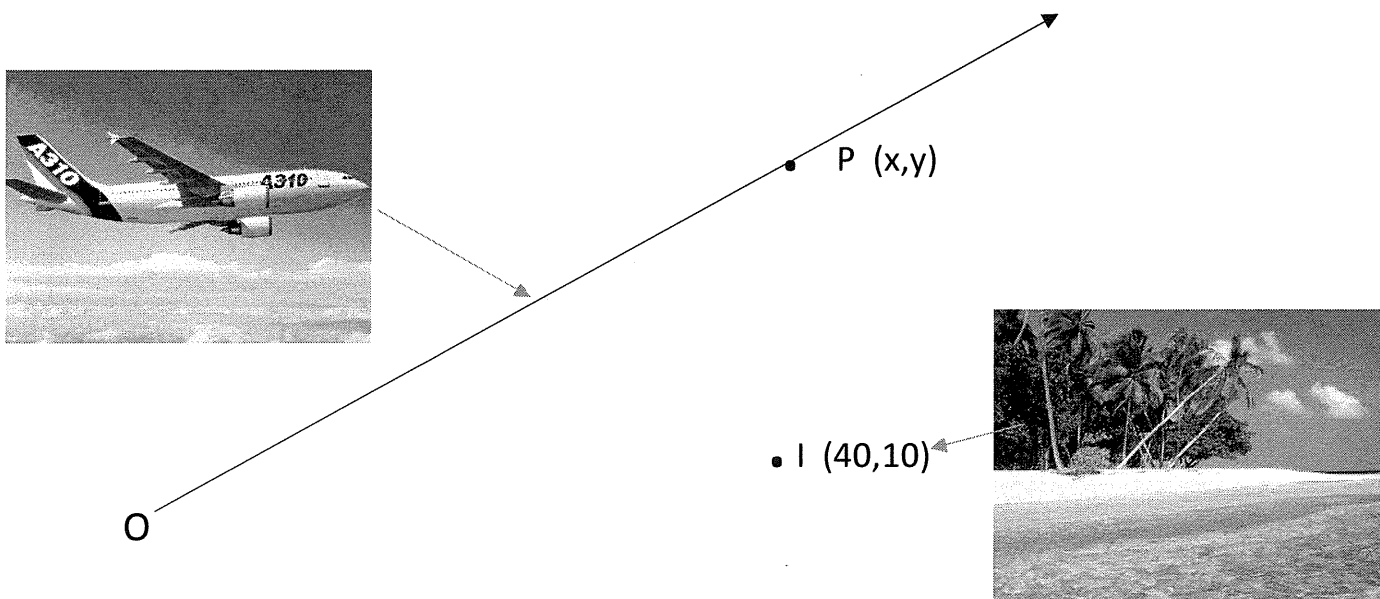
$= \underline{a} \cdot \underline{a} + \underline{b} \cdot \underline{b} = \text{LHS}$

2014 Analysis Task (12 marks)

(4 marks)



1. An airplane is travelling in the direction N  $60^\circ$  E and passes close to an island (I) as shown below:



"O" is taken as the origin. "I" is the position of the island and "P" is a general point somewhere on the path of the airplane.

- (a) Show that the vector  $\underline{p}$  joining  $\vec{OP}$  is given by  $\underline{p} = k(\sqrt{3}\underline{i} + \underline{j})$

$x = \sqrt{3}y$  Let  $y = k, x = \sqrt{3}k$

$\Rightarrow \underline{p} = k(\sqrt{3}\underline{i} + \underline{j})$  as req'd

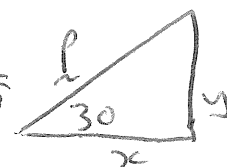
2 marks

- (b) Hence find an expression for the vector  $\vec{IP}$  in terms of  $k$ .

$\vec{IP} = \vec{IO} + \vec{OP} = -40\underline{i} - 10\underline{j} + k\sqrt{3}\underline{i} + k\underline{j}$

$= (\sqrt{3}k - 40)\underline{i} + (k - 10)\underline{j}$

2 marks



(c) When the plane is at its *closest* to the island, write down the angle  $\angle OPI$ .

$90^\circ$

1 mark

(d) Use the dot product to find the exact value of  $k$  when the plane is closest to the island.

$$\begin{aligned} \vec{IP} \cdot \vec{OP} &= 0 \Rightarrow [(\sqrt{3}k - 40)\underline{i} + (k - 10)\underline{j}] \cdot [\sqrt{3}k\underline{i} + k\underline{j}] = 0 \\ \sqrt{3}k(\sqrt{3}k - 40) + k(k - 10) &= 0 \\ 4k^2 + (-10 - 40\sqrt{3})k &= 0 \Rightarrow k = \cancel{0}, \frac{40\sqrt{3} + 10}{4} \quad (\text{since } k \neq 0) \end{aligned}$$

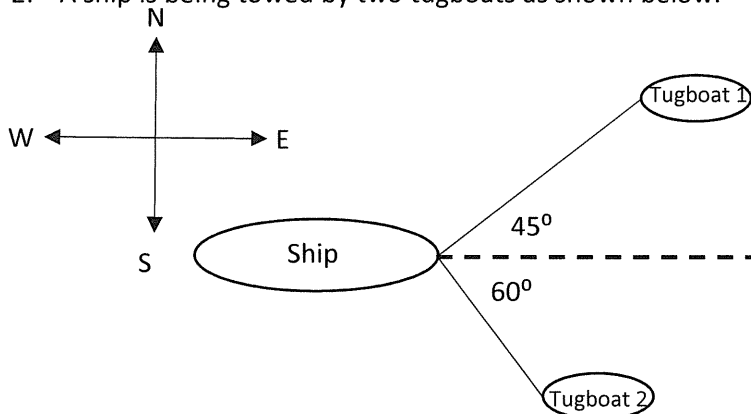
2 marks

(e) Hence find the shortest distance (to two decimal places) between the plane's path and the island.

$$\begin{aligned} |\vec{IP}| &= \sqrt{[\sqrt{3}(10\sqrt{3} + 5/2) - 40]^2 + [10\sqrt{3} + 5/2 - 10]^2} \\ &\approx 11.34 \text{ km} \end{aligned}$$

2 marks

2. A ship is being towed by two tugboats as shown below:



The ship is moving at constant speed towards the East. Take  $\underline{i}$  to be in the direction east and  $\underline{j}$  to be in the direction north.

Assume that the tugboats apply a force on the ship in the same direction as the ropes attaching them.

The vector sum of the forces applied by tugboat 1,  $\vec{F}_1$  and tugboat 2,  $\vec{F}_2$  is to the east.

Use vector resolution to find the ratio  $|\vec{F}_1| : |\vec{F}_2|$ .

$$\begin{aligned} |\vec{F}_1| \times \sin 45^\circ &= |\vec{F}_2| \times \sin 60^\circ \\ \therefore |\vec{F}_1| : |\vec{F}_2| &= \sin 60^\circ : \sin 45^\circ \\ &= \frac{\sqrt{3}}{2} : \frac{1}{\sqrt{2}} \\ &= \sqrt{6} : 2 \end{aligned}$$

3 marks

# 2012 Analysis Task (6 marks)

## Question 2

A bushwalker heads off from base camp, but after 500m gets injured. He estimates his current position is  $300\hat{i} + 400\hat{j}$ , relative to the base camp.

A search party sets off from the base camp in the direction parallel to  $4\hat{i} + 5\hat{j}$ .

- a) How far is the search party from the base camp when it is closest to the injured bushwalker? Express your answer to the nearest metre.

$$\begin{aligned}\vec{CB} \cdot \vec{CR} &= (300\hat{i} + 400\hat{j}) \cdot \frac{1}{\sqrt{41}} (4\hat{i} + 5\hat{j}) \\ &= \frac{1}{\sqrt{41}} (1200 + 2000) = 499.76 \\ &\approx 500 \text{ m}\end{aligned}$$

(4 marks)

- b) How far is the search party from the bushwalker at this time? Express your answer to the nearest metre.

$$\begin{aligned}d &= \sqrt{500^2 - 499.76^2} \quad (\text{NOTE: USE EXACT VALUE}) \\ d &\approx 15.61 \text{ m} \approx \underline{16 \text{ m}}\end{aligned}$$

(2 marks)

# 2009 Analysis Task (19 marks)

## Question 3

Point A has position vector  $\vec{a} = m\hat{i} + 2\hat{j}$ , point B has position vector  $\vec{b} = \hat{i} + 6\hat{j}$ , point C has position vector  $\vec{c} = n\hat{i} + 6\hat{j}$  and point D has position vector  $\vec{d} = 3\hat{i} + 2\hat{j}$  relative to the origin O, where m and n are real numbers.

- a) Find  $\vec{AB}$  and  $\vec{AD}$  in terms of m.

$$\begin{aligned}\vec{AB} &= -\vec{OA} + \vec{OB} = -(m\hat{i} + 2\hat{j}) + \hat{i} + 6\hat{j} \\ &= (1-m)\hat{i} + 4\hat{j} \\ \vec{AD} &= -\vec{OA} + \vec{OD} = -(m\hat{i} + 2\hat{j}) + 3\hat{i} + 2\hat{j} \\ &= (3-m)\hat{i}\end{aligned}$$

(2 marks)

- b) Hence use vector methods to find the values of m and n such that ABCD is a rhombus.

$$\begin{aligned}\vec{AB} &= \vec{DC} \Rightarrow (1-m)\hat{i} + 4\hat{j} = (n-3)\hat{i} + 4\hat{j} \Rightarrow 1-m = n-3 \\ &\Rightarrow n = 4-m \\ |\vec{AB}| &= |\vec{AD}| \Rightarrow \sqrt{(1-m)^2 + 4^2} = \sqrt{(3-m)^2} \\ 1 - 2m + m^2 + 16 &= 9 - 6m + m^2 \\ 4m &= -8 \\ m &= -2, n = 6\end{aligned}$$

(4 marks)

c) Show that  $\vec{AC}$  is perpendicular to  $\vec{BD}$ .

$$\begin{aligned}\vec{AC} \cdot \vec{BD} &= [(-2\hat{i} + 2\hat{j}) - (6\hat{i} + 6\hat{j})] \cdot [(3\hat{i} + 2\hat{j}) - (\hat{i} + 6\hat{j})] \\ &= (-8\hat{i} - 4\hat{j}) \cdot (2\hat{i} - 4\hat{j}) \\ &= -16 + 16 = 0 \quad \therefore \vec{AC} \perp \vec{BD}\end{aligned}$$

(2 marks)

d) Let  $\theta^\circ$  be the angle between vector  $\vec{AB}$  and vector  $\vec{AD}$  where  $\theta < 90^\circ$ .

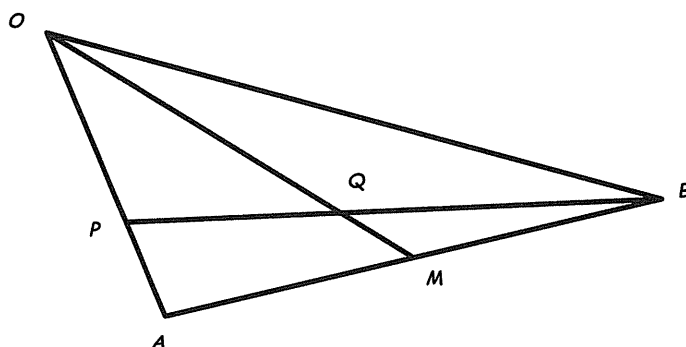
Use a vector method to find  $\cos \theta^\circ$

$$\begin{aligned}\vec{AB} &= 3\hat{i} + 4\hat{j} & \vec{AD} &= 5\hat{i} & \vec{AB} \cdot \vec{AD} &= 15 \\ |\vec{AB}| &= 5 & |\vec{AD}| &= 5 & \cos \theta &= \frac{\vec{AB} \cdot \vec{AD}}{|\vec{AB}| |\vec{AD}|} = \frac{15}{5 \times 5} = \frac{3}{5}\end{aligned}$$

(2 marks)

#### Question 4

A triangle OAB is such that  $\vec{OA} = \underline{a}$  and  $\vec{OB} = \underline{b}$ . The point M is the mid-point of  $\vec{AB}$  and the point P divides  $\vec{OA}$  in the ratio 2:1. OM and BP intersect at Q.



a) show that

$$\begin{aligned}\text{i) } \vec{OM} &= \frac{1}{2}(\underline{a} + \underline{b}) & \vec{OM} &= \vec{OA} + \frac{1}{2}\vec{AB} \\ & & &= \underline{a} + \frac{1}{2}(\underline{b} - \underline{a}) \\ & & &= \frac{1}{2}\underline{a} + \frac{1}{2}\underline{b} \\ & & &= \frac{1}{2}(\underline{a} + \underline{b})\end{aligned}$$

$$\begin{aligned}\text{ii) } \vec{BP} &= \frac{2}{3}\underline{a} - \underline{b} & \vec{BP} &= \vec{BO} + \vec{OP} \\ & & &= -\underline{b} + \frac{2}{3}\underline{a} \\ & & &= \frac{2}{3}\underline{a} - \underline{b}\end{aligned}$$

(2 + 2 = 4 marks)

b) Let  $\vec{OQ} = \lambda \vec{OM}$  and  $\vec{QP} = \ell \vec{BP}$ , where  $\lambda, \ell \in \mathbb{R}$ .

i) Express  $\vec{OQ}$  in terms of  $\lambda$ ,  $\vec{a}$  and  $\vec{b}$ .

$$\vec{OQ} = \lambda/2 (\vec{a} + \vec{b})$$

ii) Express  $\vec{QP}$  in terms of  $\ell$ ,  $\vec{a}$  and  $\vec{b}$ .

$$\vec{QP} = \ell (2/3 \vec{a} - \vec{b})$$

iii) Show that  $\left(\frac{2\ell}{3} + \frac{\lambda}{2} - \frac{2}{3}\right)\vec{a} = \left(\ell - \frac{\lambda}{2}\right)\vec{b}$

$$\vec{OP} = \vec{OQ} + \vec{QP}$$

$$2/3 \vec{a} = \lambda/2 \vec{a} + \lambda/2 \vec{b} + \frac{2\ell}{3} \vec{a} - \ell \vec{b}$$

$$\vec{a} \left( \frac{2}{3} - \frac{\lambda}{2} - \frac{2\ell}{3} \right) = \vec{b} \left( \frac{\lambda}{2} - \ell \right)$$

$$\left( \frac{2\ell}{3} + \frac{\lambda}{2} - \frac{2}{3} \right) \vec{a} = \left( \ell - \frac{\lambda}{2} \right) \vec{b}$$

(1 + 1 + 3 = 5 marks)