



Glen Waverley Secondary College
2016 Semester 1 Technology Access Exam

UNIT 1 Specialist Mathematics

Teachers:

Please circle



CAL



JLS



GMN

Student Name: Solutions

Student Code: Form Group:

Instructions:

Reading Time:	10 min
Writing Time:	120 min

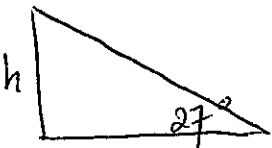
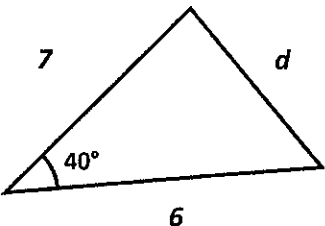
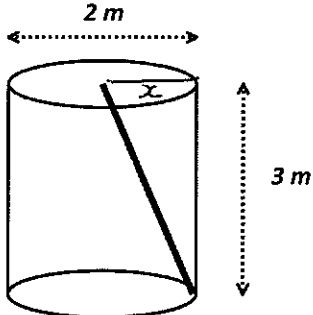
Multiple Choice	40 marks
Short Answer	60marks
Total Marks:	100 marks

This examination consists of 20 multiple choice questions and 12 short answer/extended response questions. All multiple choice questions should be answered on the **table** provided at the end this section (top of page 8). For each short answer/extended response question correct working **must be** shown to gain full marks.

A CAS calculator, a scientific calculator and a bound reference are allowed.

A formula sheet is provided.

Marks are allocated for working as well as answers.

Question 1 The angle of depression from the top of a building to the ground at a point 100 m away is 27°. The height of the building, to the nearest metre, is: A. 45 ☒ B. 51 C. 50 D. 100 E. 196	 $h = 100 \tan 27^\circ$
Question 2 The area of a sector of a circle of radius 6 cm is 50 cm^2. The length of the arc of the sector, in cm, is: A. $\frac{26}{3}$ ☒ B. $\frac{50}{3}$ C. 300 D. 150 E. $\frac{100}{3}$	$A = \frac{1}{2} r^2 \theta^\circ$ $\theta = \frac{2A}{r^2} = \frac{2 \times 50}{6^2} = \frac{100}{36}$ $l = r\theta = 6 \times \frac{100}{36} = \frac{50}{3}$
Question 3 The value of d in the diagram, correct to 1 decimal place, is: A. 5.4 B. 9 C. 4.6 D. 11.9 ☒ E. 4.5	cosine rule  $d = \sqrt{7^2 + 6^2 - 2(7)(6)\cos 40^\circ}$
Question 4 A cylindrical tank is 3 m high with a diameter of 2 m. A measuring stick is placed through a hole in the centre of the top and reaches the edge of the base as shown. What is the minimum length of the stick required so it does not fall into the tank? ☒ A. $\sqrt{10} \text{ m}$ B. $\sqrt{2} \text{ m}$ C. 2 m D. 3 m E. $\sqrt{7} \text{ m}$	 $x = \sqrt{10}$

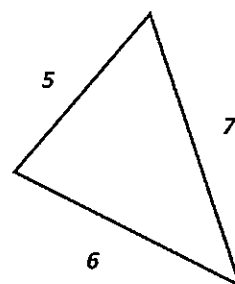
Question 5

The exact area of the triangle shown is:

- A. 12.5
- B. $12\sqrt{6}$
- C. $6\sqrt{2}$
- D. $\sqrt{105}$
- ☒ E. $6\sqrt{6}$

Heron's Formula.

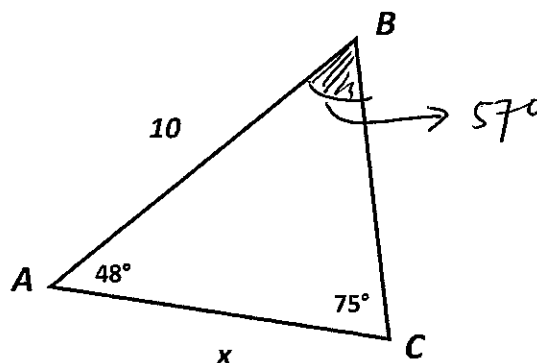
CAS



Question 6

The length of the side AC in the diagram shown can be calculated using the relationship:

- ☒ A. $\frac{10\sin 57^\circ}{\sin 75^\circ}$
- B. $\frac{10\sin 48^\circ}{\sin 75^\circ}$
- C. $\frac{10\sin 75^\circ}{\sin 48^\circ}$
- D. $\frac{10\sin 75^\circ}{\sin 57^\circ}$
- E. $\frac{10\sin 57^\circ}{\sin 48^\circ}$



$$\frac{x}{\sin 57^\circ} = \frac{10}{\sin 75^\circ}$$

$$x = \frac{10 \sin 57^\circ}{\sin 75^\circ}$$

Question 7

The 10th term of an arithmetic sequence is 48 and the 20th term is 98. The first term and common difference are, respectively:

- A. 6 and 5
- B. 4.8 and 9.8
- C. 5 and 3
- ☒ D. 3 and 5
- E. 5 and 6

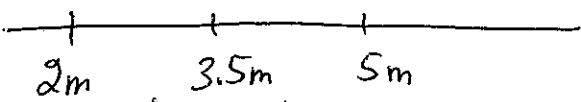
$$t_n = a + (n-1)d$$

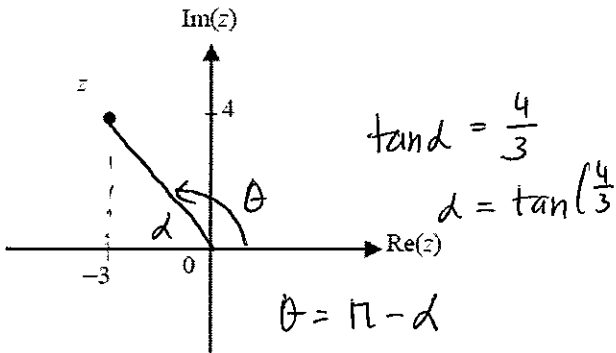
$$t_{10} = a + 9d = 48$$

$$t_{20} = a + 19d = 98$$

Solve for 'a' and 'd'

CAS

Question 8 $\frac{2x-1}{x^2-x-12}$ expressed in partial fractions is: A. $\frac{2x}{x+3} - \frac{1}{x-4}$ B. $\frac{2}{x+3} - \frac{1}{x-4}$ C. $\frac{1}{x+3} + \frac{1}{x-4}$ D. $\frac{1}{x+3} - \frac{1}{x-4}$ E. $\frac{1}{x-4} - \frac{1}{x+3}$	<u>CAS</u> expand partial fraction.
Question 9 A car is accelerating such that in the 1st second it travels 2.0 metres, in the 2nd second it travels 3.5 metres, in the 3rd second it travels 5.0 metres, and so on for a total of 15 seconds. The total distance travelled by the car is: A. 630 m B. 93.75 m C. 187.5 m D. 375 m E. 315 m	$t_0 = 2 \quad t_1 = \dots$  $a = 2$ $d = 1.5$ $S_{15} = \frac{15}{2} [4 + (15-1)1.5]$ $= 187.5$
Question 10 The terms $\{x - 2y, 3x - 4y, 4x - 7y, \dots\}$ form an <u>arithmetic sequence</u>. An expression for y in terms of x is: A. $y = x$ B. $y = -x$ C. $y = -2x$ D. $y = 2x$ E. $y = -3x$	$(3x - 4y) - (x - 2y) = (4x - 7y) - (3x - 4y)$ $2x - 2y = x - 3y$ $y = -x$

Question 11 $i^n + i^{n+1} + i^{n+2} + i^{n+3}, n \in \mathbb{Z}$, in its simplest form is: A. 1 B. -1 C. 0 D. i E. -i	n is odd then $i^{\text{odd}} + i^{\text{even}} + i^{\text{odd}} + i^{\text{even}} + \dots$ $= i - 1 - i - 1$ $= 0$ If n is even then result also '0'
Question 12 The complex number z shown in the diagram is best represented by: A. 5 cis (0.93) B. 5 cis (126.87) $\leftarrow$ degrees C. 5 cis (2.21) $\leftarrow$ radian D. 25 cis (126.87) E. 25 cis (2.21)	 $\tan \alpha = \frac{4}{3}$ $\alpha = \tan^{-1}\left(\frac{4}{3}\right)$ $\theta = \pi - \alpha$
Question 13 If $z = 2$, then $\left \frac{1}{z}\right$ is equal to: A. $\frac{1}{\sqrt{2}}$ B. $-\frac{1}{\sqrt{2}}$ C. $\frac{1}{2}$ D. $-\frac{1}{2}$ E. $\sqrt{2}$	If $z = 2 + 0i$ $\frac{1}{z} = \frac{1}{2+0i} \times \frac{2-0i}{2-0i}$ $= \frac{2-0i}{4}$ $= \frac{1}{2}$ If $z = 0+2i$ $\frac{1}{z} = -\frac{1}{2}$ $\left \frac{1}{z}\right = \frac{1}{2}$
Question 14 The equation $z^2 - 2z - 3 + 4i = 0$ is best described as having: A. two rational solutions B. two real solutions C. two complex solutions D. one complex solution E. one real solution	<u>CAS</u> $z = 1 \pm \sqrt{4-4i}$

Question 15

If $(x + iy)^2 = -32i$ for real values of x and y , then:

- A. $x = 4, y = 4$
 B. $x = -4, y = 4$
 C. $x = 4, y = -4$
☒ D. $x = 4, y = -4$ or $x = -4, y = 4$
 E. $x = 4, y = 4$ or $x = -4, y = -4$

Expand

$$(x + iy)^2 = x^2 + 2xyi - y^2 = -32i$$

$$(x^2 - y^2) = 0 \quad (x - y)(x + y) = 0$$

$$2xy = -32$$

$$x - y = 0$$

$$x = y$$

or

$$x = -y$$

$$2xy = -32$$

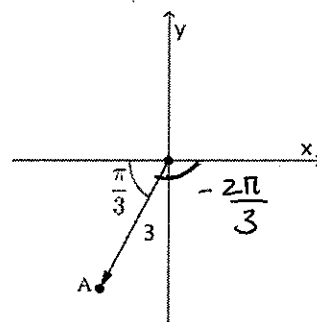
$$xy = -16$$

Hence $y \neq x$

Question 16

The polar coordinates of point A are:

- A. $\left(3, \frac{\pi}{3}\right)$
 B. $\left(3, \frac{2\pi}{3}\right)$
 C. $\left(-3, \frac{2\pi}{3}\right)$
 D. $\left(-3, -\frac{2\pi}{3}\right)$
☒ E. $\left(3, -\frac{2\pi}{3}\right)$

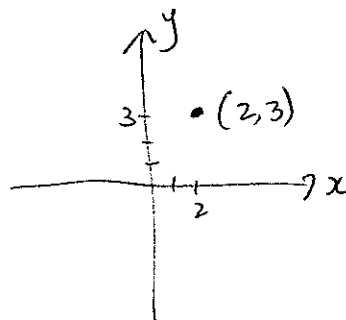
**Question 17**

Suppose a and b are both positive. Then the ellipse with equation

$$\frac{(x-2)^2}{a^2} + \frac{(y-3)^2}{b^2} = 1$$

will have two y-intercepts provided:

- A. $a > 4$
 B. $a > \sqrt{2}$
☒ C. $a > 2$
 D. $b > 3$
 E. $b > \sqrt{3}$



Question 18

The equation of the locus of the points $P(x, y)$ that satisfy $PA = PB$ given points $A(0, 7)$ and $B(7, 0)$ is:

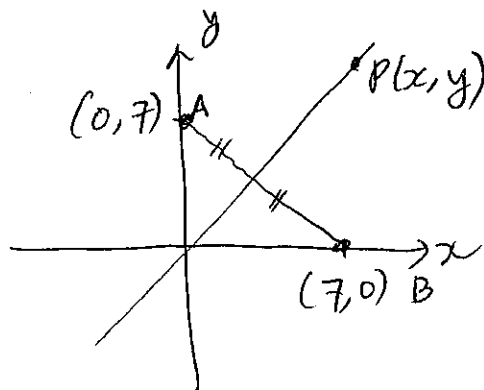
A. $x = 7$

B. $\frac{x}{7} + \frac{y}{7} = 1$

C. $y = x$

D. $y = -x - 7$

E. $y = x + 7$

**Question 19**

The circle with centre $C(2, -3)$ and radius 3 can be parameterised by the equations

A. $x = 2 + 3\cos t$ and $y = -3 + 3\sin t$

B. $x = -2 + 3\cos t$ and $y = 3 + 3\sin t$

C. $x = -2 + 9\cos t$ and $y = 3 + 9\sin t$

D. $x = 2 + 9\cos t$ and $y = -3 + 9\sin t$

E. $x = 2 + \sqrt{3}\cos t$ and $y = -3 + \sqrt{3}\sin t$

Circle.

$$(x-2)^2 + (y+3)^2 = 9(\cos^2 t + \sin^2 t)$$

$$(x-2)^2 = 9\cos^2 t$$

$$(x-2) = 3\cos t$$

$$x = 3\cos t + 2$$

$$y+3 = 3\sin t$$

$$y = 3\sin t - 3$$

Question 20

If $A = \begin{bmatrix} a & 0 & 0 \\ 0 & 0 & a \\ 0 & a & 0 \end{bmatrix}$, then A^2 is:

A. $\begin{bmatrix} a^2 & 0 & 0 \\ 0 & 0 & a^2 \\ 0 & a^2 & 0 \end{bmatrix}$

B. $a^2 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

C. $\begin{bmatrix} a & 0 & 0 \\ 0 & 0 & a \\ 0 & a & 0 \end{bmatrix}$

D. aI , where I is the 3×3 identity matrix.

E. $a^2 I$, where I is the 3×3 identity matrix.

Use CAS

$$A^2 = \begin{bmatrix} a^2 & 0 & 0 \\ 0 & a^2 & 0 \\ 0 & 0 & a^2 \end{bmatrix} = A^2 I.$$

Multiple Choice Answer Table									
1	2	3	4	5	6	7	8	9	10
B	B	E	A	E	A	D	C	C	B
11	12	13	14	15	16	17	18	19	20
C	B/c	C	C	D	E	C	C	A	E

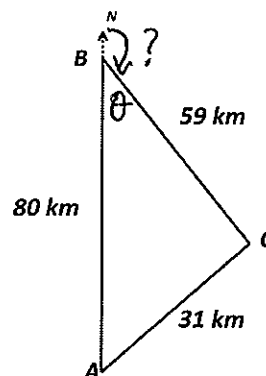
SHORT ANSWER/EXTENDED RESPONSE QUESTIONS

Question 1

Town B is 80 km due north of Town A and 59 km from Town C. Town A is 31 km from Town C. What is the true bearing, to the nearest degree, of Town C from Town B?

$$\cos \theta = \frac{59^2 + 80^2 - 31^2}{2(59)(80)} = 19.1^\circ \quad (1)^M$$

$$\text{Bearing } 180^\circ - 19.1^\circ = 160.9^\circ \approx 161^\circ T \quad (1)^A$$

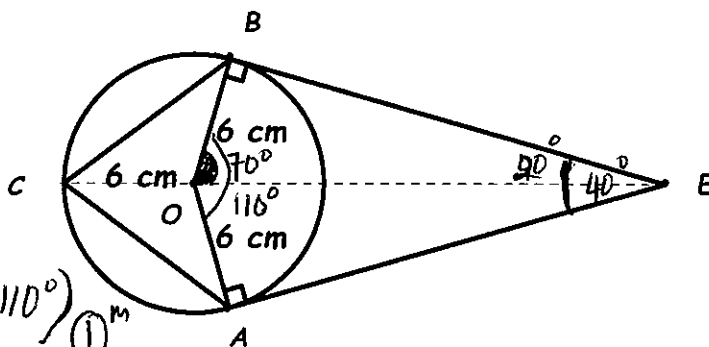


(2 marks)

Question 2

A circle (centre O) in which a quadrilateral is located has a radius of 6 cm and the tangents drawn from an external point E touch the circle at A and at B . The angle between the tangents is 40° . Determine the area of the following quadrilateral $AOBC$ in the following diagram, expressing your answer correct to 2 decimal places.

$$\begin{aligned} A_{AOBC} &= A_{BOC} + A_{AOC} \quad (1)^M \\ &= 2 \times \left(\frac{1}{2} \times 6 \times 6 \times \sin 110^\circ \right) \quad (1)^M \\ &= 33.83 \text{ cm}^2 \quad (1)^M \end{aligned}$$



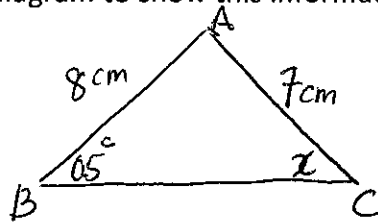
$(1)^M$ for 110°

4 marks)

Question 3

In a triangle ABC, $AB = 8 \text{ cm}$, $AC = 7 \text{ cm}$, $\angle ABC = 0.5 \text{ radians}$ and $\angle ACB = x \text{ radians}$.

- a) Sketch a diagram to show this information.



(1)^A angles

(1)^A sides

- b) Use the sine rule to find the value of $\sin(x)$, giving your answer to 3 decimal places.

$$\frac{7}{\sin 0.5^\circ} = \frac{8}{\sin x} \quad (1)^M$$

$$\sin x = \frac{8 \sin 0.5^\circ}{7} \approx 0.548 \quad (1)^A$$

- c) Find the two possible values of x , correct to 2 decimal places.

$$\sin x = \frac{8 \sin 0.5^\circ}{7}$$

$$x_1 = 0.58^\circ \quad \text{or} \quad \pi - x_1 \approx 2.56^\circ$$

(1)^A

(1/2)^M

(1/2)^A

(2 + 2 + 2 = 6 marks)

Question 4

The diagram shows a plan of a patio. The patio PQRS is in the shape of a sector of a circle with centre Q and radius 6 m.

The length of the straight line PR is $6\sqrt{3} \text{ m}$.

- a) Find the exact size of the angle PQR in radians.

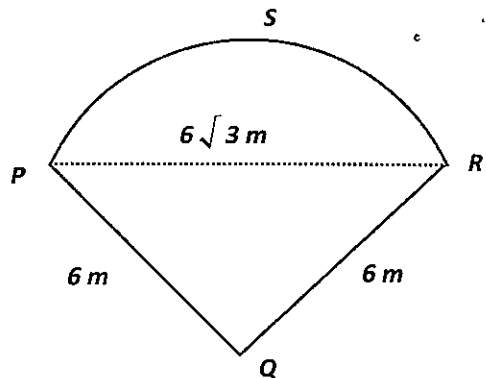
$$\angle PQR = \cos^{-1} \left(\frac{6^2 + 6^2 - (6\sqrt{3})^2}{2 \times 6 \times 6} \right) = 120^\circ$$

$$= \frac{2\pi}{3}$$

(1/2)^A

(1)^M

(1/2)^A



- b) Show that the area of the patio PQRS is $12\pi \text{ m}^2$.

No consequential.

$$\begin{aligned} \text{Area} &= \frac{1}{2} r^2 \theta^\circ \\ &= \frac{1}{2} (6 \times 6) \left(\frac{2\pi}{3} \right) \\ &= \frac{36\pi}{3} = 12\pi \text{ m}^2 \end{aligned} \quad (1)^A$$

- c) Find the exact area of the triangle PQR.

No consequential.

$$\begin{aligned} \text{Area} &= \frac{1}{2} \times 6 \times 6 \times \sin \frac{2\pi}{3} \\ &= 18 \times \frac{\sqrt{3}}{2} \\ &= 9\sqrt{3} \text{ m}^2 \end{aligned} \quad (1)^A \quad (2 + 1 + 2 = 5 \text{ marks})$$

Question 5

In a recurrence relation,

$$t_{n+2} = t_{n+1} + 2t_n, \text{ where } t_1 = 2 \text{ and } t_2 = 3.$$

- a) State the first 4 terms of the sequence.

$$t_3 = 3 + (2 \times 2) = 7 \quad \textcircled{\frac{1}{2}} \quad \therefore \{2, 3, 7, 13\} \quad \textcircled{1^A}$$
$$t_4 = 7 + (2 \times 3) = 13 \quad \textcircled{\frac{1}{2}}$$

- b) Is the sequence an arithmetic sequence? Justify your answer.

No. There is no common difference.

$$3 - 2 \neq 7 - 3 \neq 13 - 7.$$

(2 + 1 = 3 marks)

Question 6

The number of rabbits on a remote island was causing concern. A team of scientists were sent to investigate the problem. One week after their arrival they estimated the rabbit population to be 360,000.

$$\text{Hence } Q = 360\,000$$

- a) If the rabbit population on the island increases at a constant rate of 12,000 rabbits per week,

- i. how many rabbits will be on the island 4 weeks after the scientists arrive?

$$t_4 = 360,000 + (3 \times 12,000)$$
$$= 396\,000 \quad \textcircled{1^A}$$

- ii. write down a rule for the number of rabbits on the island n weeks after the arrival of the scientists

$$t_n = 360\,000 + (n-1)12\,000$$
$$= 348,000 + 12\,000n \quad \textcircled{1^A}$$

- iii. how many weeks after their arrival would it take for this rabbit population to grow to 600,000?

$$348\,000 + 12\,000n = 600,000 \quad \textcircled{1^M}$$

$$n = \frac{252\,000}{12\,000} \approx 21 \text{ wks} \quad \textcircled{1^A}$$

(1+1+2=4 marks)

- b) On a nearby island another team of scientist introduced a viral infection into its rabbit population in an attempt to limit rising numbers. The scientists kept a record of the number of rabbits killed each week by the virus with the following results for the first 3 weeks.

Week number	Number of rabbits killed by virus
1	512
2	768
3	1152

The scientists found that the number of rabbits killed each week by the virus followed a geometric sequence.

- i. Show that the common ratio, r for the sequence is 1.5

$$\frac{768}{512} = 1.5 \quad \frac{1152}{768} = 1.5 \quad (1)^A$$

Both must be shown to get full mark!

- ii. Use an appropriate sequences and series formula to calculate the number of rabbits which die from the virus in week 6.

$$\begin{aligned} t_n &= ar^{(n-1)} \\ t_6 &= 512 \cdot (1.5)^{6-1} \\ &= 3888 \quad (1)^A \end{aligned}$$

- iii. Use appropriate sequences and series formula to calculate the total number of rabbits which die from the virus during the first 8 weeks of the scientists' stay on the island.

$$\begin{aligned} S_n &= \frac{a(r^n - 1)}{r - 1} \\ S_8 &= \frac{512(1.5^8 - 1)}{1.5 - 1} \quad (1)^M \\ &= 25220 \quad (1)^A \end{aligned}$$

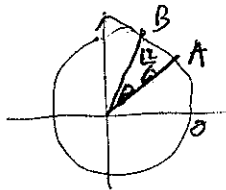
(1+1+2=4 marks)

Question 7

$\{z : |z| = 2\}$ represents all complex numbers which are two units from the origin. These points lie on a circle of radius 2 units centred at the origin (i.e. Cartesian Equation $x^2 + y^2 = 4$). Let $A = 2 \operatorname{cis} \frac{\pi}{6}$ and $B = 2 \operatorname{cis} \frac{\pi}{3}$ be two points lying on this circle.

- a) Find the exact area of the minor segment of this circle formed by the chord AB .

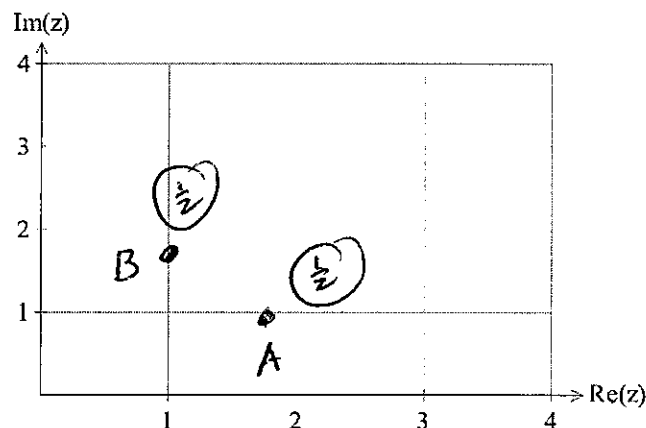
$$\theta = \frac{\pi}{3} - \frac{\pi}{6} = \frac{\pi}{6}$$



$$\begin{aligned} A &= \frac{1}{2} r^2 (\theta^c - \sin \theta) \\ &= \frac{1}{2} \times 2^2 \left(\frac{\pi}{6} - \sin \frac{\pi}{6} \right) \quad (1)^M \\ &= \frac{\pi}{3} - 1 \quad (1)^A \end{aligned}$$

- b) Convert A and B into Cartesian form using exact values. Now plot and label them on the Argand diagram below.

$$\begin{aligned} A &= 2 \operatorname{cis} \frac{\pi}{6} = 2 \left(\frac{\sqrt{3}}{2} + i \frac{1}{2} \right) = \sqrt{3} + i \quad (1)^A \\ B &= 2 \operatorname{cis} \frac{\pi}{3} = 2 \left(\frac{1}{2} + i \frac{\sqrt{3}}{2} \right) = 1 + \sqrt{3}i \quad (1)^A \end{aligned}$$



- c) Find the Cartesian equation of this line segment AB in exact form.

Using $(\sqrt{3}, 1)$ and $(1, \sqrt{3})$

$$m = \frac{\sqrt{3} - 1}{1 - \sqrt{3}} = -1 \quad \left(\frac{1}{2} \right)^M$$

$$y - y_1 = m(x - x_1) \quad \left(\frac{1}{2} \right)^M$$

$$y - 1 = -1(x - \sqrt{3})$$

$$y = -x + 1 + \sqrt{3} \quad (1)^A$$

(2+3+2=7 marks)

Question 8

The impedance of an AC circuit is the complex number ratio of sinusoidal voltage to current. The real part represents the resistance of the circuit (energy lost) and the imaginary part represents the reactance of the circuit (energy stored). The total impedance is Z_{Total} .

When two elements of a circuit are connected in series, that is one after the other, the total impedance is the sum of the individual impedance: **SERIES** $Z_{Total} = Z_1 + Z_2$.

When two elements of the circuit are connected in parallel, that is, next to each other, the total impedance is found using: **PARALLEL** $Z_{Total} = \frac{Z_1 Z_2}{Z_1 + Z_2}$.

- a) Determine the total impedance, Z_{Total} , in these circuits, in the form of $x + yi$, if the elements are connected in series and parallel when $z_1 = 3 + 2i$ and $z_2 = 2 + 4i$.

i. Series

$$(3+2i) + (2+4i) \\ = 5+6i \quad (1)^A$$

ii Parallel

$$\frac{(3+2i)(2+4i)}{5+6i} = \frac{86}{61} + \frac{92i}{61} \quad (1)^A$$

- b) Given $z_1 = 2+i$ and $z_2 = \frac{-\sqrt{3}-4}{2} - \frac{1}{2}i$ are the impedances of two elements connected in series

in a circuit, show that the total impedance, in the form of $x + yi$, is $Z_{Total} = \frac{-\sqrt{3}}{2} + \frac{1}{2}i$.

$$\begin{aligned} 2+i + \frac{-\sqrt{3}-4}{2} - \frac{1}{2}i &= 2 + \frac{-\sqrt{3}-4}{2} + (1-\frac{1}{2})i \\ (1)^M &= \frac{4-\sqrt{3}-4}{2} + (1-\frac{1}{2})i \quad (1)^M \\ &= \frac{-\sqrt{3}}{2} + \frac{1}{2}i \quad \text{as required.} \end{aligned}$$

- c) Give the solution to the part (b) in polar form.

$$\text{cis } \frac{5\pi}{6} \quad (1)$$

- d) The total impedance in a parallel circuit is actually calculated as $\frac{1}{Z_{Total}} = \frac{1}{Z_1} + \frac{1}{Z_2}$. Show

mathematically that for two impedance, Z_1 and Z_2 , in parallel circuit, the total impedance,

$$Z_{Total}, \text{ is given by } Z_{Total} = \frac{Z_1 Z_2}{Z_1 + Z_2}$$

$$\frac{1}{Z_t} = \frac{1}{Z_1} + \frac{1}{Z_2}$$

$$\frac{1}{Z_t} = \frac{Z_2 + Z_1}{Z_1 Z_2} \quad (1)^M$$

$$Z_t = \frac{Z_1 Z_2}{Z_2 + Z_1}$$

reciprocate $(1)^M$
as required.
(2+2+1+2=7 marks)

Question 9

Consider the matrix:

$$A = \begin{bmatrix} \cos \theta - z & -\sin \theta \\ \sin \theta & \cos \theta - z \end{bmatrix}$$

- a) Find $\det(A)$ when $z=0$. Hence show that A has an inverse when $z=0$.

(Note: $\sin^2(\theta) + \cos^2(\theta) = 1$)

When $z=0$

$$A = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \quad \det(A) = \cos^2 \theta + \sin^2 \theta = 1$$

As $\det(A) \neq 0$, A has an inverse

- b) Now $z \neq 0$. Find an expression for $\det(A)$ as a quadratic polynomial in terms of z with coefficients as functions of θ .

$$\begin{aligned} \det(A) &= (\cos \theta - z)^2 + \sin^2 \theta \\ &= \cos^2 \theta - 2z \cos \theta + z^2 + \sin^2 \theta \\ &= z^2 - 2z \cos \theta + 1 \end{aligned}$$

- c) Hence, considering A to be a matrix with complex entries, show that for $\det(A)=0$, there are two possible complex numbers: $z = \cos \theta \pm i \sin \theta$. (Hint: use the quadratic formula).

$$\text{Solve } z^2 - 2z \cos \theta + 1 = 0$$

$$z = \frac{2 \cos \theta \pm \sqrt{4 \cos^2 \theta - 4}}{2}$$

$$= \frac{2 \cos \theta \pm 2 \sin \theta (+\sqrt{-1})}{2}$$

$$z = \cos \theta \pm i \sin \theta$$

Note: $\sqrt{4 \cos^2 \theta - 4} = 2 \sqrt{\cos^2 \theta - 1}$
 $= 2 \sqrt{-\sin^2 \theta}$
 $= 2 \sin \theta \times i$

- d) Hence, determine all the possible values of θ for which A is a singular matrix with real entries.

$$\text{Real part } \operatorname{Im}(z) = 0$$

$$\text{i.e. } \sin \theta = 0$$

$$\theta = n\pi, \text{ where } n \in \mathbb{Z}$$

- e) If $z = \frac{1}{\sqrt{2}}(1+i)$, then determine one possible angle, θ , for which $\det(A)=0$. (Hint: Use c).

$$z = \frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}} = \cos \theta + i \sin \theta$$

$$\cos \theta = \frac{1}{\sqrt{2}} \quad \text{and} \quad \sin \theta = \frac{1}{\sqrt{2}} \text{ or } -\frac{1}{\sqrt{2}}$$

$$\theta = \frac{\pi}{4} \quad \theta = \pm \frac{\pi}{4}$$

$$\therefore \theta = \frac{\pi}{4} \text{ or } -\frac{\pi}{4} \quad (\text{only 1 angle required})$$

(2+2+2+2+2=10 marks)

Question 10

Solve the matrix equation below.

$$\begin{bmatrix} 1 & 2 & 5 & 7 \\ 3 & 2 & 3 & -10 \\ 4 & 5 & 15 & 8 \\ -5 & 8 & 11 & -2 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$$

In CAS

$$\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 1 & 2 & 5 & 7 \\ 3 & 2 & 3 & -10 \\ 4 & 5 & 15 & 8 \\ -5 & 8 & 11 & -2 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$$

$$a = \frac{38}{245}$$

$$b = \frac{31}{49}$$

$$c = -\frac{8}{245}$$

$$d = -\frac{9}{245}$$

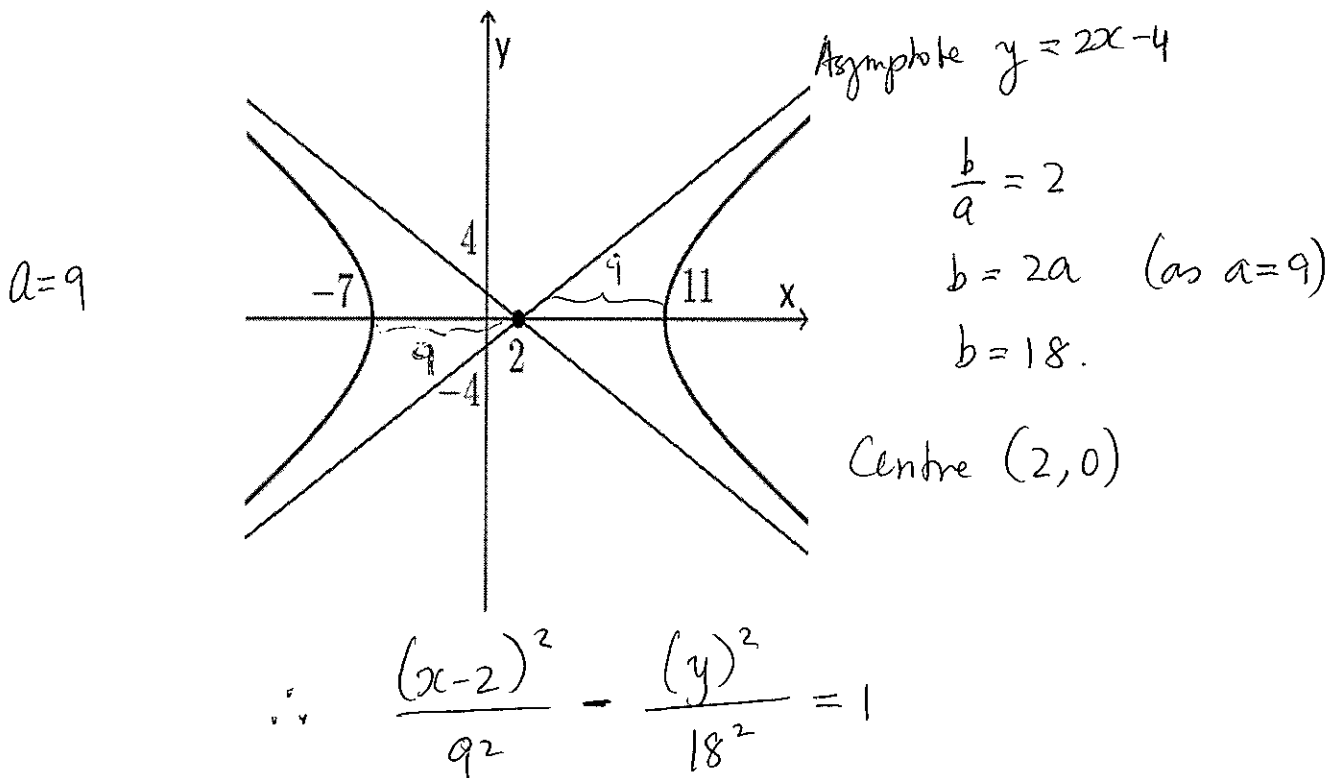
$\left(\frac{1}{2}\right)^A$ for each.

(2 marks)

Question 11

Find the equation of the hyperbola shown below, in the form $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$.

The vertices are $(-7, 0)$ and $(11, 0)$. The asymptotes have y -intercepts of $(0, -4)$ and $(0, 4)$ and an x -intercept of $(2, 0)$.



(3 marks)

Question 12

Write the equation of this ellipse $4x^2 + 9y^2 - 48x + 72y + 144 = 0$ in the form:

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

$$4x^2 + 9y^2 - 48x + 72y = -144 \quad \left(\frac{1}{2}\right)^M$$

$\left(\frac{1}{2}\right)^M$ for attempting to complete the square.

$$4x^2 - 48x + 9y^2 + 72y = -144$$

$$4(x^2 - 12x) + 9(y^2 + 8y) = -144$$

$$4[x^2 - 12x + 6^2 - 6^2] + 9[y^2 + 8y + 4^2 - 4^2] = -144$$

$$4[(x-6)^2 - 36] + 9[(y+4)^2 - 16] = -144 \quad \left(\frac{1}{2}\right)^M$$

$$4(x-6)^2 - 144 + 9(y+4)^2 - 144 = -144$$

$$4(x-6)^2 + 9(y+4)^2 = 144$$

$\left(\frac{1}{2}\right)^A$

$$\frac{(x-6)^2}{36} + \frac{(y+4)^2}{16} = 1$$

$\left(\frac{1}{2}\right)^A$

(3 marks)

END OF EXAM