



**Glen Waverley Secondary College  
2016 Unit 2 Technology Access Exam**

**Specialist Mathematics**

**Teachers:**

*Please circle*



**CAL**



**JLS**



**GMN**

**Student Name:** ..... SOLUTIONS .....

**Student Code:** ..... **Form Group:** .....

**Instructions:**

|                      |         |
|----------------------|---------|
| <b>Reading Time:</b> | 10min   |
| <b>Writing Time:</b> | 120 min |

|                         |           |
|-------------------------|-----------|
| <b>Section A Marks:</b> | 40 marks  |
| <b>Section B Marks:</b> | 60 marks  |
| <b>Total Marks:</b>     | 100 marks |

This examination consists of 20 multiple choice questions and 12 short answer/extended response questions. All multiple choice questions should be answered on the **table** provided at the end this section (bottom of page 8). For each short answer/extended response question correct working **must be** shown to gain full marks.

A CAS calculator, a scientific calculator and a bound reference are allowed.

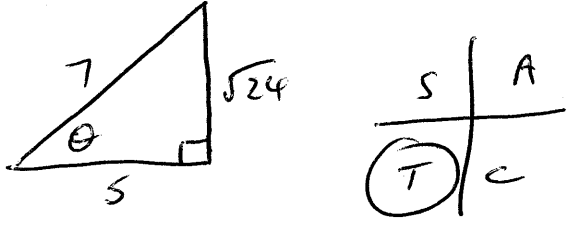
A formula sheet is provided.

*Marks are allocated for working as well as answers.*

Answer multiple choice questions on the table at the end of this section.

| Question 1                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |                                                                                                                                                                                                                          |
|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <p>If <math>A = \begin{bmatrix} 2 &amp; -1 \\ 3 &amp; 4 \end{bmatrix}</math> and <math>B = \begin{bmatrix} 3 &amp; 1 \\ 3 &amp; 2 \end{bmatrix}</math><br/> and <math>AX + B = \begin{bmatrix} -2 &amp; -2 \\ 1 &amp; 14 \end{bmatrix}</math> then X is:</p> <p>A. <math>\begin{bmatrix} -5 &amp; -3 \\ -2 &amp; 12 \end{bmatrix}</math><br/> B. <math>\begin{bmatrix} -8 &amp; -18 \\ -23 &amp; 39 \end{bmatrix}</math><br/> C. <math>\begin{bmatrix} -1 &amp; -1 \\ -4 &amp; 2 \end{bmatrix}</math><br/> D. <math>\begin{bmatrix} -2 &amp; 0 \\ 1 &amp; 3 \end{bmatrix}</math><br/> E. <math>\begin{bmatrix} -19 &amp; -7 \\ 32 &amp; 50 \end{bmatrix}</math></p> | $X = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}^{-1} \left( \begin{bmatrix} -2 & -2 \\ 1 & 14 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ 3 & 2 \end{bmatrix} \right)$ $= \begin{bmatrix} -2 & 0 \\ 1 & 3 \end{bmatrix}$ |
| Question 2                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |                                                                                                                                                                                                                          |
| <p>A transformation is given by:<br/> <math>\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 2 &amp; 0 \\ 0 &amp; 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 3 \\ 1 \end{bmatrix}</math></p> <p>The point (3,-1) is mapped to:</p> <p>A. (9,-3)<br/> B. (6,-4)<br/> C. (5,5)<br/> D. (7,3)<br/> E. (1,13)</p>                                                                                                                                                                                                                                                                                                                   | $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix} \begin{bmatrix} 3 \\ -1 \end{bmatrix} + \begin{bmatrix} 3 \\ 1 \end{bmatrix}$ $= \begin{bmatrix} 9 \\ -3 \end{bmatrix}$           |
| Question 3                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |                                                                                                                                                                                                                          |
| <p>The unit square is mapped to a parallelogram of area 6 units by the matrix: <math>\begin{bmatrix} n &amp; 4 \\ -n &amp; 2n \end{bmatrix}</math><br/> Possible values for n are:</p> <p>A. <math>-1 \pm \sqrt{2}</math><br/> B. -3,1<br/> C. <math>1 \pm \sqrt{2}</math><br/> D. -1,3<br/> E. -1,1</p>                                                                                                                                                                                                                                                                                                                                                            | $ 2n^2 + 4n  = 6$ $2n^2 + 4n - 6 = 0 \quad \left  \quad 2n^2 + 4n + 6 = 0 \right.$ $x = -3, x = 1 \quad \left  \quad \text{NO REAL SOLUTIONS} \right.$                                                                   |

| Question 4                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |                                                                                  |
|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------|
| <p>The acceleration of an object is given by:</p> $a(t) = 6t^2 - 2t, t \geq 0$ <p>Initially the object is at rest at the origin. The position of the object is given by:</p> <p><b>A.</b> <math>x(t) = \frac{1}{2}t^4 - \frac{1}{3}t^3, t \geq 0</math></p> <p><b>B.</b> <math>x(t) = 2t^4 - 3t^3, t \geq 0</math></p> <p><b>C.</b> <math>x(t) = t^4 - t^3, t \geq 0</math></p> <p><b>D.</b> <math>x(t) = \frac{1}{2}t^4 - t^3, t \geq 0</math></p> <p><b>E.</b> <math>x(t) = t^4 - \frac{1}{3}t^3, t \geq 0</math></p> | $v(t) = 2t^3 - t^2, t \geq 0$ $x(t) = \frac{1}{2}t^4 - \frac{1}{3}t^3, t \geq 0$ |
| Question 5                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |                                                                                  |
| <p>The expression <math>\sin(\theta) - \sqrt{3}\cos(\theta)</math> can be expressed as:</p> <p><b>A.</b> <math>2\sin\left(\theta + \frac{\pi}{3}\right)</math></p> <p><b>B.</b> <math>2\sin\left(\theta + \frac{\pi}{6}\right)</math></p> <p><b>C.</b> <math>2\cos\left(\theta + \frac{\pi}{3}\right)</math></p> <p><b>D.</b> <math>2\cos\left(\theta + \frac{\pi}{6}\right)</math></p> <p><b>E.</b> <math>2\sin\left(\theta - \frac{\pi}{3}\right)</math></p>                                                          |                                                                                  |

|                                                                                                                                                                                                                                                                                                                                                                                                                         |                                                                                                                                                                                                            |
|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <p><b>Question 6</b></p> <p>If <math>\cos(\theta) = -\frac{5}{7}</math> and <math>\pi \leq \theta \leq \frac{3\pi}{2}</math>, then <math>\cot(\theta)</math> is:</p> <p>A. <math>-\frac{7}{5}</math></p> <p>B. <math>-\frac{\sqrt{24}}{5}</math></p> <p>C. <math>\frac{\sqrt{24}}{5}</math></p> <p>D. <math>-\frac{5}{\sqrt{24}}</math></p> <p><input checked="" type="radio"/> E. <math>\frac{5}{\sqrt{24}}</math></p> |                                                                                                                          |
| <p><b>Question 7</b></p> <p>A motorcycle accelerates uniformly from 20 m/s to 40 m/s over a distance of 120 metres. The acceleration is closest to:</p> <p>A. 0.167 m/s<sup>2</sup></p> <p>B. 1.2 m/s<sup>2</sup></p> <p><input checked="" type="radio"/> C. 5 m/s<sup>2</sup></p> <p>D. 12 m/s<sup>2</sup></p> <p>E. 10 m/s<sup>2</sup></p>                                                                            | <p> <math>u = 20</math><br/> <math>v = 40</math><br/> <math>a = ?</math><br/> <math>s = 120</math> </p> <p> <math>v^2 = u^2 + 2as</math><br/> <math>40^2 = 20^2 + 240a</math><br/> <math>a = 5</math> </p> |
| <p><b>Question 8</b></p> <p>In Australia the proportion of adults with red hair is 12%. In a randomly selected sample of eight people, the probability that none of them have red hair is closest to:</p> <p>A. 0.3923</p> <p>B. 0.3454</p> <p>C. 0.8163</p> <p>D. 0.9600</p> <p><input checked="" type="radio"/> E. 0.3596</p>                                                                                         | <p> <math>x = 0, n = 8, p = 0.12</math><br/>         BINOMIAL       </p>                                                                                                                                   |

|                                                                                                                                                                                                                                                                                                                                                                                                                    |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <p><b>Question 9</b></p> <p>A chocolate box contains five soft centres and three hard centres. Five chocolates are randomly drawn without replacement. The probability that the proportion of soft-centre chocolates is greater than 0.75 is closest to:</p> <p>A. 0.2679</p> <p><b>B. 0.2857</b></p> <p>C. 0.0179</p> <p>D. 0.7143</p> <p>E. 0.9821</p>                                                           | <p>HYPERGEO CDF</p> <p>LOWER = 4, UPPER = 5</p> <p>N = 8, O = 5, n = 5</p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               |
| <p><b>Question 10</b></p> <p>A boat has an initial velocity of 4 m/s and accelerates uniformly for three seconds and covers a distance of 80 metres. How far did the boat travel in the third second?</p> <p>A. <math>\frac{308}{9}</math> m</p> <p>B. <math>\frac{80}{3}</math> m</p> <p><b>C. <math>\frac{376}{9}</math> m</b></p> <p>D. <math>\frac{148}{3}</math> m</p> <p>E. <math>\frac{448}{9}</math> m</p> | <p> <math>u = 4</math><br/> <math>v =</math><br/> <math>a = ?</math><br/> <math>s = 80</math><br/> <math>t = 3</math> </p> <p> <math>s = ut + \frac{1}{2}at^2</math><br/> <math>80 = 12 + \frac{9}{2}a</math><br/> <math>a = \frac{136}{9}</math> </p> <p> <math>u = 4</math><br/> <math>v =</math><br/> <math>a = \frac{136}{9}</math><br/> <math>s = ?</math><br/> <math>t = 2</math> </p> <p> <math>s = ut + \frac{1}{2}at^2</math><br/> <math>= 4 \times 2 + \frac{1}{2} \times \frac{136}{9} \times 4</math><br/> <math>= \frac{344}{9}</math> </p> <p> <math>DIST = 80 - \frac{344}{9} = \frac{376}{9}</math> </p> |

**Question 11**

The partial fraction decomposition of

$\frac{3x+7}{(x-2)(x+2)^2}$  has the form:

A.  $\frac{A}{x-2} + \frac{B}{x+2} + \frac{C}{x+2}$

B.  $\frac{A}{x-2} + \frac{Bx+C}{(x+2)}$

☒ C.  $\frac{A}{x-2} + \frac{B}{x+2} + \frac{C}{(x+2)^2}$

D.  $\frac{A}{x-2} + \frac{B}{(x+2)^2} + \frac{C}{(x-2)^2}$

E.  $\frac{Ax+B}{(x-2)(x+2)} + \frac{C}{x+2}$

**Question 12**

The third term of an arithmetic sequence is 10 and the eighth term is 30. The first term and common difference are, respectively:

☒ A. 2 and 4

B. 10 and 30

C. 10 and 3

D. 3 and 5

E. 2 and 3

$$a + 2d = 10$$

$$a + 7d = 30$$

---


$$5d = 20$$

$$d = 4$$

$$a = 2$$

**Question 13**

If  $z = 2 - 2i$ , then  $|z^3|$  and  $\text{Arg}(z^3)$  are, respectively:

A. 16 and  $\frac{5\pi}{4}$

B. 16 and  $\frac{-3\pi}{4}$

C.  $16\sqrt{2}$  and  $\frac{5\pi}{4}$

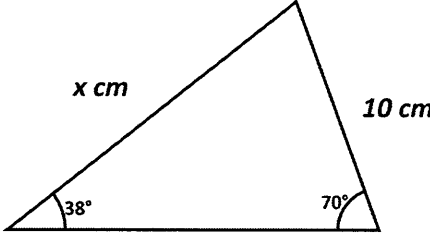
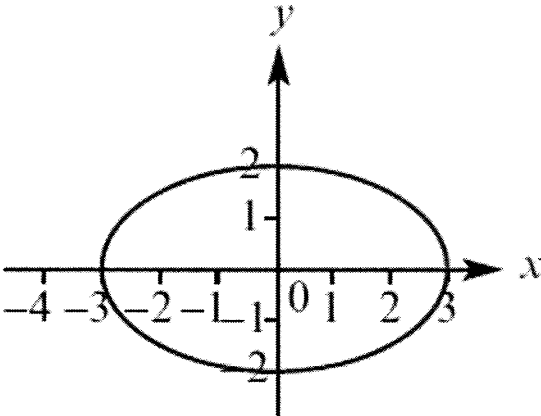
☒ D.  $16\sqrt{2}$  and  $\frac{-3\pi}{4}$

E. 16 and  $\frac{-\pi}{4}$

$$z^3 = -16 - 16i$$

$$|z^3| = 16\sqrt{2}$$

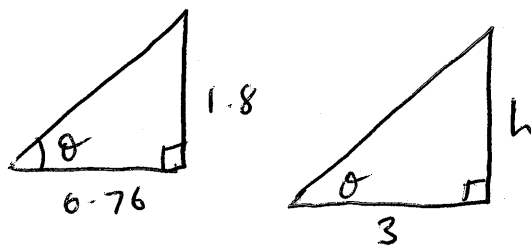
$$\text{Arg}(z^3) = \frac{-3\pi}{4}$$

|                                                                                                                                                                                                                                                                                                                                                                  |                                                                                                                                         |
|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------|
| <p><b>Question 14</b></p> <p>The value of <math>x</math>, to 2 decimal places, in the diagram shown is:</p> <p>A. 14.25 cm<br/> B. 10 cm<br/> C. 6.55 cm<br/> <b>D. 15.26 cm</b><br/> E. 23.04 cm</p>                                                                                                                                                            |  $\frac{x}{\sin 70^\circ} = \frac{10}{\sin 38^\circ}$ |
| <p><b>Question 15</b></p> <p>The equation of the graph shown is:</p> <p>A. <math>\frac{x^2}{3} + \frac{y^2}{2} = 1</math><br/> <b>B. <math>\frac{x^2}{9} + \frac{y^2}{4} = 1</math></b><br/> C. <math>\frac{x^2}{3} - \frac{y^2}{2} = 1</math><br/> D. <math>\frac{x^2}{9} - \frac{y^2}{4} = 1</math><br/> E. <math>\frac{x^2}{4} + \frac{y^2}{9} = 1</math></p> |  <p><math>a = 3, b = 2</math></p>                    |
| <p><b>Question 16</b></p> <p>The surface areas of two similar shapes are in the ratio 2:3. The ratio of their volumes is:</p> <p>A. <math>\frac{3}{2}:1</math><br/> B. 4:9<br/> C. 1:4.5<br/> <b>D. <math>2\sqrt{2}:3\sqrt{3}</math></b><br/> E. <math>\sqrt[3]{2}:\sqrt[3]{3}</math></p>                                                                        | $k^2 = \frac{2}{3}$ $k^3 = \left(\frac{2}{3}\right)^{3/2} = \frac{2\sqrt{2}}{3\sqrt{3}}$                                                |

### Question 17

A man who is 1.8 m tall casts a shadow of 0.76 m in length. If at the same time a telephone pole casts a 3 m shadow. The height of the pole is closest to:

- A. 4.10m
- B. 2.19m
- C. 1.27m
- D. 7.11m**
- E. 3.00m



$$\frac{h}{1.8} = \frac{3}{0.76}$$

### Question 18

If a vector  $\underline{v}$  of magnitude 6 is parallel to the vector  $2\underline{i} + 2\underline{j} - \underline{k}$  then  $\underline{v}$  is represented by the vector:

- A.  $12\underline{i} + 12\underline{j} - 6\underline{k}$
- B.  $\left(\frac{1}{3}\right)\underline{i} + \left(\frac{1}{3}\right)\underline{j} - \left(\frac{1}{6}\right)\underline{k}$
- C.  $4\underline{i} + 4\underline{j} - 2\underline{k}$**
- D.  $-2\underline{i} - 2\underline{j} + 2\underline{k}$
- E.  $2\underline{i} + 4\underline{j} - 4\underline{k}$

$$|2\underline{i} + 2\underline{j} - \underline{k}| = \sqrt{2^2 + 2^2 + 1^2} = 3$$

$$\underline{v} = 2(2\underline{i} + 2\underline{j} - \underline{k}) = 4\underline{i} + 4\underline{j} - 2\underline{k}$$

### Question 19

The vector expression  $\overrightarrow{2AB} + \overrightarrow{CD} + \overrightarrow{BA} + \overrightarrow{BC}$  can be simplified to

- A.  $\overrightarrow{3AB} + \overrightarrow{CD} + \overrightarrow{BC}$
- B.  $\overrightarrow{2AB} + \overrightarrow{CD} + \overrightarrow{AC}$
- C.  $\overrightarrow{AD}$**
- D.  $\overrightarrow{3AB} + \overrightarrow{2AC}$
- E.  $\underline{0}$

$$\begin{aligned} \overrightarrow{AB} + \overrightarrow{BA} &= \underline{0} \\ &= \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} \\ &= \overrightarrow{AD} \end{aligned}$$

**Question 20**

The vector component of  $\underline{a}$  in the direction of  $\underline{b}$  is  $5\underline{i} + 2\underline{j}$ . If  $\underline{a} = 7\underline{i} - 3\underline{j}$  then the vector component of  $\underline{a}$  perpendicular to the direction of  $\underline{b}$  is

- A.  $-2\underline{i} + 5\underline{j}$
- ☒ B.  $2\underline{i} - 5\underline{j}$
- C.  $-5\underline{i} + 2\underline{j}$
- D.  $7\underline{i} - \underline{j}$
- E.  $12\underline{i} - \underline{j}$

$$\underline{a} = \underline{a}_{\parallel} + \underline{a}_{\perp}$$

$$\begin{aligned}\underline{a}_{\perp} &= (7\underline{i} - 3\underline{j}) - (5\underline{i} + 2\underline{j}) \\ &= 2\underline{i} - 5\underline{j}\end{aligned}$$

**Multiple Choice Answer Table**

| 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  | 10 |
|----|----|----|----|----|----|----|----|----|----|
| D  | A  | B  | A  | E  | E  | C  | E  | B  | C  |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| C  | A  | D  | D  | B  | D  | D  | C  | C  | B  |

**SHORT ANSWER/EXTENDED RESPONSE QUESTIONS****Question 1**

The unit square is transformed by the matrix  $T = \begin{bmatrix} -2 & 3 \\ -1 & 4 \end{bmatrix}$

a) What is the determinant of the matrix  $T$ ?

$$\begin{aligned} \det(T) &= -2 \times 4 - (-1) \times 3 && (1/2) \\ &= -5 && (1/2) \end{aligned}$$

b) What is the area of the resulting parallelogram?

$$5 \quad (1)$$

c) Find the inverse of the matrix  $(T^{-1})$ .

$$T^{-1} = \begin{bmatrix} -4/5 & 3/5 \\ -1/5 & 2/5 \end{bmatrix} \quad (1)$$

d) The line  $y = 3x - 5$  is transformed by matrix  $T$ . Find the equation of the image.

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -4/5 & 3/5 \\ -1/5 & 2/5 \end{bmatrix} \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -4/5 x' + 3/5 y' \\ -1/5 x' + 2/5 y' \end{bmatrix} \quad (1)$$

$$\therefore -1/5 x' + 2/5 y' = 3(-4/5 x' + 3/5 y') - 5 \quad (1)$$

$$-x + 2y = -12x + 9y - 25$$

$$7y - 13x = 25 \quad (1)$$

e) Find the image of the point  $(2, -1)$  which is first reflected in the line  $y = x$  and then undergoes transformation by the matrix  $T$ . Find the image of the point.

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -2 & 3 \\ -1 & 4 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 8 \\ 9 \end{bmatrix} \quad (1)$$

$$\therefore (2, -1) \rightarrow (8, 9) \quad (1)$$

(1 + 1 + 1 + 3 + 2 = 8 marks)

## Question 2

- a) If  $\cos(2A) = \frac{7}{9}$ ,  $0 \leq A \leq \frac{\pi}{2}$  find the exact value for  $\tan(A)$ .

$$2\cos^2 A - 1 = 7/9 \quad (1/2)$$

$$\cos^2 A = 8/9$$

$$\cos A = \pm \frac{2\sqrt{2}}{3} \quad (1/2)$$

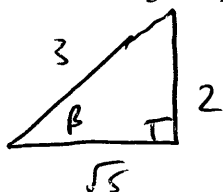
$$0 \leq A \leq \pi/2 \Rightarrow \cos A = \frac{2\sqrt{2}}{3} \quad (1/2)$$

$$\sin A = \sqrt{1 - 8/9} = 1/3 \quad (1/2)$$

$$\tan A = \frac{1/3}{2\sqrt{2}/3} \quad (1/2)$$

$$\tan A = \frac{1}{2\sqrt{2}} \quad (1/2)$$

- b) If  $\sin(B) = \frac{2}{3}$ ,  $\frac{\pi}{2} \leq B \leq \pi$  find the exact value for  $\cos(A-B)$ .



(1/2)

$$\cos B = -\frac{\sqrt{5}}{3} \quad (1/2)$$

$$\cos(A-B) = \cos A \cos B + \sin A \sin B$$

$$= \frac{2\sqrt{2}}{3} \times -\frac{\sqrt{5}}{3} + \frac{1}{3} \times \frac{2}{3} = \frac{2 - 2\sqrt{10}}{9}$$

- c) Prove the identity:  $\sec(x) + \tan(x) = \frac{\cos(x)}{1 - \sin(x)}$

(1/2)

(1/2)

$$RHS = \frac{\cos(x)(1 + \sin(x))}{(1 - \sin(x))(1 + \sin(x))} \quad (1)$$

$$= \frac{\cos(x)(1 + \sin(x))}{1 - \sin^2(x)} \quad (1/2)$$

$$= \frac{\cos(x)(1 + \sin(x))}{\cos^2(x)} \quad (1/2)$$

$$= \frac{1 + \sin(x)}{\cos(x)} = \sec(x) + \tan(x) \quad (3 + 2 + 3 = 8 \text{ marks})$$

(1/2)

= RHS AS REQUIRED  
(1/2)

### Question 3

Two rockets are launched from the ground. Both rockets start from rest at the origin. Rocket A has an acceleration of  $45 \text{ m/s}^2$  up. Rocket B is launched 22 seconds after Rocket A and has an acceleration of  $60 \text{ m/s}^2$  up. (Express all answers in this question to two decimal places)

- a) Set up two equations and use them to show that 164.21 seconds after Rocket A is launched, Rocket B will pass Rocket A.

$$x_A = \frac{45}{2} t^2 \quad x_B = 30(t-22)^2, \quad t \geq 22 \quad (1)$$

Solving  $x_A = x_B \Rightarrow t = 11.79, 164.21 \quad (1/2)$

Since  $t \geq 22 \Rightarrow \underline{t = 164.21}$   
 $(1/2)$  AS REQUIRED

- b) At what height will the rockets be when Rocket B passes Rocket A?

$$x = \frac{45}{2} \times 164.21^2 \quad (1)$$

$$= \underline{606,712.53 \text{ m}} \quad (1)$$

- c) At the instant Rocket B passes Rocket A, it turns off its engine. What is its speed at this time?

$$\begin{aligned} u &= 0 \\ v &= ? \\ a &= 60 \\ s &= \\ t &= 164.21 - 22 = 142.21 \end{aligned} \quad \begin{aligned} v &= u + at \\ &= 0 + 60 \times 142.21 \\ &= \underline{8532.61 \text{ m/s}} \end{aligned} \quad (1)$$

- d) How long after its engine turns off does Rocket B take to hit the ground?

$$u = 8532.61 \quad s = 30(142.21)^2 = 606,712.53 \quad (1)$$

$$v =$$

$$a = -9.8$$

$$s = -606,712.53$$

$$t = ?$$

$$\begin{aligned} s &= ut + \frac{1}{2}at^2 \\ -606,712.53 &= 8532.61t - 4.9t^2 \end{aligned} \quad (2+2+2+2=8 \text{ marks})$$

12

Solving  $\Rightarrow \underline{t = 1809.77} \quad (1)$

#### Question 4

The following table gives the sample proportion of defectives in a random sample of five washing machines:

|                          |      |               |               |               |               |      |
|--------------------------|------|---------------|---------------|---------------|---------------|------|
| $\hat{p}$                | 0    | $\frac{1}{5}$ | $\frac{2}{5}$ | $\frac{3}{5}$ | $\frac{4}{5}$ | 1    |
| $\Pr(\hat{P} = \hat{p})$ | 0.62 | 0.24          | 0.07          | 0.04          | 0.02          | 0.01 |

- a) Find the probability that three or more washing machines are defective.

$$\Pr(X \geq 3) = 0.04 + 0.02 + 0.01 = \underline{0.07} \quad \left(\frac{1}{2}\right) \quad \left(\frac{1}{2}\right)$$

- b) In a particular batch it is known that between one and four (inclusive) washing machines are defective. Find the probability that three or more washing machines are defective.

$$\begin{aligned} \Pr(X \geq 3 \mid 1 \leq X \leq 4) &= \frac{\Pr(X = 3, 4)}{\Pr(X = 1, 2, 3, 4)} \quad (1) \\ &= \frac{0.04 + 0.02}{0.24 + 0.07 + 0.04 + 0.02} \quad \left(\frac{1}{2}\right) \\ &= \frac{6}{37} \quad \left(\frac{1}{2}\right) \end{aligned}$$

- c) At the warehouse there are 5000 washing machines of which 45 are defective. Use a binomial approximation to determine the probability that a sample of six washing machines will contain exactly one defective washing machine. (Express your answer to four decimal places)

$$n = 6, p = \frac{45}{5000}, x = 1 \quad (1)$$

$$\Pr(X = 1) = 0.0516 \quad (1)$$

- d) Explain why using a binomial approximation is suitable in this calculation.

$$\text{POPULATION IS LARGE} \quad (1)$$

(1 + 2 + 2 + 1 = 6 marks)

### Question 5

Express  $12x^2 + 4x + 4$  in the form  $a(2x+1)^2 + b(2x+1) + c$

$$= a(4x^2 + 4x + 1) + b(2x+1) + c \quad (1/2)$$

$$= 4ax^2 + (4a+2b)x + (a+b+c) \quad (1/2)$$

EQUATING COEFFICIENTS  $\Rightarrow a = 3, 12 + 2b = 4 \quad (1/2)$   
 $b = -4 \quad (1/2)$

$$3 - 4 + c = 4$$

$$c = 5$$

$(1/2)$  (3 marks)

### Question 6

$$\therefore 12x^2 + 4x + 4 = 3(2x+1)^2 - 4(2x+1) + 5 \quad (1/2)$$

A sequence is given by:  $t_1 = 1, t_{n+1} = t_n(p + t_n)$  where  $p$  is a constant and  $p \neq 0$

a) Find  $t_2$  in terms of  $p$ .

$$t_2 = 1(p+1) \quad (1/2)$$

$$= p+1 \quad (1/2)$$

b) Given that  $t_3 = 1$ , find the value of  $p$ .

$$t_3 = (p+1)(p+p+1) \quad (1/2)$$

$$= (p+1)(2p+1)$$

$$= 2p^2 + 3p + 1 \quad (1/2)$$

$$\therefore 2p^2 + 3p + 1 = 1$$

$$p(2p+3) = 0$$

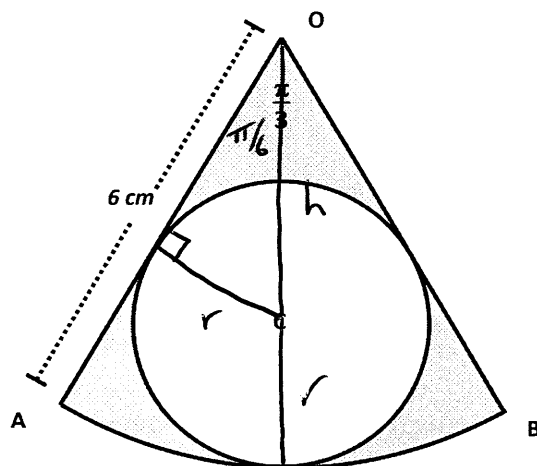
$$p = 0, -3/2 \quad (1/2)$$

(1 + 2 = 3 marks)

SINCE  $p \neq 0, \underline{p = -3/2} \quad (1/2)$

### Question 7

The shape shown in the diagram is a pattern for a pendant. It consists of a sector  $OAB$  of a circle centre  $O$ , of radius  $6\text{ cm}$ , and angle  $AOB = \frac{\pi}{3}$ . The circle,  $C$ , inside the sector, touches the two straight edges,  $OA$  and  $OB$ , and the arc  $AB$  as shown. Leave your answers in exact form.



- a) Find the area of the sector  $OAB$  in terms of  $\pi$ .

$$A = \frac{1}{2} r^2 \theta = \frac{1}{2} \times 6^2 \times \frac{\pi}{3} = 6\pi$$

(1/2) (1/2)

- b) Show that the radius of the circle  $C$  is  $2\text{ cm}$ .

$$h + r = 6 \Rightarrow h = 6 - r \quad (1/2)$$

$$\sin \frac{\pi}{6} = \frac{1}{2} = \frac{r}{6 - r} \quad (1/2)$$

$$6 - r = 2r \quad (1/2)$$

$$3r = 6$$

$$r = 2\text{ cm} \quad \text{As required} \quad (1/2) \quad (1+2=3 \text{ marks})$$

### Question 8

Given that  $z = x + yi$ , find the value of  $x$  and the value of  $y$  such that  $z + 3i\bar{z} = -1 + 13i$ , where  $\bar{z}$  is the conjugate of  $z$ .

$$x + iy + 3i(x - iy) = -1 + 13i \quad (1/2)$$

$$(x + 3y) + (3x + y)i = -1 + 13i \quad (1/2)$$

$$\left. \begin{array}{l} x + 3y = -1 \\ 3x + y = 13 \end{array} \right\} \text{Solving} \Rightarrow \underline{x = 5, y = -2}$$

(1)

(1) (3 marks)

### Question 9

An ellipse is expressed in parametric equations as:  $x = 4 \cos t + 1$  and  $y = 2 \sin t - 1$ . Rewrite it in the form

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1 \quad \sin(t) = \frac{y+1}{2} \quad \cos(t) = \frac{x-1}{4} \quad (1)$$

$$\sin^2(t) + \cos^2(t) = 1 \quad (1)$$

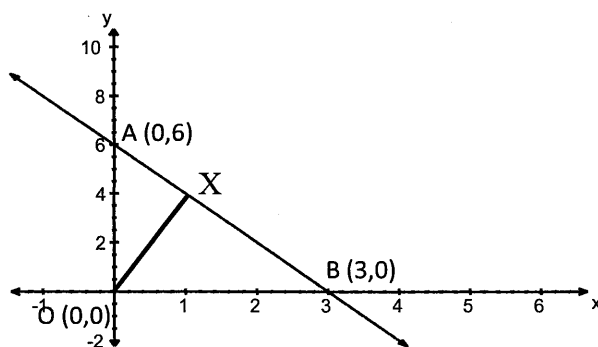
$$\frac{(x-1)^2}{16} + \frac{(y+1)^2}{4} = 1 \quad (1)$$

(3 marks)

### Question 10

A **vector method** is to be used to determine the minimum distance between the line  $y = 6 - 2x$  and the origin.

Let  $\underline{i}$  and  $\underline{j}$  be unit vectors directed along the x and y axes respectively and X be the point on the line segment BA closest to O (0, 0).



- a) Show that the vector  $\overrightarrow{BA}$  can be expressed in component form as  $-3\underline{i} + 6\underline{j}$ .

$$\overrightarrow{BA} = \overrightarrow{OA} - \overrightarrow{OB} = 6\underline{j} - 3\underline{i} = -3\underline{i} + 6\underline{j} \quad (1/2)$$

Let  $\overrightarrow{BX} = m \overrightarrow{BA}$ .

- b) Determine an expression for  $\overrightarrow{BX}$  in terms of  $m$ ,  $\underline{i}$  and  $\underline{j}$ .

$$\overrightarrow{BX} = m(-3\underline{i} + 6\underline{j}) \quad (1)$$

- c) Determine an expression for  $\overrightarrow{OX}$  in terms of  $m$ ,  $\underline{i}$  and  $\underline{j}$ .

$$\begin{aligned} \overrightarrow{OX} &= \overrightarrow{OB} + \overrightarrow{BX} = 3\underline{i} + m(-3\underline{i} + 6\underline{j}) \quad (1/2) \\ &= (3 - 3m)\underline{i} + 6m\underline{j} \quad (1/2) \end{aligned}$$

- d) Determine the value for  $m$ , using the fact that if  $X$  is the closest point on  $\overline{BA}$  to  $O$ , then  $\angle OXB$  will be a right angle.

$$\vec{OX} \cdot \vec{BA} = 0 \quad (1/2)$$

$$[(3-3m)\hat{i} + 6m\hat{j}] \cdot [-3\hat{i} + 6\hat{j}] = 0 \quad (1)$$

$$-3(3-3m) + 36m = 0 \quad (1/2)$$

$$45m = 9 \quad (1/2)$$

$$m = 1/5 \quad (1/2)$$

- e) Substitute the value of  $m$  calculated above into the expression for  $\vec{OX}$  and hence calculate the minimum distance between the origin and the line that passes through  $A$  and  $B$ . Give your answer to 1 decimal place.

$$\vec{OX} = (3 - 3/5)\hat{i} + 6/5\hat{j} \quad (1/2)$$

$$= 12/5\hat{i} + 6/5\hat{j} \quad (1/2)$$

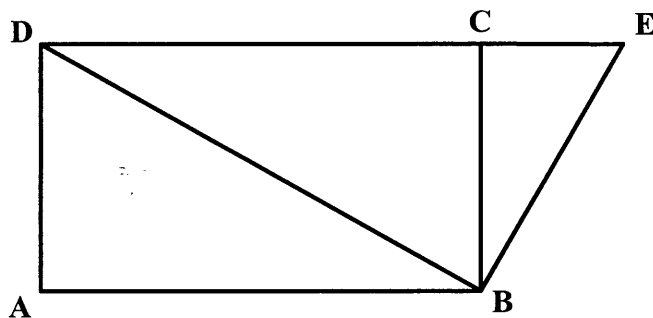
$$\text{MIN DIST} = |\vec{OX}| = \sqrt{(12/5)^2 + (6/5)^2} \quad (1/2)$$

$$\approx 2.7 \quad (1/2)$$

(1+1+1+3+2=8 marks)

### Question 11

$ABCD$  is a rectangle.  $AB = 4$  cm and  $AD = 3$  cm.  $DCE$  is a straight line. Angle  $DBE$  is  $90^\circ$ .



- a) Explain why triangles  $ADB$  and  $BDE$  are similar.

$$\angle DAB = \angle DBE = 90^\circ \quad (1/2)$$

$$\angle ABO = \angle BDE \quad (\text{ALTERNATE ANGLES}) \quad (1/2)$$

$$\angle ADB = \angle DEB \quad (\text{ANGLES IN } \Delta \text{ ADD TO } 180^\circ) \quad (1/2)$$

$$\therefore \triangle ADB \sim \triangle BDE \quad (\text{AAA}) \quad (1/2)$$

b) Find the length of  $BE$ .

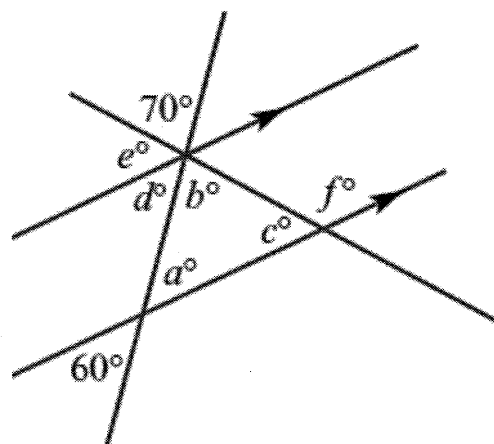
$$\frac{BE}{3} = \frac{5}{4} \quad (1)$$

$$BE = \frac{15}{4} \text{ cm} \quad (1)$$

(2 + 2 = 4 marks)

### Question 12

Calculate the values of the unknowns in the diagram below.



$$a^\circ = 60^\circ$$

$$b^\circ = 70^\circ$$

$$c^\circ = 50^\circ$$

$$d^\circ = 60^\circ$$

$$e^\circ = 50^\circ$$

$$f^\circ = 130^\circ$$

( $\frac{1}{2}$  M each)

(3 marks)

END OF EXAM