



**Yr 11 Specialist Maths
Sequences and Series
2017 Technology Access Test**

Name: SOLUTIONS

Time allowed: 45 minutes

Total Marks: / 45

Section A. Multiple Choice Questions. Clearly circle your answer (10 x 2 marks = 20 marks total)

1. If $t_n = 2t_{n-1} + 3$, $t_1 = 2$ then t_4 is: A. 29 B. 17 C. 11 D. 37 E. 33	<i>Working space</i> $2, 7, 17, 37$
2. A sequence is defined recursively as: $a_n = 1.5a_{n-1}$, $a_0 = 3$ then the sequence can be defined explicitly as: A. 1.5^n B. 1.5^{n+1} C. 2×1.5^n D. 3×1.5^n E. 4.5×1.5^n	<i>Working space</i> $a_0 = 3$ $a_1 = 4.5 = 3 \times 1.5$ $a_2 = 3 \times 1.5^2$ $a_3 = 3 \times 1.5^3$ etc
3. A sequence is defined by: $t_n = 2t_{n-1} + t_{n-2}$. If $t_3 = 16$ and $t_4 = 38$ then t_1 is: A. 1 B. 2 C. 3 D. 4 E. 5	<i>Working space</i> $38 = 2 \times 16 + t_2$ $t_2 = 6$ $16 = 2 \times 6 + t_1$ $t_1 = 4$
4. In an arithmetic sequence the fourth term is zero and the ninth term is 15. The first term is: A. -15 B. -9 C. -6 D. -4 E. -2	<i>Working space</i> $a + 3d = 0$ $a + 8d = 15$ $5d = 15$ $d = 3$ $a = -9$
5. The sum of the first six terms of an arithmetic sequence is 12 and the sum of the next six terms is -60. The first term and common difference respectively are: A. 7, -2 B. 10, -3 C. 0, 2 D. 5, -1 E. 18, -3	<i>Working space</i> $S_6 = 3(2a + 5d) = 12$ $2a + 5d = 4 \text{ --- (1)}$ $S_{12} = 6(2a + 11d) = 12 - 60$ $2a + 11d = -8 \text{ --- (2)}$ $(2) - (1) \Rightarrow 6d = -12 \quad d = -2$

Sub in (1) $\Rightarrow 2a - 10 = 4$
 $a = 7$

6. For the arithmetic sequence $\{a, 4a, 7a, 10a, \dots\}$ the sum of the first twenty terms is given by: A. $58a$ B. $59a$ C. $290a$ D. $580a$ E. $590a$	Working space $a = a \quad d = 3a$ $S_{20} = 10(2a + 19 \times 3a)$ $= 590a$
7. A geometric series has $t_3 = 36$ and $t_6 = 972$. The second term is: A. 9 B. 12 C. 18 D. 6 E. 24	Working space $ar^2 = 36 \quad \text{--- (1)}$ $ar^5 = 972 \quad \text{--- (2)}$ $\textcircled{2} \div \textcircled{1} \Rightarrow r^3 = 27$ $r = 3$ $a \times 9 = 36 \Rightarrow a = 4$
8. The geometric mean of a^2bc^3 and a^6b^3c is: A. $a^8b^4c^4$ B. a^6b^3c C. $a^4b^2c^2$ D. ab^2c^2 E. a^5b^3c	Working space $\sqrt{a^8b^4c^4}$ $= a^4b^2c^2$
9. A geometric sequence has $a = 3$ and $S_5 = 93$. If $r > 1$, then r is: A. 1.5 B. 2 C. 2.5 D. 3 E. 2.5568	Working space $S_5 = \frac{3(r^5 - 1)}{r - 1} = 93$ $3(r^5 - 1) = 93(r - 1)$ $\text{CAS} \Rightarrow r = -2.5568, 1, \textcircled{2}$
10. A drink bottle initially has 1,000,000 bacteria. Each time it is washed 80% of the bacteria are removed. After five washes, the amount of bacteria remaining is closest to: A. 1600 B. 409600 C. 320 D. 327680 E. 0	Working space $a = 1,000,000 \times 0.2$ $r = 0.2$ $t_5 = 1,000,000 \times 0.2^5$ $= 320$

Section B. Short Answer Questions (total 25 marks)

1. For the arithmetic sequence $\{19.8, 19.6, 19.4, \dots, 0\}$ find:

a) The number of terms.

$$a = 19.8 \quad d = -0.2 \quad (1/2)$$

$$t_n = 19.8 - 0.2(n-1) = 0 \quad (1/2)$$

$$n-1 = \frac{19.8}{0.2} = 99 \quad (1/2)$$

$$\underline{n = 100} \quad (1/2)$$

b) The sum of the sequence.

$$(1/2) \quad S_n = \frac{n}{2}(a+d)$$

$$(1/2) \quad = \frac{100}{2}(19.8+0)$$

$$(1) \quad = 990$$

$$\begin{aligned} \text{OR } S_n &= \frac{n}{2} [2a + (n-1)d] \\ &= \frac{100}{2} [2 \times 19.8 + 99 \times -0.2] \\ &= 990 \end{aligned}$$

(2 + 2 = 4 marks)

2. For the fibonacci type sequence $t_n = 2t_{n-1} + 3t_{n-2}$, $t_1 = 1$, $t_2 = 3$ the value of t_5 is:

$$(1/2) \quad t_3 = 2 \times 3 + 3 \times 1 = 9$$

$$(1/2) \quad t_4 = 2 \times 9 + 3 \times 3 = 27$$

$$(1) \quad t_5 = 2 \times 27 + 3 \times 9 = \underline{81}$$

(2 marks)

3. For the geometric sequence $\{\sqrt{2}, \sqrt{6}, 3\sqrt{2}, \dots\}$

a. Write down the values of a and r .

$$a = \sqrt{2} \quad (1/2)$$

$$r = \sqrt{3} \quad (1/2)$$

b. Find the exact value of the sum of the first eight terms.

$$S_8 = \frac{\sqrt{2}((\sqrt{3})^8 - 1)}{\sqrt{3} - 1} = \frac{80\sqrt{2}}{\sqrt{3} - 1} \times \frac{\sqrt{3} + 1}{\sqrt{3} + 1} \quad (1)$$

$$= \frac{80\sqrt{6} + 80\sqrt{2}}{2}$$

$$= 40\sqrt{6} + 40\sqrt{2} \quad (1)$$

(1 + 2 = 3 marks)

4. Express $2.\dot{1}7$ as an improper fraction.

$$= 2 + 0.17 + 0.0017 + \dots \quad (1)$$

$$= 2 + \frac{0.17}{1-0.01} \quad (1)$$

$$= 2 + \frac{0.17}{0.99}$$

$$= \frac{215}{99} \quad (1)$$

(3 marks)

5. If 2, x, y are the first three terms in a geometric sequence and 4, x, y-10 are the first three terms in an arithmetic sequence, find all possible values of x and y.

$$\frac{y}{x} = \frac{x}{2} \Rightarrow 2y = x^2 \quad - (1)$$

$$y-10-x = x-4 \Rightarrow y = 2x+6 \quad - (2)$$

$$4x+12 = x^2 \quad (1)$$

$$x^2 - 4x - 12 = 0$$

$$(x-6)(x+2) = 0 \quad (1)$$

$$x = -2, 6$$

$$y = 18, 2$$

$$\text{ANS : } \left. \begin{array}{l} x = -2, y = 2 \\ \text{or } x = 6, y = 18 \end{array} \right\} \quad (1) \quad (4 \text{ marks})$$

6. An arithmetic sequence has a sum given by: $S_n = 2n^2 + 5n$. Find the first term and common difference.

$$n=1 \Rightarrow S_1 = 7 \Rightarrow \underline{a=7} \quad (1)$$

$$n=2 \Rightarrow S_2 = 2 \times 4 + 10 = 18 \quad (1)$$

$$2 \times 7 + d = 18$$

$$\underline{d=4} \quad (1)$$

NOTE : OTHER METHODS POSSIBLE

(3 marks)

7. A ball is dropped from a height of 12 metres. Every time it hits the ground it rebounds to 60% of the previous height. It continues to do this indefinitely. This can be expressed as a geometric sequence.

Find the total distance travelled.

HINT: The first drop is only downwards. Subsequent bounces are upwards and downwards.

$$S_{\infty} = 12 + 2 \times [12 \times 0.6 + 12 \times 0.6^2 + \dots] \quad (1)$$

$$= 12 + 2 \times \frac{7.2}{1-0.6} \quad (1)$$

$$= 48 \text{ m} \quad (1)$$

(3 marks)

8. Two factories produce cars.

Factory A produces 100 cars in the first month and increases production by 10 cars each month.

Factory B produces 20 cars in the first month and increases production by 20% each month.

After how many months will the **total** number of cars (S_n) produced by Factory B exceed those produced by Factory A. (You may use your CAS to solve equations but all working must be shown)

$$\begin{aligned} A: S_n &= \frac{n}{2} [200 + (n-1) \times 10] \\ &= 100n + 5n^2 - 5n \\ &= 95n + 5n^2 \end{aligned} \quad (1)$$

$$\begin{aligned} B: S_n &= \frac{20(1.2^n - 1)}{1.2 - 1} \\ &= 100(1.2^n - 1) \end{aligned} \quad (1)$$

$$\text{SOLVE ON CAS: } 95n + 5n^2 = 100(1.2^n - 1)$$

$$n \approx 20.389$$

ANS: WILL EXCEED AFTER 21 MONTHS (1)

(3 marks)

END OF TEST

