

SOLUTIONS

Question 19															
In Australia the probability of a person having red hair is 0.15. In a random selection of 15 people the probability that exactly two have red hair is: A. 0.9126 B. 0.2312 C. 0.6814 D. 0.6042 E. 0.2856															
Question 20															
The following table represents the sampling distribution for a random variable. $Pr\left(\hat{p} > 0.4 \mid \hat{p} < 0.8\right)$ is: <table><tr><td>$\hat{p}$</td><td>0</td><td>0.2</td><td>0.4</td><td>0.6</td><td>0.8</td><td>1</td></tr><tr><td>$Pr\left(\hat{P} = \hat{p}\right)$</td><td>0.1</td><td>0.3</td><td>0.2</td><td>0.2</td><td>0.1</td><td>0.1</td></tr></table> A. $\frac{3}{8}$ B. $\frac{1}{4}$ C. $\frac{5}{9}$ D. $\frac{2}{9}$ E. $\frac{1}{2}$	$\hat{p}$	0	0.2	0.4	0.6	0.8	1	$Pr\left(\hat{P} = \hat{p}\right)$	0.1	0.3	0.2	0.2	0.1	0.1	
$\hat{p}$	0	0.2	0.4	0.6	0.8	1									
$Pr\left(\hat{P} = \hat{p}\right)$	0.1	0.3	0.2	0.2	0.1	0.1									

Multiple Choice Answer Table									
1	2	3	4	5	6	7	8	9	10
A	C	D	E	B	D	B	B	C	A
11	12	13	14	15	16	17	18	19	20
C	C	D	D	B	E	A	A	E	B

SHORT ANSWER/EXTENDED RESPONSE QUESTIONS

Question 1

A rabbit jumps several times, and the distance it travels each jump is measured. Its 3rd, 4th, and 5th jumps were $\frac{8}{9}$, $\frac{16}{27}$, and $\frac{32}{81}$ m, respectively. (Assume a geometric progression)

- a) Find the common ratio of the jumps.

$$r = \frac{16/27}{8/9} = 2/3 \quad (1/2)$$

- b) Assuming that every jump follows this pattern, calculate the distance of the 12th jump.

$$8/9 = a \times (2/3)^2 \quad (1/2)$$

$$a = 2 \quad (1/2)$$

$$t_{12} = 2 \times (2/3)^{11} \quad (1/2)$$

$$= \frac{4096}{177147} \text{ m} \quad (1/2)$$

- c) If the rabbit continued to jump forever, how far would it travel in total?

$$S_{\infty} = \frac{2}{1-2/3} \quad (1)$$

$$= 6 \text{ m} \quad (1)$$

(1 + 2 + 2 = 5 marks)

Question 2

Let $z = \frac{2k-3i}{1+ki}$, where $k \in \mathbb{R}$, and $\text{Im}(z) = -\frac{5}{2}$. Find all possible values of $\text{Re}(z)$.

$$z = x - 5/2 i$$

$$(x - 5/2 i)(1 + ki) = 2k - 3i \quad (1)$$

$$(x + 5k/2) + (xk - 5/2)i = 2k - 3i \quad (1)$$

$$(1) \quad \left\{ \begin{array}{l} x + 5k/2 = 2k \\ xk - 5/2 = -3 \end{array} \right\} \quad \begin{array}{l} \text{SOLVING ON CAS} \\ x = \pm 0.5, k = \mp 1 \end{array}$$

$$(1) \quad \text{Re}(z) = \pm 0.5$$

(4 marks)

Question 3

A circle is defined by the locus of points $P(x, y)$ whose distance from $A = (2, 5)$ is 5.

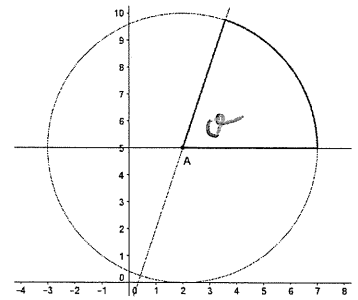
- a) A minor arc is bounded by the lines $y = 5$ and $y = 3x - 1$, for $2 \leq x \leq 7$. Find the length of this arc (shown in bold on the diagram) to 2 decimal places.

$$m = \tan \theta$$

$$\theta = \tan^{-1}(3) \quad (1)$$

$$\theta \approx 1.249^\circ \quad (1)$$

$$l = r\theta = 5 \times 1.249 = 6.25 \quad (1)$$



- b) Calculate the area of the sector bounded by the same lines to 2 decimal places.

$$A = \frac{1}{2} r^2 \theta$$

$$= \frac{1}{2} \times 5^2 \times 1.249 \quad (1)$$

$$\approx 15.61 \text{ unit}^2 \quad (1)$$

- c) Write the equation of the circle in Cartesian form.

$$(x-2)^2 + (y-5)^2 = 25 \quad (1)$$

- d) The circle is transformed by the matrix $\begin{bmatrix} 3 & 0 \\ 0 & \frac{1}{4} \end{bmatrix}$. Write the Cartesian equation of the resulting ellipse.

$$x' = 3x \Rightarrow x = x'/3 \quad (1/2)$$

$$y' = \frac{1}{4}y \Rightarrow y = 4y' \quad (1/2)$$

$$\left(\frac{x'}{3} - 2\right)^2 + (4y' - 5)^2 = 25 \quad (1)$$

$$\frac{(x-6)^2}{225} + \frac{16(y-\frac{5}{4})^2}{25} = 1 \quad (1)$$

- e) Find the area of the transformed sector from part b), to 2 decimal places.

$$A' = |\det(A)| \times A$$

$$= \frac{3}{4} \times 15.61 \quad (1)$$

$$\approx 11.71 \text{ unit}^2 \quad (1)$$

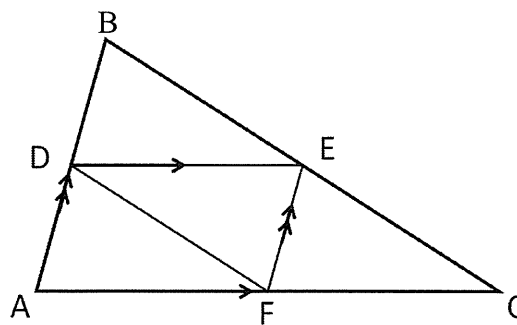
(3 + 2 + 1 + 3 + 2 = 11 marks)

Question 4

a) Show that $\triangle ADF \cong \triangle FED$

- (1/2) $AD = EF$ (PARALLELOGRAM)
 (1/2) $DE = AF$ (PARALLELOGRAM)
 (1/2) $DF = DF$ (COMMON)

(1/2) $\triangle ADF \cong \triangle FED$ (SSS)



b) Hence, show that $\triangle ABC \sim \triangle FED$

$$\left. \begin{array}{l} AB = 2AD \\ AC = 2AF \\ \angle A \text{ common} \end{array} \right\} (2)$$

$\triangle ABC \sim \triangle FED$ (SAS) (1)

c) If $\angle BAC = 50^\circ$, $AB = 5$, and $AC = 8$, find the total area of triangles $\triangle ADF$, $\triangle FEC$ and $\triangle DBE$. (Assume point D is the midpoint of AB) (Answer to two decimal places)

$$\begin{aligned} \text{Area } \triangle ABC &= \frac{1}{2} \times 5 \times 8 \times \sin 50^\circ & (1/2) \\ &= 15.321 & (1) \end{aligned}$$

$$\begin{aligned} \triangle ADF + \triangle FEC + \triangle DBE &= \frac{3}{4} \times 15.321 & (1/2) \\ &= 11.49 \text{ unit}^2 & (1) \end{aligned}$$

(2 + 3 + 3 = 8 marks)

Question 5

The rule for a transformation is $(x, y) \rightarrow (-x + 2y, 4x - y)$.

a) Find the image of the point $(2, -4)$.

$$(2, -4) \rightarrow (-2 - 8, 8 + 4) = (-10, 12) \quad (1)$$

b) Find the matrix of this transformation.

$$\begin{bmatrix} -1 & 2 \\ 4 & -1 \end{bmatrix} \quad (1)$$

c) Find the matrix of the inverse transformation.

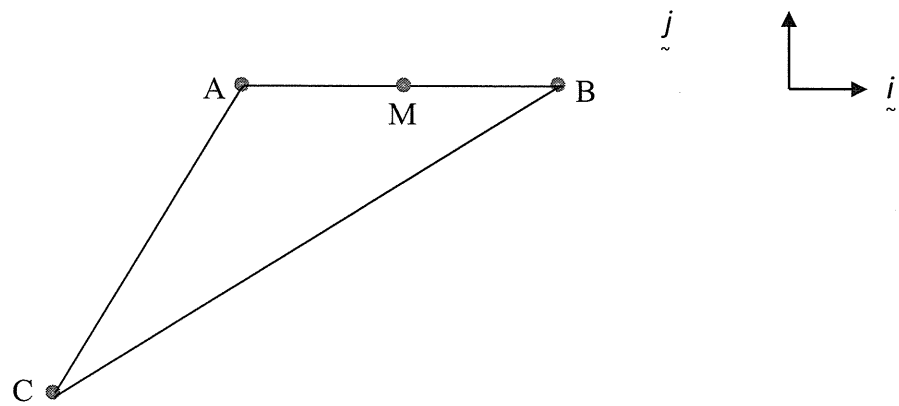
$$\frac{1}{7} \begin{bmatrix} 1 & 2 \\ 4 & -1 \end{bmatrix} \quad \text{OR} \quad \begin{bmatrix} 1/7 & 2/7 \\ 4/7 & -1/7 \end{bmatrix} \quad (1)$$

d) Find the point (x, y) that is mapped to the point $(-7, 14)$.

$$\begin{bmatrix} 1/7 & 2/7 \\ 4/7 & -1/7 \end{bmatrix} \begin{bmatrix} -7 \\ 14 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \end{bmatrix} \quad (1) \quad (1/2) \quad (3, -2)$$

(1 + 1 + 1 + 2 = 5 marks)

Question 6



In the diagram above A and B are two goal posts, M is the midpoint of the two goal posts and C is a footballer trying to kick a ball between the posts.

$$\vec{CA} = (10\hat{i} + 30\hat{j}) \text{ m and } \vec{AB} = (8\hat{i}) \text{ m}$$

- a) Find vector $\vec{CM}$.
- $$\begin{aligned}\vec{CM} &= \vec{CA} + \frac{1}{2}\vec{AB} \\ &= 10\hat{i} + 30\hat{j} + 4\hat{i} \quad (1) \\ &= 14\hat{i} + 30\hat{j} \quad (1)\end{aligned}$$
- b) Hence find the straight line distance from C to M. (Answer to two decimal places)

$$\begin{aligned}d &= \sqrt{14^2 + 30^2} \quad (1) \\ &\approx 33.11 \text{ m} \quad (1)\end{aligned}$$

A wind is blowing with a velocity $\vec{w} = (10\hat{i} + \lambda\hat{j})$ m/s. The footballer finds that if he kicks the ball with a velocity $\vec{v}$ which has a magnitude of 10 and at an angle 30° measured clockwise from $\hat{j}$ the ball moves directly towards the goal post A. Assume the total velocity of the ball will be $\vec{v} + \vec{w}$.

Let $\hat{a}$ be a unit vector in the direction of $\vec{CA}$

- c) Find $\hat{a}$.
- $$\begin{aligned}\hat{a} &= \frac{1}{\sqrt{10^2 + 30^2}} (10\hat{i} + 30\hat{j}) \quad (1) \\ &= \frac{1}{\sqrt{10}} \hat{i} + \frac{3}{\sqrt{10}} \hat{j} \quad \text{OR} \quad \frac{\sqrt{10}}{10} \hat{i} + \frac{3\sqrt{10}}{10} \hat{j} \quad (1)\end{aligned}$$

- d) If the ball travels directly from C to A, then $\underline{v} + \underline{w} = k\hat{a}$. Use this information to determine k and λ

$$\underline{v} = 10 \sin 30^\circ \underline{i} + 10 \cos 30^\circ \underline{j} = 5 \underline{i} + 5\sqrt{3} \underline{j} \quad (1/2)$$

$$\underline{v} + \underline{w} = 15 \underline{i} + (\lambda + 5\sqrt{3}) \underline{j} = k \left(\frac{1}{\sqrt{10}} \underline{i} + \frac{3}{\sqrt{10}} \underline{j} \right) \quad (1/2)$$

$$15 = k/\sqrt{10} \Rightarrow k = 15\sqrt{10} \quad (1/2)$$

$$\& \lambda + 5\sqrt{3} = \frac{3}{\sqrt{10}} k \Rightarrow \lambda = 45 - 5\sqrt{3} \quad (1/2)$$

- e) Hence find $\underline{w}$.

$$\underline{w} = 10 \underline{i} + (45 - 5\sqrt{3}) \underline{j} \quad (1)$$

(2 + 2 + 2 + 2 + 1 = 9 marks)

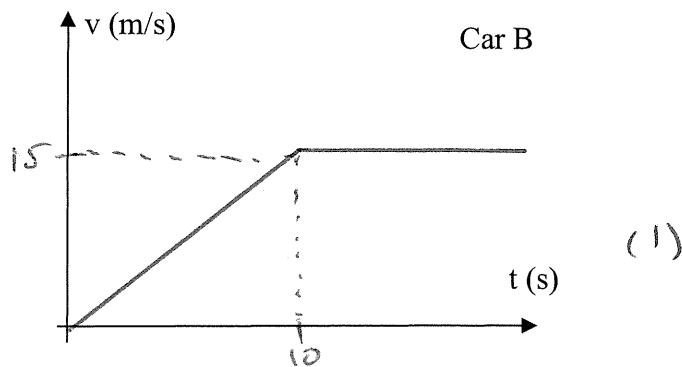
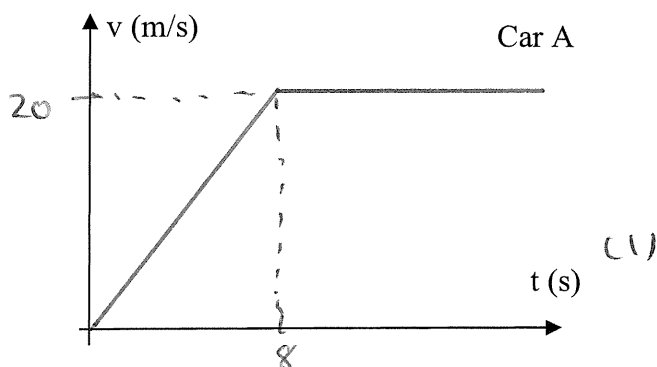
Question 7

Two traffic lights are 400 metres apart.

Car A starts from one traffic light and accelerates from rest uniformly at 2.5 m/s^2 up to a maximum speed of 20 m/s and then maintains a constant speed.

Car B starts from the other traffic light in the opposite direction. It accelerates from rest uniformly at 1.5 m/s^2 up to a maximum speed of 15 m/s and then maintains a constant speed.

- a) Sketch velocity – time graphs on the axes below for both cars.



- b) Hence find an expression for the distance covered by each car $d_A(t)$ and $d_B(t)$.

$$d_A(t) = \frac{1}{2} \times 8 \times 20 + 20(t-8) = 20t - 80, \quad t \geq 8 \quad (1)$$

$$d_B(t) = \frac{1}{2} \times 10 \times 15 + 15(t-10) = 15t - 75, \quad t \geq 10 \quad (1)$$

- c) At what time will they pass each other?

$$20t - 80 + 15t - 75 = 400 \quad (1)$$

$$t = \frac{11}{7} \text{ s} \quad (1)$$

- d) At this time how far is Car A from its starting point? (Answer to three decimal places)

$$d_A\left(\frac{11}{7}\right) = 20 \times \frac{11}{7} - 80 \quad (1/2)$$

$$\approx 237.143 \text{ m} \quad (1/2)$$

- e) When Car A is 40 metres from the other traffic light it brakes uniformly to a stop at the lights. How long does it take to stop?

$$u = 20$$

$$s = \frac{1}{2}(u+v)t$$

$$v = 0$$

$$a =$$

$$40 = \frac{1}{2}(20+0)t \quad (1)$$

$$s = 40$$

$$t = ?$$

$$t = 4 \text{ s} \quad (1)$$

(2 + 2 + 2 + 1 + 2 = 9 marks)

Question 8

A school has 1600 students of whom 850 are female. A random selection of six students is selected.

- a) Using a hypergeometric distribution calculate the probability that the sample has equal numbers of boys and girls. (Answer to four decimal places)

$$N = 1600, D = 850, n = 6, x = 3 \quad (1)$$

$$Pr = 0.3094 \quad (1)$$

- b) Now use a binomial approximation to estimate the probability that the sample has equal numbers of boys and girls. (Answer to four decimal places)

$$p = \frac{850}{1600}, n = 6, x = 3 \quad (1)$$

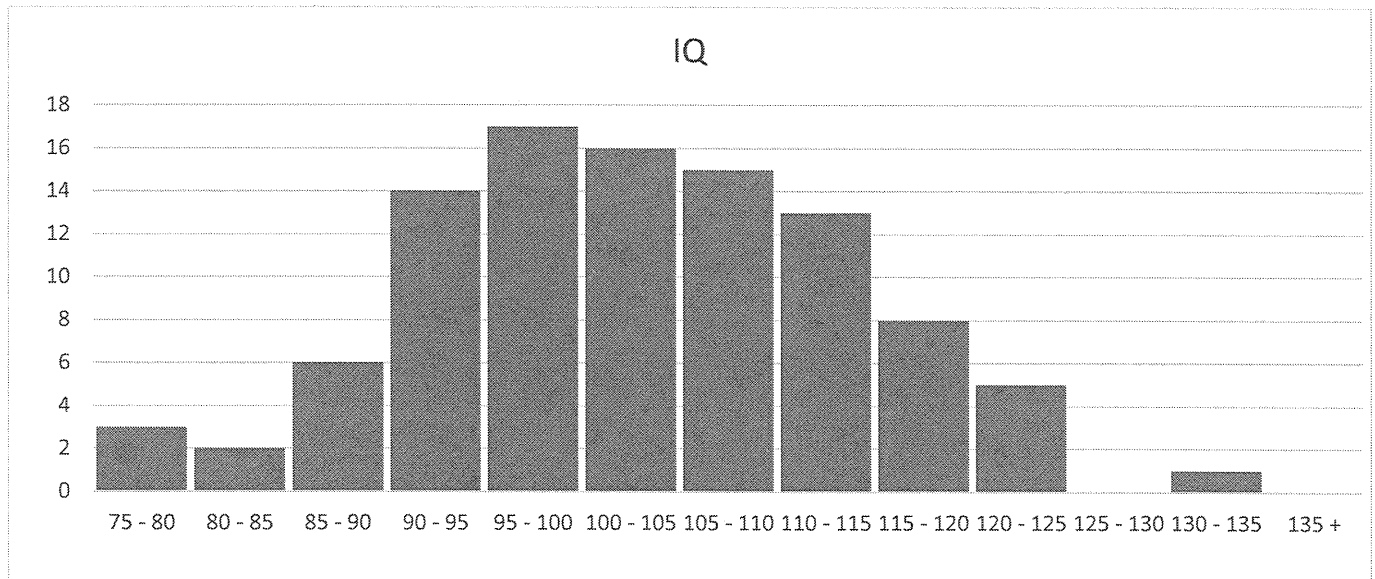
$$Pr = 0.3089 \quad (1)$$

- c) Comment on the suitability of using a binomial approximation in this case.

REASONABLE APPROXIMATION AS THE POPULATION
IS LARGE COMPARED TO THE SAMPLE (1)
(2 + 2 + 1 = 5 marks)

Question 9

The histogram below shows the distribution of the sample mean of IQ for 100 random samples of 20 people.



- a) State the proportion of the sample means which are greater than 120.

$$\frac{6}{100} = 0.06 \quad (1)$$

- b) Using the table, find the probability that the sample mean will be greater than 120 given that it is greater than 100.

$$\begin{aligned} \Pr(\bar{X} > 120 \mid \bar{X} > 100) &= \frac{\Pr(\bar{X} > 120)}{\Pr(\bar{X} > 100)} \\ (1) \quad &= \frac{0.06}{0.58} = \frac{3}{29} \quad (1) \end{aligned}$$

- c) How would you expect the distribution to be different if a sample size of 100 was used?

SAME MEAN, SMALLER SPREAD

(1/2)

(1/2)

(1 + 2 + 1 = 4 marks)

END OF EXAM