



Australian Mathematics Competition

AUSTRALIAN MATHEMATICS TRUST



AUSTRALIAN MATHEMATICS COMPETITION

AN ACTIVITY OF THE AUSTRALIAN MATHEMATICS TRUST

NAME _____

YEAR _____

TEACHER _____

2017

SENIOR DIVISION

AUSTRALIAN SCHOOL YEARS 11 and 12

TIME ALLOWED: 75 MINUTES

INSTRUCTIONS AND INFORMATION

GENERAL

1. Do not open the booklet until told to do so by your teacher.
2. NO calculators, maths stencils, mobile phones or other calculating aids are permitted. Scribbling paper, graph paper, ruler and compasses are permitted, but are not essential.
3. Diagrams are NOT drawn to scale. They are intended only as aids.
4. There are 25 multiple-choice questions, each requiring a single answer, and 5 questions that require a whole number answer between 0 and 999. The questions generally get harder as you work through the paper. There is no penalty for an incorrect response.
5. This is a competition not a test; do not expect to answer all questions. You are only competing against your own year in your own country/Australian state so different years doing the same paper are not compared.
6. Read the instructions on the answer sheet carefully. Ensure your name, school name and school year are entered. It is your responsibility to correctly code your answer sheet.
7. When your teacher gives the signal, begin working on the problems.

THE ANSWER SHEET

1. Use only lead pencil.
2. Record your answers on the reverse of the answer sheet (not on the question paper) by FULLY colouring the circle matching your answer.
3. Your answer sheet will be scanned. The optical scanner will attempt to read all markings even if they are in the wrong places, so please be careful not to doodle or write anything extra on the answer sheet. If you want to change an answer or remove any marks, use a plastic eraser and be sure to remove all marks and smudges.

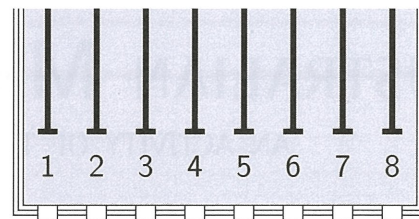
INTEGRITY OF THE COMPETITION

The AMT reserves the right to re-examine students before deciding whether to grant official status to their score.

Senior Division

Questions 1 to 10, 3 marks each

1. Eight children in a swimming race are John, Iain, Hans, Ivan, Giovanni, Beth, Liz and Elisa. They are put in lanes 1 to 8 randomly. What is the probability that Beth, Liz or Elisa is in lane 1?



(A) $\frac{3}{8}$ (B) $\frac{1}{2}$ (C) $\frac{3}{5}$ (D) $\frac{5}{3}$ (E) $\frac{1}{3}$

2. What is the value of $17a + 20b$ when $a = 20$ and $b = 17$?

(A) 680 (B) 689 (C) 1720 (D) 2017 (E) 3737

3. 1000% of 1 is

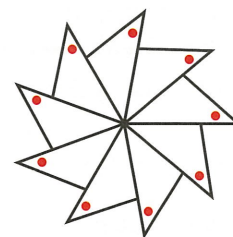
(A) 0.1 (B) 1 (C) 10 (D) 100 (E) 1000

4. $4^2 + 3^3 + 2^4 =$

(A) 29 (B) 33 (C) 43 (D) 59 (E) 73

5. This pinwheel star is formed by rotating a right-angled triangle around one of its corners. What is the angle at each of the nine tips that are marked with dots?

(A) 30° (B) 40° (C) 45°
(D) 50° (E) 60°



6. Alice is playing with words. At each tick of her grandfather's clock she swaps two letters. What is the smallest number of clock ticks during which she can change WORDS to SWORD?

(A) 3 (B) 4 (C) 6 (D) 7 (E) 8

7. $200 - 199 + 198 - 197 + 196 - \dots + 2 - 1$ is equal to

(A) 1 (B) 99 (C) 100 (D) 101 (E) 200

8. I type 360 characters per minute. I am typing a long document that has exactly 50 characters per line. After 8 minutes of typing, which line number am I on?

(A) 54 (B) 55 (C) 56 (D) 57 (E) 58

9. I make a multiplication table for the numbers from 1 to 5. What is the sum of the 25 numbers inside this table?

(A) 100 (B) 200 (C) 225
(D) 250 (E) 256

$\times$	1	2	3	4	5
1					
2					
3					
4					
5					

10. Evaluate $\sqrt{2} \times \sqrt[4]{2} \times \sqrt{800} \times \sqrt[4]{8}$.

(A) 2 (B) 4 (C) 8 (D) 80 (E) 800

Questions 11 to 20, 4 marks each

11. Each row, column and diagonal of this grid should add up to 30. What number goes in the bottom-right corner?

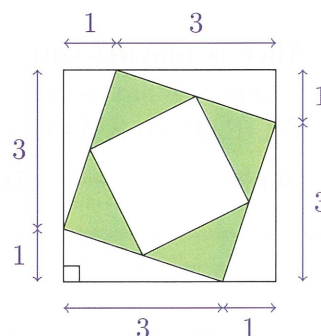
(A) 3 (B) 15 (C) 5 (D) 13 (E) 4

	2	
8		12
		?

12. The vertices of the innermost white square bisect the sides of the shaded square.

What is the area of the innermost white square?

- (A) 6 (B) 4 (C) 10
(D) 3 (E) 5



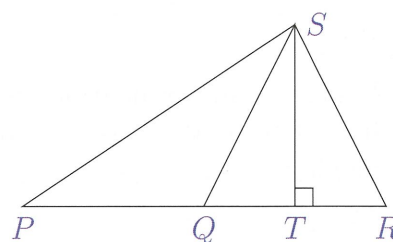
13. Two cars leaving Sale for Melbourne are travelling at 60 km/h and are 240 m apart travelling west on the Princes Highway. When each car reaches the 80 km/h sign it starts travelling at 80 km/h. How far apart are they once they are both past the sign?

- (A) 180 m (B) 240 m (C) 360 m (D) 300 m (E) 320 m

14. In the triangle shown, $PQ = SQ = SR = QR$ and $\angle STR = 90^\circ$.

The ratio $TR : PS$ is equal to

- (A) $1 : \sqrt{3}$ (B) $1 : 2\sqrt{3}$ (C) $1 : 2$
(D) $1 : \sqrt{2}$ (E) $1 : (3\sqrt{3})/2$



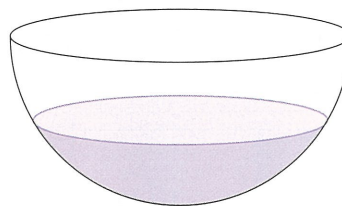
15. The solution to the equation $2017x^2 - 2017^{2017} = 0$ is

- (A) $x = \pm 2017$ (B) $x = \pm 2017^{2017}$ (C) $x = \pm 2017^{1008}$
(D) $x = \pm 2017^{1009}$ (E) $x = \pm \sqrt{2017^{2017}}$

16. A hemispherical bowl is filled to half its depth.

The maximum angle through which the bowl may be tilted from horizontal without spilling is

- (A) $22\frac{1}{2}^\circ$ (B) 30° (C) 45°
(D) 60° (E) $67\frac{1}{2}^\circ$



17. I spent exactly \$4000 on books which cost \$25 and \$26 each. I bought at least one book at each price. What is the maximum number of books I could have bought?

- (A) 153 (B) 157 (C) 155 (D) 159 (E) 161

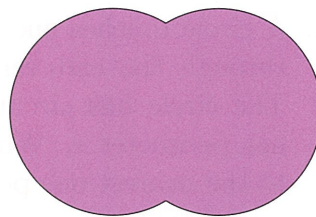
18. All of the digits from 0 to 9 are used to form two 5-digit numbers. What is the smallest possible difference between these two numbers?

(A) 1 (B) 9 (C) 99 (D) 247 (E) 315

19. Two circles of diameter 12 cm overlap, forming the shape shown with perimeter 14π cm.

What is the area of the shape, in cm^2 ?

(A) $42\pi + 18$ (B) $42\pi + 9\sqrt{3}$ (C) $72\pi - 18$
 (D) $72\pi - 9\sqrt{3}$ (E) $24\pi\sqrt{3}$



20. Five spherical balls of diameter 10 cm fit inside a closed cylindrical tin with internal diameter 16 cm.

What is the smallest height the tin can be?

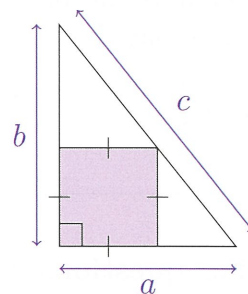
(A) 39 cm (B) 42 cm (C) 45 cm (D) 48 cm (E) 50 cm

Questions 21 to 25, 5 marks each

21. A square is drawn in the corner of a right-angled triangle with side lengths a , b and c , as shown.

Which expression gives the ratio of the unshaded area to the shaded area in all cases?

(A) 1 : 1 (B) $c : (a + b)$ (C) $ab : c^2$
 (D) $(a + b)^2 : 2c^2$ (E) $c^2 : 2ab$



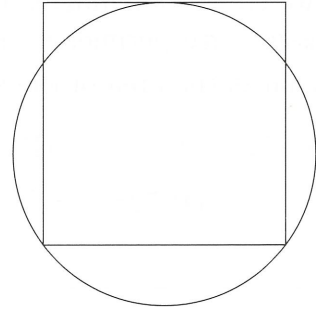
22. Andy and Brandy have decided to share a cake in an unusual way. Andy will take some fraction of the cake first. From then on, Brandy and Andy will alternate taking half of what remains each time. Eventually there is so little left they decide they are finished. What fraction of the cake should Andy take first to ensure that they end up with half each?

(A) $\frac{1}{6}$ (B) $\frac{1}{4}$ (C) $\frac{2}{5}$ (D) $\frac{1}{3}$ (E) $\frac{1}{2}$

23. Each student in a class sends ten Christmas cards, one to each of ten other students in the class. It is known that no two students sent Christmas cards to each other. What is the smallest number of students who could be in the class?

(A) 15 (B) 20 (C) 21 (D) 25 (E) 30

24. A circle touches one side of a square and passes through the end vertices of the opposite side. The circle has circumference c and the square has perimeter s . Which of the following is true to the nearest one percent?



- (A) c is approximately 5% larger than s .
(B) c is approximately 2% larger than s .
(C) c and s are equal.
(D) c is approximately 2% smaller than s .
(E) c is approximately 5% smaller than s .

25. I have two blue socks, two red socks, and two yellow socks in a drawer. On Monday, I randomly pick two of the socks to wear. On Tuesday, I randomly pick two of the remaining socks to wear. On Wednesday, I wear the two remaining socks from the drawer.

What is the probability that I didn't wear a matching pair of socks on any of the three days?

(A) $\frac{1}{8}$ (B) $\frac{2}{5}$ (C) $\frac{1}{2}$ (D) $\frac{64}{125}$ (E) $\frac{8}{15}$

For questions 26 to 30, shade the answer as an integer from 0 to 999 in the space provided on the answer sheet.

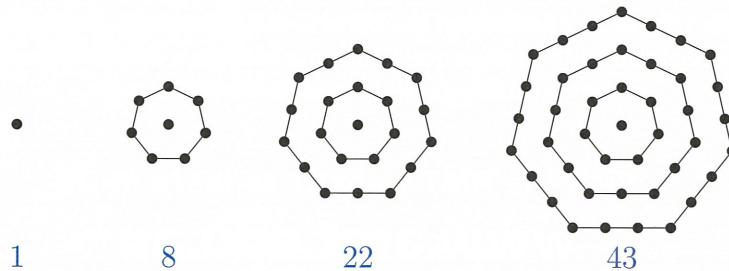
Question 26 is 6 marks, question 27 is 7 marks, question 28 is 8 marks, question 29 is 9 marks and question 30 is 10 marks.

26. A river runs from west to east. Colin's hat has fallen in 28 metres upstream from the waterfall. Pepper the wonder dog is admiring the view 36 metres directly south of the waterfall. Pepper can run exactly three times as fast as the hat is being carried downstream. What is the minimum distance in metres that Colin's hat must travel before Pepper can retrieve it?

27. Mike multiplied at least two consecutive integers together. He obtained a six-digit number N . The first two digits of N are 47 and the last two digits of N are 74. What is the sum of the integers that Mike multiplied together?

28. For $n \geq 3$, the sequence of *centred n -gon numbers* is found by starting with a central dot, then adding layers consisting of n -gons of dots around this centre, where the number of dots on each side increases by 1 for each layer.

For instance, the sequence of centred 7-gon numbers starts 1, 8, 22, 43, ... as shown.

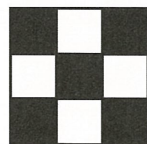


What is the smallest n for which 2017 is in the sequence of centred n -gon numbers?

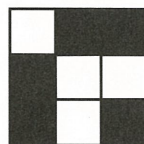
29. A friend and I play a game. We each start with two coins. We take it in turns to toss a coin; if it comes down heads, we keep it, if tails, we give it to the other. I always go first, and the game ends when one of us wins by having all four coins.

If we play this game 840 times, what is the expected number of games that I would win?

30. The squares of an 8×8 grid are coloured black or white. A colouring is called *balanced* if each 2×2 subgrid contains exactly two squares of each colour. An example of a balanced colouring of a 3×3 grid is shown on the left. The *unbalanced* colouring on the right fails this requirement since the 2×2 subgrid on the bottom right contains three white squares.



balanced



unbalanced

Counting rotations and reflections of a pattern as different, how many balanced colourings of the 8×8 grid are there?

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Check your answers!

2017 Solutions includes the problems and complete solutions to all five papers of the Australian Mathematics Competition (AMC).

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AUSTRALIAN MATHEMATICS TRUST

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