



SOLUTIONS

2017 Yr 11 Specialist Maths Unit 1 Algebra Test – Technology Free Assessment

Name: _____

Time allowed: 30 minutes

Total: / 30 marks

1.	Solve the equation for x where $\frac{x+2}{3} = 5 - \frac{x}{2}$ Note 2^m method 1^m answer $\frac{x+2}{3} = \frac{10-x}{2}$ $2x+4 = 30-3x$ $5x = 26$ $x = \frac{26}{5} \text{ or } 5\frac{1}{5} \text{ or } 5.2$ (3 marks)
2.	Solve for x $b = ab - \frac{1}{x+a}$ Note 2^m method 1^m answer $b(x+a) = ab(x+a) - 1$ $(b-ab)x = a^2b - ab - 1$ $x = \frac{ab(a-1)-1}{b(1-a)}$ or $\frac{a^2b - ab - 1}{b - ab}$ (3 marks)
3.	Transpose $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$ to make v the subject $\frac{v+u}{uv} = \frac{1}{f}$ $fv + fu = uv$ $(f-u)v = -fu$ $v = \frac{-f \cdot u}{f-u}$ or $\frac{fu}{u-f}$ (3 marks)
4.	Find the values of a and b so that $\frac{6}{ax+b} = \frac{5}{x-4}$ $6x-24 = 5ax+5b$ $5a=6$ $5b=-24$ $a = \frac{6}{5}$ $b = -\frac{24}{5}$ $\therefore a = \frac{6}{5} \text{ and } b = -\frac{24}{5}$ (3 marks)

Mark method accordingly!

5. Express $\frac{3}{x-2} - \frac{5}{x+4}$ as a single fraction.

$$\frac{3}{x-2} - \frac{5}{x+4} = \frac{3(x+4) - 5(x-2)}{(x-2)(x+4)} \quad (1)^M$$

$$= \frac{3x - 5x + 12 + 10}{(x-2)(x+4)}$$

$$= \frac{-2x + 22}{(x-2)(x+4)} \quad \text{or} \quad \frac{-2(x-11)}{(x-2)(x+4)} \quad (1)^A$$

(2 marks)

6. Express $2x^2 + 11x + 10$ in the form $a(x+2)^2 + b(x+2) + c$

$$2x^2 + 11x + 10 = a\bar{x}^2 + 4ax + 4a + bx + 2b + c \quad (1)$$

$$a = 2 \quad (1/2)$$

$$4a + 2b + c = 10$$

$$4a + b = 11$$

$$c = 10 - 8 - 6$$

$$b = 11 - 8 = 3 \quad (1/2)$$

$$= -4$$

(1)

$$\therefore 2x^2 + 11x + 10 = 2(x+2)^2 + 3(x+2) - 4 \quad (1)$$

(4 marks)

7. Express $\frac{2x^2 - 9x + 13}{(x+5)(x-1)^2}$ in partial fractions.

$$\frac{2x^2 - 9x + 13}{(x+5)(x-1)^2} = \frac{A}{x+5} + \frac{B}{x-1} + \frac{C}{(x-1)^2} \quad (1)$$

$$2x^2 - 9x + 13 = A(x-1)^2 + B(x+5)(x-1) + C(x+5) \quad (1)$$

$$\text{Let } x = 1 \quad -6 = 6C$$

$$C = 1 \quad (1/2)$$

$$\text{Let } x = -5 \quad 108 = 36A \quad (1/2)$$

$$A = 3$$

$$\text{Let } x = 0 \quad 13 = A - 5B + 5C$$

$$5B = 3 + 5 - 13$$

$$B = -1 \quad (1)$$

$$\therefore \frac{2x^2 - 9x + 13}{(x+5)(x-1)^2} = \frac{3}{x+5} - \frac{1}{x-1} + \frac{1}{(x-1)^2} \quad (1)$$

(5 marks)

8. Solve the following simultaneous equations.

$$2x + ay = b \quad \dots \textcircled{1}$$

$$x - 2ay = -b \quad \dots \textcircled{2}$$

$$\begin{aligned} \textcircled{1} - 2 \times \textcircled{2} \text{ gives } & \begin{array}{r} 2x + ay = b \\ \underline{2x - 4ay = -2b} \\ 5ay = 3b \\ y = \frac{3b}{5a} \end{array} \quad \textcircled{1} \end{aligned}$$

$$\begin{aligned} \text{Sub } y = \frac{3b}{5a} \text{ into } \textcircled{2} \text{ gives: } & \begin{array}{r} x = -b + 2a\left(\frac{3b}{5a}\right) \\ = -b + \frac{6b}{5} \\ = \frac{b}{5} \end{array} \quad \textcircled{1} \end{aligned}$$

$$\boxed{\therefore x = \frac{b}{5} \quad y = \frac{3b}{5a}} \quad \textcircled{1}^A$$

(3 marks)

9. Calculate the coordinates of the points of intersection of the straight line with equation $x + y = 2$ and the circle with equation $x^2 + y^2 = 8$

$$\text{Sub } y = 2 - x \text{ into } x^2 + y^2 = 8 \quad \textcircled{\frac{1}{2}} \quad \textcircled{\frac{1}{2}}$$

$$x^2 + (2 - x)^2 = 8$$

$$x^2 + 4 - 4x + x^2 = 8$$

$$2x^2 - 4x - 4 = 0$$

$$x^2 - 2x - 2 = 0$$

$$x_{1,2} = \frac{2 \pm \sqrt{4 + 8}}{2}$$

$$= \frac{2 \pm 2\sqrt{3}}{2}$$

$$= 1 \pm \sqrt{3} \quad \textcircled{1}$$

$$\text{When } x = 1 + \sqrt{3}$$

$$y = 1 - \sqrt{3}$$

$$\text{When } x = 1 - \sqrt{3}$$

$$y = 1 + \sqrt{3}$$

$$\therefore (1 + \sqrt{3}, 1 - \sqrt{3}) \quad \textcircled{1}$$

$$(1 - \sqrt{3}, 1 + \sqrt{3}) \quad \textcircled{1}$$

(4 marks)