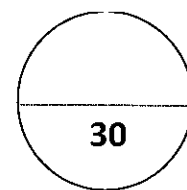




Yr 11 Specialist 2016
Trigonometric Identities
Technology Free Test – 35 minutes



Name: Solutions

Multiple Choice (5 x 2 = 10 marks)

Question 1 Given that $\cos x = \frac{-1}{10}$ and $x \in \left(\frac{\pi}{2}, \pi\right)$, the value of $\operatorname{cosec} x$ is: ☒ A $\frac{10}{3\sqrt{11}}$ B $3\sqrt{11}$ C $-3\sqrt{11}$ D $\frac{\sqrt{11}}{33}$ E $-\frac{\sqrt{11}}{33}$	
Question 2 The minimum value of $2 \cos(x) + 7$ is: A 2 B -3 ☒ C 5 D 9 E 7	
Question 3 $1 - \cos 4A$ is equal to: A $2(\cos A - 1)^2$ ☒ B $2 \sin^2 2A$ C $\sin^2 A$ D $\sin 2A$ E $1 - \sin 2A$	

Question 4

Given that $\cos(A) = \frac{-1}{9}$ the exact value of $\cos(2A)$ is:

- A $-\frac{2}{9}$
 B $-\frac{83}{81}$
 C $\frac{2}{81}$
 D $-\frac{79}{81}$
 E $-\frac{80}{81}$

Question 5

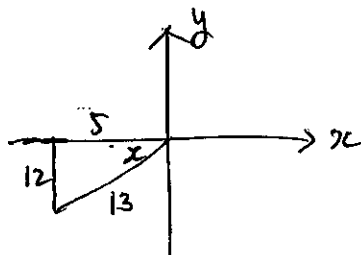
$(\cos \theta + 2 \sin \theta)^2 + (2 \cos \theta - \sin \theta)^2 = n$ where n is an integer. The value of n is:

- A 1
 B 2
 C 3
 D 4
 E 5

Short answer questions

1. If $\cos x = \frac{-5}{13}$ and $\pi < x < \frac{3\pi}{2}$, then the exact value of $\tan x$ is

Quadrant III angle



$$\tan x = \frac{12}{5}$$

(1)

①^M showing
some sort
of working out.

(2 marks)

2. If $\tan u = \frac{4}{3}$ and $\tan v = \frac{5}{12}$, and both u and v are acute angles, evaluate $\tan(u+v)$

$$\begin{aligned}\tan(u+v) &= \frac{\tan u + \tan v}{1 - \tan u \tan v} \\ &= \frac{\frac{4}{3} + \frac{5}{12}}{1 - \left(\frac{4}{3} \times \frac{5}{12}\right)} \quad (1) \\ &= \frac{\frac{16+5}{12}}{\frac{9-5}{9}} \quad (1) \\ &= \frac{21}{12} \times \frac{9}{4} \quad (1)\end{aligned}$$

$$\therefore \tan(u+v) = \frac{63}{16} \quad (1)$$

(3 marks)

3. Find in exact form the value of $\cos(105^\circ)$

$$\begin{aligned}\cos 105^\circ &= \cos(60^\circ + 45^\circ) \quad (1) \\ &= \cos 60^\circ \cos 45^\circ - \sin 60^\circ \sin 45^\circ \quad (1) \\ &= \frac{1}{2} \times \frac{1}{\sqrt{2}} - \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} \quad (1) \\ &= \frac{1 - \sqrt{3}}{2\sqrt{2}} \quad (1) \\ &= \frac{(1 - \sqrt{3})\sqrt{2}}{2\sqrt{2} \times \sqrt{2}} \\ &= \frac{\sqrt{2} - \sqrt{6}}{4} \quad (1)\end{aligned}$$

(4 marks)

4. Simplify $\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right) + \cos\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$

$$\text{Let } \frac{x+y}{2} = A \quad \text{and} \quad \frac{x-y}{2} = B$$

$$\begin{aligned}\sin A \cos B + \cos A \sin B &= \sin(A+B) \quad (1) \\ &= \sin\left(\frac{x+y}{2} + \frac{x-y}{2}\right) \\ &= \sin\left(\frac{x+y+x-y}{2}\right) \quad (1)\end{aligned}$$

$$\therefore \sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right) + \cos\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right) = \sin(x) \quad (1)$$

(3 marks)

5. Prove that $\cos^2 x + \cos(2x) = 2 - 3 \sin^2 x$

$$\begin{aligned} \text{LHS } \cos^2 x + \cos(2x) &= 1 - \sin^2 x + 1 - 2\sin^2 x \\ &= 2 - 3\sin^2 x \end{aligned}$$

$$\text{LHS} = \text{RHS.}$$

QED

(2 marks)

6. a. Express $\sin x - \sqrt{3} \cos x$ as a single trigonometric function $r \sin(x + \alpha)$

$$\sin x - \sqrt{3} \cos x = r \sin(x + \alpha)$$

$$r = \sqrt{1+3}$$

$$r = 2 \quad (1)$$

$$\alpha = \tan^{-1}\left(-\frac{\sqrt{3}}{1}\right) = -\frac{\pi}{3} \quad (1)$$

quadrant 4

$$\therefore \sin x - \sqrt{3} \cos x = 2 \sin\left(x - \frac{\pi}{3}\right) \quad (1)$$

b. Hence determine the minimum value of $\sin x - \sqrt{3} \cos x$ and the value of the x in the interval $[0, 2\pi]$ at which the minimum occurs

$$\text{Minimum value} = 2 \times -1 = -2. \quad (1)$$

$$\sin\left(x - \frac{\pi}{3}\right) = -1 \quad (1/2)$$

$$x - \frac{\pi}{3} = \frac{3\pi}{2} \quad (1/2)$$

$$x = \frac{3\pi}{2} + \frac{\pi}{3}$$

$$= \frac{11\pi}{6} \quad (1)$$

(3+3=5 mark)

END OF TEST