



Glen Waverley Secondary College
2016 Unit 2 Technology Access Exam

Specialist Mathematics

Teachers:

Please circle



CAL



JLS



GMN

Student Name: SOLUTIONS

Student Code: Form Group:

Instructions:

Reading Time:	10min
Writing Time:	120 min

Section A Marks:	40 marks
Section B Marks:	60 marks
Total Marks:	100 marks

This examination consists of 20 multiple choice questions and 12 short answer/extended response questions. All multiple choice questions should be answered on the **table** provided at the end this section (bottom of page 8). For each short answer/extended response question correct working **must be** shown to gain full marks.

A CAS calculator, a scientific calculator and a bound reference are allowed.

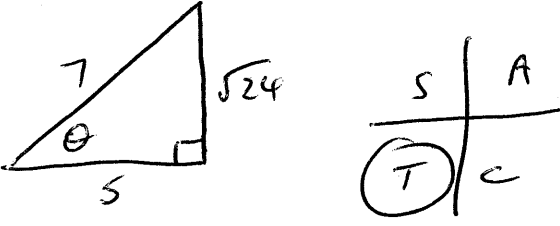
A formula sheet is provided.

Marks are allocated for working as well as answers.

Answer multiple choice questions on the table at the end of this section.

Question 1	
If $A = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 1 \\ 3 & 2 \end{bmatrix}$ and $AX + B = \begin{bmatrix} -2 & -2 \\ 1 & 14 \end{bmatrix}$ then X is: A. $\begin{bmatrix} -5 & -3 \\ -2 & 12 \end{bmatrix}$ B. $\begin{bmatrix} -8 & -18 \\ -23 & 39 \end{bmatrix}$ C. $\begin{bmatrix} -1 & -1 \\ -4 & 2 \end{bmatrix}$ D. $\begin{bmatrix} -2 & 0 \\ 1 & 3 \end{bmatrix}$ E. $\begin{bmatrix} -19 & -7 \\ 32 & 50 \end{bmatrix}$	$X = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}^{-1} \left(\begin{bmatrix} -2 & -2 \\ 1 & 14 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ 3 & 2 \end{bmatrix} \right)$ $= \begin{bmatrix} -2 & 0 \\ 1 & 3 \end{bmatrix}$
Question 2	
A transformation is given by: $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 3 \\ 1 \end{bmatrix}$ The point (3,-1) is mapped to: A. (9,-3) B. (6,-4) C. (5,5) D. (7,3) E. (1,13)	$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix} \begin{bmatrix} 3 \\ -1 \end{bmatrix} + \begin{bmatrix} 3 \\ 1 \end{bmatrix}$ $= \begin{bmatrix} 9 \\ -3 \end{bmatrix}$
Question 3	
The unit square is mapped to a parallelogram of area 6 units by the matrix: $\begin{bmatrix} n & 4 \\ -n & 2n \end{bmatrix}$ Possible values for n are: A. $-1 \pm \sqrt{2}$ B. -3,1 C. $1 \pm \sqrt{2}$ D. -1,3 E. -1,1	$ 2n^2 + 4n = 6$ $2n^2 + 4n - 6 = 0 \quad \left \quad 2n^2 + 4n + 6 = 0 \right.$ $x = -3, x = 1 \quad \left \quad \text{NO REAL SOLUTIONS} \right.$

Question 4	
The acceleration of an object is given by: $a(t) = 6t^2 - 2t, t \geq 0$ Initially the object is at rest at the origin. The position of the object is given by: A. $x(t) = \frac{1}{2}t^4 - \frac{1}{3}t^3, t \geq 0$ B. $x(t) = 2t^4 - 3t^3, t \geq 0$ C. $x(t) = t^4 - t^3, t \geq 0$ D. $x(t) = \frac{1}{2}t^4 - t^3, t \geq 0$ E. $x(t) = t^4 - \frac{1}{3}t^3, t \geq 0$	$v(t) = 2t^3 - t^2, t \geq 0$ $x(t) = \frac{1}{2}t^4 - \frac{1}{3}t^3, t \geq 0$
Question 5	
The expression $\sin(\theta) - \sqrt{3}\cos(\theta)$ can be expressed as: A. $2\sin\left(\theta + \frac{\pi}{3}\right)$ B. $2\sin\left(\theta + \frac{\pi}{6}\right)$ C. $2\cos\left(\theta + \frac{\pi}{3}\right)$ D. $2\cos\left(\theta + \frac{\pi}{6}\right)$ E. $2\sin\left(\theta - \frac{\pi}{3}\right)$	

Question 6 If $\cos(\theta) = -\frac{5}{7}$ and $\pi \leq \theta \leq \frac{3\pi}{2}$, then $\cot(\theta)$ is: A. $-\frac{7}{5}$ B. $-\frac{\sqrt{24}}{5}$ C. $\frac{\sqrt{24}}{5}$ D. $-\frac{5}{\sqrt{24}}$ E. $\frac{5}{\sqrt{24}}$	
Question 7 A motorcycle accelerates uniformly from 20 m/s to 40 m/s over a distance of 120 metres. The acceleration is closest to: A. 0.167 m/s² B. 1.2 m/s² C. 5 m/s² D. 12 m/s² E. 10 m/s²	$u = 20$ $v = 40$ $a = ?$ $s = 120$ $v^2 = u^2 + 2as$ $40^2 = 20^2 + 240a$ $a = 5$
Question 8 In Australia the proportion of adults with red hair is 12%. In a randomly selected sample of eight people, the probability that none of them have red hair is closest to: A. 0.3923 B. 0.3454 C. 0.8163 D. 0.9600 E. 0.3596	$x = 0, n = 8, p = 0.12$ BINOMIAL

Question 9 A chocolate box contains five soft centres and three hard centres. Five chocolates are randomly drawn without replacement. The probability that the proportion of soft-centre chocolates is greater than 0.75 is closest to: A. 0.2679 (B.) 0.2857 C. 0.0179 D. 0.7143 E. 0.9821	HYPERGEO CDF LOWER = 4, UPPER = 5 N = 8, O = 5, n = 5
Question 10 A boat has an initial velocity of 4 m/s and accelerates uniformly for three seconds and covers a distance of 80 metres. How far did the boat travel in the third second? A. $\frac{308}{9}$ m B. $\frac{80}{3}$ m (C.) $\frac{376}{9}$ m D. $\frac{148}{3}$ m E. $\frac{448}{9}$ m	$u = 4$ $v =$ $a = ?$ $s = 80$ $t = 3$ $s = ut + \frac{1}{2}at^2$ $80 = 12 + \frac{9}{2}a$ $a = \frac{136}{9}$ $u = 4$ $v =$ $a = \frac{136}{9}$ $s = ?$ $t = 2$ $s = ut + \frac{1}{2}at^2$ $= 4 \times 2 + \frac{1}{2} \times \frac{136}{9} \times 4$ $= \frac{344}{9}$ $DIST = 80 - \frac{344}{9} = \frac{376}{9}$

Question 11

The partial fraction decomposition of

$\frac{3x+7}{(x-2)(x+2)^2}$ has the form:

A. $\frac{A}{x-2} + \frac{B}{x+2} + \frac{C}{x+2}$

B. $\frac{A}{x-2} + \frac{Bx+C}{(x+2)}$

☒ C. $\frac{A}{x-2} + \frac{B}{x+2} + \frac{C}{(x+2)^2}$

D. $\frac{A}{x-2} + \frac{B}{(x+2)^2} + \frac{C}{(x-2)^2}$

E. $\frac{Ax+B}{(x-2)(x+2)} + \frac{C}{x+2}$

Question 12

The third term of an arithmetic sequence is 10 and the eighth term is 30. The first term and common difference are, respectively:

☒ A. 2 and 4

B. 10 and 30

C. 10 and 3

D. 3 and 5

E. 2 and 3

$$a + 2d = 10$$

$$a + 7d = 30$$

$$5d = 20$$

$$d = 4$$

$$a = 2$$

Question 13

If $z = 2 - 2i$, then $|z^3|$ and $\text{Arg}(z^3)$ are, respectively:

A. 16 and $\frac{5\pi}{4}$

B. 16 and $\frac{-3\pi}{4}$

C. $16\sqrt{2}$ and $\frac{5\pi}{4}$

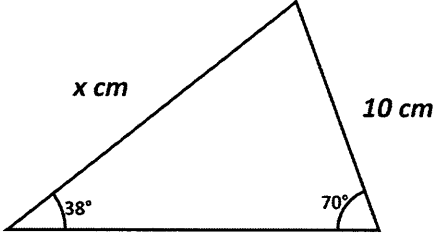
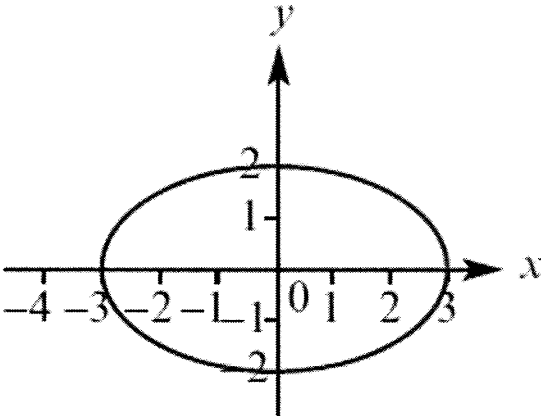
☒ D. $16\sqrt{2}$ and $\frac{-3\pi}{4}$

E. 16 and $\frac{-\pi}{4}$

$$z^3 = -16 - 16i$$

$$|z^3| = 16\sqrt{2}$$

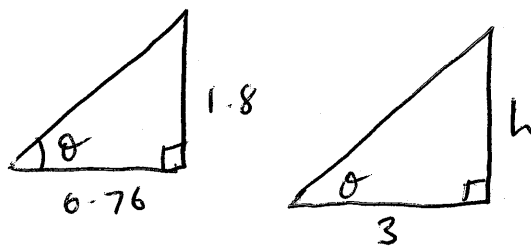
$$\text{Arg}(z^3) = \frac{-3\pi}{4}$$

Question 14 The value of x, to 2 decimal places, in the diagram shown is: A. 14.25 cm B. 10 cm C. 6.55 cm D. 15.26 cm E. 23.04 cm	 $\frac{x}{\sin 70^\circ} = \frac{10}{\sin 38^\circ}$
Question 15 The equation of the graph shown is: A. $\frac{x^2}{3} + \frac{y^2}{2} = 1$ B. $\frac{x^2}{9} + \frac{y^2}{4} = 1$ C. $\frac{x^2}{3} - \frac{y^2}{2} = 1$ D. $\frac{x^2}{9} - \frac{y^2}{4} = 1$ E. $\frac{x^2}{4} + \frac{y^2}{9} = 1$	 $a = 3, b = 2$
Question 16 The surface areas of two similar shapes are in the ratio 2:3. The ratio of their volumes is: A. $\frac{3}{2}:1$ B. 4:9 C. 1:4.5 D. $2\sqrt{2}:3\sqrt{3}$ E. $\sqrt[3]{2}:\sqrt[3]{3}$	$k^2 = \frac{2}{3}$ $k^3 = \left(\frac{2}{3}\right)^{3/2} = \frac{2\sqrt{2}}{3\sqrt{3}}$

Question 17

A man who is 1.8 m tall casts a shadow of 0.76 m in length. If at the same time a telephone pole casts a 3 m shadow. The height of the pole is closest to:

- A. 4.10m
- B. 2.19m
- C. 1.27m
- D. 7.11m**
- E. 3.00m



$$\frac{h}{1.8} = \frac{3}{0.76}$$

Question 18

If a vector $\underline{v}$ of magnitude 6 is parallel to the vector $2\underline{i} + 2\underline{j} - \underline{k}$ then $\underline{v}$ is represented by the vector:

- A. $12\underline{i} + 12\underline{j} - 6\underline{k}$
- B. $\left(\frac{1}{3}\right)\underline{i} + \left(\frac{1}{3}\right)\underline{j} - \left(\frac{1}{6}\right)\underline{k}$
- C. $4\underline{i} + 4\underline{j} - 2\underline{k}$**
- D. $-2\underline{i} - 2\underline{j} + 2\underline{k}$
- E. $2\underline{i} + 4\underline{j} - 4\underline{k}$

$$|2\underline{i} + 2\underline{j} - \underline{k}| = \sqrt{2^2 + 2^2 + 1^2} = 3$$

$$\underline{v} = 2(2\underline{i} + 2\underline{j} - \underline{k}) = 4\underline{i} + 4\underline{j} - 2\underline{k}$$

Question 19

The vector expression $\overrightarrow{2AB} + \overrightarrow{CD} + \overrightarrow{BA} + \overrightarrow{BC}$ can be simplified to

- A. $\overrightarrow{3AB} + \overrightarrow{CD} + \overrightarrow{BC}$
- B. $\overrightarrow{2AB} + \overrightarrow{CD} + \overrightarrow{AC}$
- C. $\overrightarrow{AD}$**
- D. $\overrightarrow{3AB} + \overrightarrow{2AC}$
- E. $\underline{0}$

$$\begin{aligned} \overrightarrow{AB} + \overrightarrow{BA} &= \underline{0} \\ &= \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} \\ &= \overrightarrow{AD} \end{aligned}$$

Question 20

The vector component of $\underline{a}$ in the direction of $\underline{b}$ is $5\underline{i} + 2\underline{j}$. If $\underline{a} = 7\underline{i} - 3\underline{j}$ then the vector component of $\underline{a}$ perpendicular to the direction of $\underline{b}$ is

- A. $-2\underline{i} + 5\underline{j}$
 B. $2\underline{i} - 5\underline{j}$
 C. $-5\underline{i} + 2\underline{j}$
 D. $7\underline{i} - \underline{j}$
 E. $12\underline{i} - \underline{j}$

$$\underline{a} = \underline{a}_{\parallel} + \underline{a}_{\perp}$$

$$\begin{aligned}\underline{a}_{\perp} &= (7\underline{i} - 3\underline{j}) - (5\underline{i} + 2\underline{j}) \\ &= 2\underline{i} - 5\underline{j}\end{aligned}$$

Multiple Choice Answer Table

1	2	3	4	5	6	7	8	9	10
D	A	B	A	E	E	C	E	B	C
11	12	13	14	15	16	17	18	19	20
C	A	D	D	B	D	D	C	C	B

SHORT ANSWER/EXTENDED RESPONSE QUESTIONS**Question 1**

The unit square is transformed by the matrix $T = \begin{bmatrix} -2 & 3 \\ -1 & 4 \end{bmatrix}$

a) What is the determinant of the matrix T ?

$$\begin{aligned} \det(T) &= -2 \times 4 - (-1) \times 3 && (1/2) \\ &= -5 && (1/2) \end{aligned}$$

b) What is the area of the resulting parallelogram?

$$5 \quad (1)$$

c) Find the inverse of the matrix (T^{-1}) .

$$T^{-1} = \begin{bmatrix} -4/5 & 3/5 \\ -1/5 & 2/5 \end{bmatrix} \quad (1)$$

d) The line $y = 3x - 5$ is transformed by matrix T . Find the equation of the image.

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -4/5 & 3/5 \\ -1/5 & 2/5 \end{bmatrix} \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -4/5 x' + 3/5 y' \\ -1/5 x' + 2/5 y' \end{bmatrix} \quad (1)$$

$$\therefore -1/5 x' + 2/5 y' = 3(-4/5 x' + 3/5 y') - 5 \quad (1)$$

$$-x + 2y = -12x + 9y - 25$$

$$7y - 13x = 25 \quad (1)$$

e) Find the image of the point $(2, -1)$ which is first reflected in the line $y = x$ and then undergoes transformation by the matrix T . Find the image of the point.

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -2 & 3 \\ -1 & 4 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 8 \\ 9 \end{bmatrix} \quad (1)$$

$$\therefore (2, -1) \rightarrow (8, 9) \quad (1)$$

(1 + 1 + 1 + 3 + 2 = 8 marks)

Question 2

- a) If $\cos(2A) = \frac{7}{9}$, $0 \leq A \leq \frac{\pi}{2}$ find the exact value for $\tan(A)$.

$$2\cos^2 A - 1 = 7/9 \quad (1/2)$$

$$\cos^2 A = 8/9$$

$$\cos A = \pm \frac{2\sqrt{2}}{3} \quad (1/2)$$

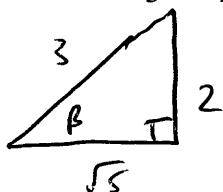
$$0 \leq A \leq \pi/2 \Rightarrow \cos A = \frac{2\sqrt{2}}{3} \quad (1/2)$$

$$\sin A = \sqrt{1 - 8/9} = 1/3 \quad (1/2)$$

$$\tan A = \frac{1/3}{2\sqrt{2}/3} \quad (1/2)$$

$$\tan A = \frac{1}{2\sqrt{2}} \quad (1/2)$$

- b) If $\sin(B) = \frac{2}{3}$, $\frac{\pi}{2} \leq B \leq \pi$ find the exact value for $\cos(A-B)$.



(1/2)

$$\cos B = -\frac{\sqrt{5}}{3} \quad (1/2)$$

$$\cos(A-B) = \cos A \cos B + \sin A \sin B$$

$$= \frac{2\sqrt{2}}{3} \times -\frac{\sqrt{5}}{3} + \frac{1}{3} \times \frac{2}{3} = \frac{2 - 2\sqrt{10}}{9}$$

- c) Prove the identity: $\sec(x) + \tan(x) = \frac{\cos(x)}{1 - \sin(x)}$

(1/2)

(1/2)

$$RHS = \frac{\cos(x)(1 + \sin(x))}{(1 - \sin(x))(1 + \sin(x))} \quad (1)$$

$$= \frac{\cos(x)(1 + \sin(x))}{1 - \sin^2(x)} \quad (1/2)$$

$$= \frac{\cos(x)(1 + \sin(x))}{\cos^2(x)} \quad (1/2)$$

$$= \frac{1 + \sin(x)}{\cos(x)} = \sec(x) + \tan(x) \quad (3 + 2 + 3 = 8 \text{ marks})$$

(1/2)

= RHS AS REQUIRED
(1/2)

Question 3

Two rockets are launched from the ground. Both rockets start from rest at the origin. Rocket A has an acceleration of 45 m/s^2 up. Rocket B is launched 22 seconds after Rocket A and has an acceleration of 60 m/s^2 up. (Express all answers in this question to two decimal places)

- a) Set up two equations and use them to show that 164.21 seconds after Rocket A is launched, Rocket B will pass Rocket A.

$$x_A = \frac{45}{2} t^2 \quad x_B = 30(t-22)^2, \quad t \geq 22 \quad (1)$$

Solving $x_A = x_B \Rightarrow t = 11.79, 164.21 \quad (1/2)$

Since $t \geq 22 \Rightarrow \underline{t = 164.21}$
 $(1/2)$ AS REQUIRED

- b) At what height will the rockets be when Rocket B passes Rocket A?

$$x = \frac{45}{2} \times 164.21^2 \quad (1)$$

$$= \underline{606,712.53 \text{ m}} \quad (1)$$

- c) At the instant Rocket B passes Rocket A, it turns off its engine. What is its speed at this time?

$$\begin{aligned} u &= 0 \\ v &= ? \\ a &= 60 \\ s &= \\ t &= 164.21 - 22 = 142.21 \end{aligned} \quad \begin{aligned} v &= u + at \\ &= 0 + 60 \times 142.21 \\ &= \underline{8532.61 \text{ m/s}} \end{aligned} \quad (1)$$

- d) How long after its engine turns off does Rocket B take to hit the ground?

$$u = 8532.61 \quad s = 30(142.21)^2 = 606,712.53 \quad (1)$$

$$v =$$

$$a = -9.8$$

$$s = -606,712.53$$

$$t = ?$$

$$\begin{aligned} s &= ut + \frac{1}{2}at^2 \\ -606,712.53 &= 8532.61t - 4.9t^2 \end{aligned} \quad (2+2+2+2=8 \text{ marks})$$

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Solving $\Rightarrow \underline{t = 1809.77} \quad (1)$

Question 4

The following table gives the sample proportion of defectives in a random sample of five washing machines:

$\hat{p}$	0	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{3}{5}$	$\frac{4}{5}$	1
$\Pr(\hat{P} = \hat{p})$	0.62	0.24	0.07	0.04	0.02	0.01

- a) Find the probability that three or more washing machines are defective.

$$\Pr(X \geq 3) = 0.04 + 0.02 + 0.01 = \underline{0.07} \quad \left(\frac{1}{2}\right) \quad \left(\frac{1}{2}\right)$$

- b) In a particular batch it is known that between one and four (inclusive) washing machines are defective. Find the probability that three or more washing machines are defective.

$$\begin{aligned} \Pr(X \geq 3 \mid 1 \leq X \leq 4) &= \frac{\Pr(X = 3, 4)}{\Pr(X = 1, 2, 3, 4)} \quad (1) \\ &= \frac{0.04 + 0.02}{0.24 + 0.07 + 0.04 + 0.02} \quad \left(\frac{1}{2}\right) \\ &= \frac{6}{37} \quad \left(\frac{1}{2}\right) \end{aligned}$$

- c) At the warehouse there are 5000 washing machines of which 45 are defective. Use a binomial approximation to determine the probability that a sample of six washing machines will contain exactly one defective washing machine. (Express your answer to four decimal places)

$$n = 6, p = \frac{45}{5000}, x = 1 \quad (1)$$

$$\Pr(X = 1) = 0.0516 \quad (1)$$

- d) Explain why using a binomial approximation is suitable in this calculation.

$$\text{POPULATION IS LARGE} \quad (1)$$

(1 + 2 + 2 + 1 = 6 marks)

Question 5

Express $12x^2 + 4x + 4$ in the form $a(2x+1)^2 + b(2x+1) + c$

$$= a(4x^2 + 4x + 1) + b(2x+1) + c \quad (1/2)$$

$$= 4ax^2 + (4a+2b)x + (a+b+c) \quad (1/2)$$

EQUATING COEFFICIENTS $\Rightarrow a = 3, 12 + 2b = 4 \quad (1/2)$
 $b = -4 \quad (1/2)$

$$3 - 4 + c = 4$$

$$c = 5$$

$(1/2)$ (3 marks)

Question 6

$$\therefore 12x^2 + 4x + 4 = 3(2x+1)^2 - 4(2x+1) + 5 \quad (1/2)$$

A sequence is given by: $t_1 = 1, t_{n+1} = t_n(p + t_n)$ where p is a constant and $p \neq 0$

a) Find t_2 in terms of p .

$$t_2 = 1(p+1) \quad (1/2)$$

$$= p+1 \quad (1/2)$$

b) Given that $t_3 = 1$, find the value of p .

$$t_3 = (p+1)(p+p+1) \quad (1/2)$$

$$= (p+1)(2p+1)$$

$$= 2p^2 + 3p + 1 \quad (1/2)$$

$$\therefore 2p^2 + 3p + 1 = 1$$

$$p(2p+3) = 0$$

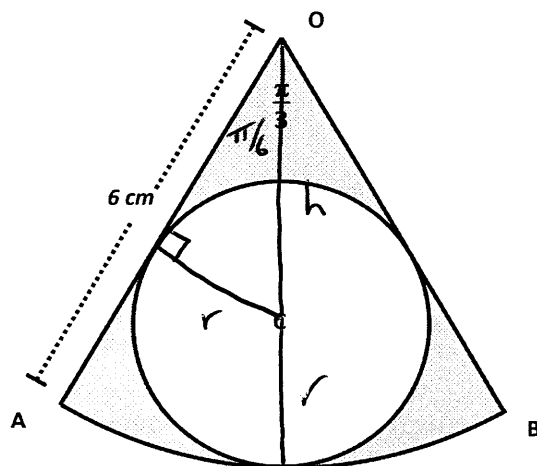
$$p = 0, -3/2 \quad (1/2)$$

(1 + 2 = 3 marks)

SINCE $p \neq 0, \underline{p = -3/2} \quad (1/2)$

Question 7

The shape shown in the diagram is a pattern for a pendant. It consists of a sector OAB of a circle centre O , of radius 6 cm , and angle $AOB = \frac{\pi}{3}$. The circle, C , inside the sector, touches the two straight edges, OA and OB , and the arc AB as shown. Leave your answers in exact form.



- a) Find the area of the sector OAB in terms of π .

$$A = \frac{1}{2} r^2 \theta = \frac{1}{2} \times 6^2 \times \frac{\pi}{3} = 6\pi$$

(1/2) (1/2)

- b) Show that the radius of the circle C is 2 cm .

$$h + r = 6 \Rightarrow h = 6 - r \quad (1/2)$$

$$\sin \frac{\pi}{6} = \frac{1}{2} = \frac{r}{6 - r} \quad (1/2)$$

$$6 - r = 2r \quad (1/2)$$

$$3r = 6$$

$$r = 2\text{ cm} \quad \text{As required} \quad (1/2) \quad (1+2=3\text{ marks})$$

Question 8

Given that $z = x + yi$, find the value of x and the value of y such that $z + 3i\bar{z} = -1 + 13i$, where $\bar{z}$ is the conjugate of z .

$$x + iy + 3i(x - iy) = -1 + 13i \quad (1/2)$$

$$(x + 3y) + (3x + y)i = -1 + 13i \quad (1/2)$$

$$\left. \begin{array}{l} x + 3y = -1 \\ 3x + y = 13 \end{array} \right\} \text{Solving} \Rightarrow \underline{x = 5, y = -2}$$

(1)

(1) (3 marks)

Question 9

An ellipse is expressed in parametric equations as: $x = 4 \cos t + 1$ and $y = 2 \sin t - 1$. Rewrite it in the form

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

$$\sin(t) = \frac{y+1}{2}$$

$$\cos(t) = \frac{x-1}{4} \quad (1)$$

$$\sin^2(t) + \cos^2(t) = 1 \quad (1)$$

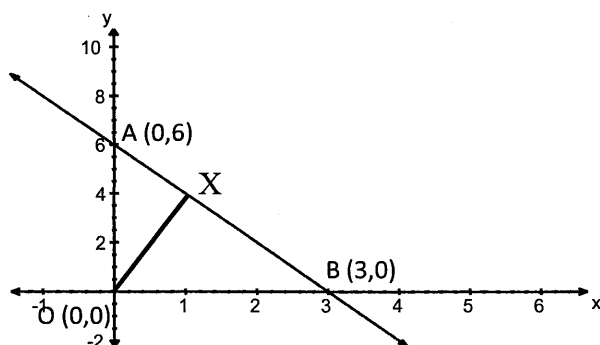
$$\frac{(x-1)^2}{16} + \frac{(y+1)^2}{4} = 1 \quad (1)$$

(3 marks)

Question 10

A **vector method** is to be used to determine the minimum distance between the line $y = 6 - 2x$ and the origin.

Let $\underline{i}$ and $\underline{j}$ be unit vectors directed along the x and y axes respectively and X be the point on the line segment BA closest to O (0, 0).



- a) Show that the vector $\overrightarrow{BA}$ can be expressed in component form as $-3\underline{i} + 6\underline{j}$.

$$\overrightarrow{BA} = \overrightarrow{OA} - \overrightarrow{OB} = 6\underline{j} - 3\underline{i} = -3\underline{i} + 6\underline{j} \quad (1/2)$$

Let $\overrightarrow{BX} = m \overrightarrow{BA}$.

- b) Determine an expression for $\overrightarrow{BX}$ in terms of m , $\underline{i}$ and $\underline{j}$.

$$\overrightarrow{BX} = m(-3\underline{i} + 6\underline{j}) \quad (1)$$

- c) Determine an expression for $\overrightarrow{OX}$ in terms of m , $\underline{i}$ and $\underline{j}$.

$$\overrightarrow{OX} = \overrightarrow{OB} + \overrightarrow{BX} = 3\underline{i} + m(-3\underline{i} + 6\underline{j}) \quad (1/2)$$

$$= (3 - 3m)\underline{i} + 6m\underline{j} \quad (1/2)$$

- d) Determine the value for m , using the fact that if X is the closest point on $\overline{BA}$ to O , then $\angle OXB$ will be a right angle.

$$\vec{OX} \cdot \vec{BA} = 0 \quad (1/2)$$

$$[(3-3m)\hat{i} + 6m\hat{j}] \cdot [-3\hat{i} + 6\hat{j}] = 0 \quad (1)$$

$$-3(3-3m) + 36m = 0 \quad (1/2)$$

$$45m = 9 \quad (1/2)$$

$$m = 1/5 \quad (1/2)$$

- e) Substitute the value of m calculated above into the expression for $\vec{OX}$ and hence calculate the minimum distance between the origin and the line that passes through A and B . Give your answer to 1 decimal place.

$$\vec{OX} = (3 - 3/5)\hat{i} + 6/5\hat{j} \quad (1/2)$$

$$= 12/5\hat{i} + 6/5\hat{j} \quad (1/2)$$

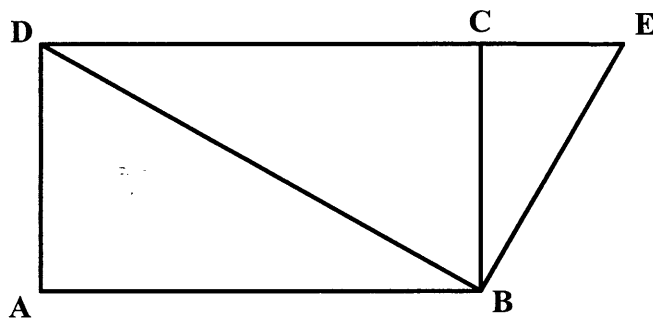
$$\text{MIN DIST} = |\vec{OX}| = \sqrt{(12/5)^2 + (6/5)^2} \quad (1/2)$$

$$\approx 2.7 \quad (1/2)$$

(1+1+1+3+2=8 marks)

Question 11

$ABCD$ is a rectangle. $AB = 4$ cm and $AD = 3$ cm. DCE is a straight line. Angle DBE is 90° .



- a) Explain why triangles ADB and BDE are similar.

$$\angle DAB = \angle DBE = 90^\circ \quad (1/2)$$

$$\angle ABO = \angle BDE \quad (\text{ALTERNATE ANGLES}) \quad (1/2)$$

$$\angle ADB = \angle DEB \quad (\text{ANGLES IN } \Delta \text{ ADD TO } 180^\circ) \quad (1/2)$$

$$\therefore \triangle ADB \sim \triangle BDE \quad (\text{AAA}) \quad (1/2)$$

b) Find the length of BE .

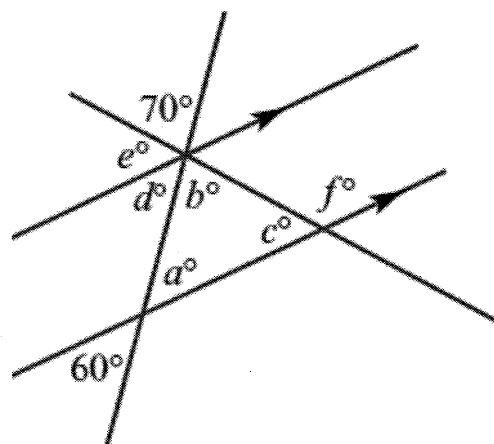
$$\frac{BE}{3} = \frac{5}{4} \quad (1)$$

$$BE = \frac{15}{4} \text{ cm} \quad (1)$$

(2 + 2 = 4 marks)

Question 12

Calculate the values of the unknowns in the diagram below.



$$a^\circ = 60^\circ$$

$$b^\circ = 70^\circ$$

$$c^\circ = 50^\circ$$

$$d^\circ = 60^\circ$$

$$e^\circ = 50^\circ$$

$$f^\circ = 130^\circ$$

($\frac{1}{2}$ M each)

(3 marks)

END OF EXAM