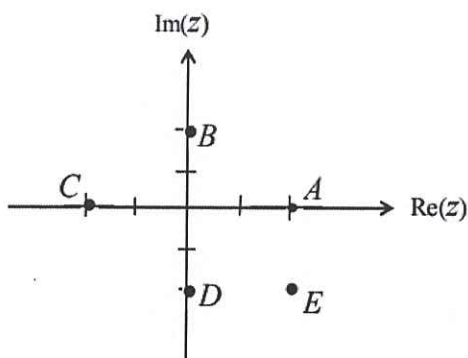


Question 9

The points A , B , C , D and E represent complex numbers on the Argand diagram below. Which points are conjugate pairs?



- A A and B
- B B and E
- C D and E
- D C and D
- E B and D**

Question 10

If $z_1 = 4\text{cis}(130^\circ)$ and $z_2 = 4\text{cis}(30^\circ)$, then $z_1 z_2$ is equal to:

- A $8\text{cis}(100^\circ)$
- B $8\text{cis}(160^\circ)$
- C $16\text{cis}(100^\circ)$
- D $16\text{cis}(160^\circ)$**
- E Cannot be calculated

$$z_1 z_2 = 16 \text{cis}(160^\circ)$$

MULTIPLE CHOICE ANSWERS

1	2	3	4	5	6	7	8	9	10
D	E	D	B	E	B	A	E	E	D

PART B: SHORT ANSWER

Question 1

Express $2\sqrt{2} \operatorname{cis} \left(\frac{-3\pi}{4} \right)$ in cartesian form.

$$\begin{aligned} 2\sqrt{2} \operatorname{cis} \left(\frac{-3\pi}{4} \right) &= 2\sqrt{2} \left(-\frac{1}{\sqrt{2}} + \frac{j}{\sqrt{2}} \right) \quad (1) \\ &= -2 - 2j \quad (1) \end{aligned}$$

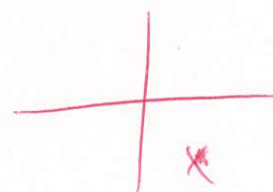
(2 marks)

Question 2

Express $\frac{1}{\sqrt{2}} + \frac{-1}{\sqrt{2}}j$ in polar form.

$$\begin{aligned} \frac{1}{\sqrt{2}} + \frac{-1}{\sqrt{2}}j \\ = \operatorname{cis} \left(\frac{-\pi}{4} \right) \quad (1) \end{aligned}$$

$$\begin{aligned} |z| = r &= \sqrt{\frac{1}{2} + \frac{1}{2}} \\ &= 1 \quad (1) \\ \tan \theta &= -1 \\ \theta &= \frac{-\pi}{4} \quad (1) \end{aligned}$$



(3 marks)

Question 3

Solve $(2z - 1)^2 + 16 = 0$.

$$(2z - 1)^2 + 16 = 0$$

$$(2z - 1 - 4j)(2z - 1 + 4j) = 0 \quad (1)$$

$$2z = 4j + 1$$

$$z = \frac{4j + 1}{2}$$

$$2z = -4j + 1$$

$$z = \frac{-4j + 1}{2} \quad (1)$$

$$\therefore z = 2j + \frac{1}{2} \quad (1)$$

$$z = \frac{1}{2} - 2j \quad (1)$$

(4 marks)

Question 4

If $z = 3 - 2i$, evaluate:

a) $(z - \bar{z})^2$

$$\begin{aligned} &= [(3 - 2i) - (3 + 2i)]^2 \quad (1) \\ &= (-4i)^2 \\ &= 16i^2 = -16 \quad (1) \end{aligned}$$

(2 marks)

b) $(z + i)(z - 1)$

$$\begin{aligned} &= (3 - 2i + i)(3 - 2i - 1) \quad (1) \\ &= (3 - i)(2 - 2i) \\ &= 6 - 6i - 2i + 2i^2 \\ &= 4 - 8i \quad (1) \end{aligned}$$

(2 marks)

c) $\frac{1}{z}$

$$= \frac{1}{(3 - 2i)} \times \frac{3 + 2i}{3 + 2i} \quad (1)$$

$$= \frac{3 + 2i}{9 - 4i^2}$$

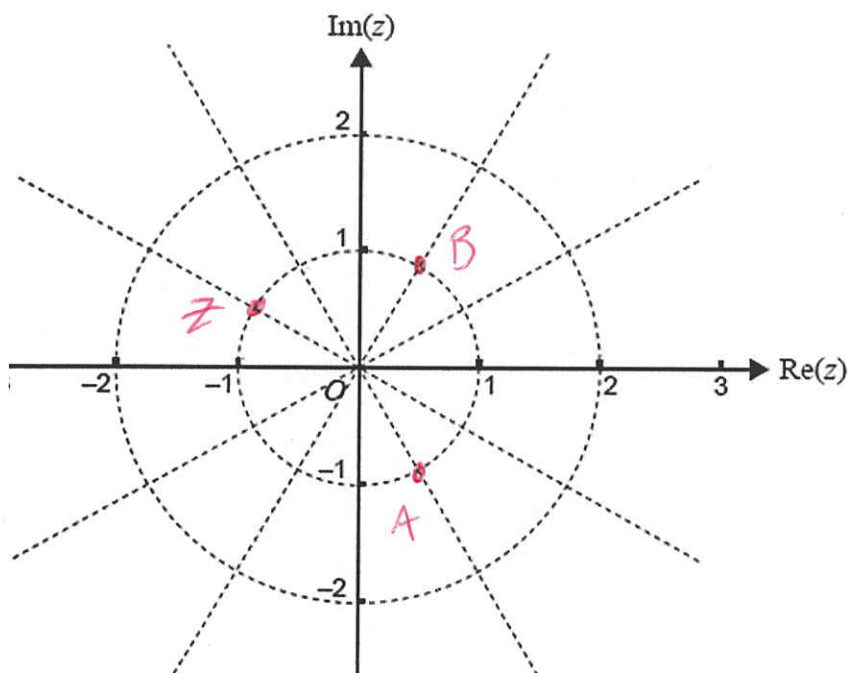
$$= \frac{3 + 2i}{13} \quad (1)$$

(3 marks)

$$= \frac{3}{13} + \frac{2}{13}i \quad (1)$$

Question 5

- a) On the Argand diagram below, plot and clearly label the complex number $z = \text{cis}\left(\frac{5\pi}{6}\right)$.



- b) On the Argand plane above, also plot and clearly label $A = z^2$ and $B = -i^5 z$.

$$A = z^2 = \text{cis}\left(\frac{5\pi}{3}\right)$$

$$B = -i^5 z = -iz$$

(1 + 3 = 4 marks)

Question 6

Factorise $P(z) = z^4 + 10z^2 + 9$.

$$= (z^2 + 9)(z^2 + 1) \quad (1)$$

$$= (z + 3i)(z - 3i)(z + i)(z - i) \quad (1)$$

(3 marks)

Question 7

Solve $z^3 + 64 = 0$. Express your answers in Cartesian form.

$$z^3 + 4^3 = 0$$

$$(z+4)(z^2-4z+16)=0 \quad (1)$$

$$(z+4)((z-2)^2+12)=0$$

$$(z+4)(z-2-2\sqrt{3}i)(z-2+2\sqrt{3}i)=0 \quad (1)$$

$$\therefore z = -4, \quad 2+2\sqrt{3}i, \quad 2-2\sqrt{3}i$$

(1) (1)

(4 marks)

Question 8

Use De Moivre's theorem to express $(\sqrt{3} + i)^6$ in the form $a + bi$.

$$z^n = r^n \cos n\theta$$

$$z = \sqrt{3} + i \quad r = \sqrt{3+1} = 2 \quad (1)$$

$$z = 2 \cos\left(\frac{\pi}{6}\right)$$

$$z^6 = 2^6 \cos\left(6 \times \frac{\pi}{6}\right) \quad (1)$$

$$= 64 \cos(\pi)$$

$$= 64(-1 + 0i)$$

$$= -64 + 0i$$

$$= -64 \quad (1)$$

END OF TEST

(3 marks)