



**Glen Waverley Secondary College
2016 Unit 2 Technology Access Exam**

Specialist Mathematics

Teachers:

Please circle



CAL



JLS



GMN

Student Name:

Student Code: **Form Group:**

Instructions:

Reading Time:	10min
Writing Time:	120 min

Section A Marks:	40 marks
Section B Marks:	60 marks
Total Marks:	100 marks

This examination consists of 20 multiple choice questions and 12 short answer/extended response questions. All multiple choice questions should be answered on the **table** provided at the end this section (bottom of page 9). For each short answer/extended response question correct working **must be** shown to gain full marks.

A CAS calculator, a scientific calculator and a bound reference are allowed.

A formula sheet is provided.

Marks are allocated for working as well as answers.

Answer multiple choice questions on the table at the end of this section.

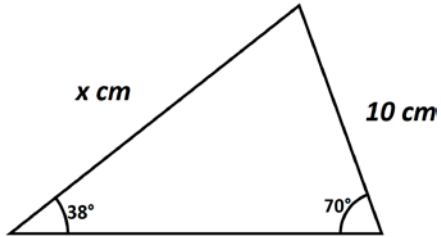
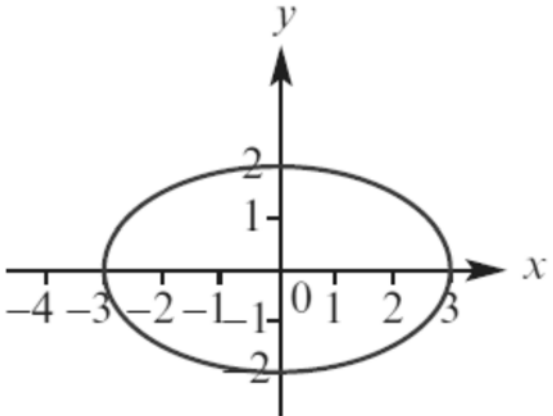
Question 1	
If $A = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 1 \\ 3 & 2 \end{bmatrix}$ and $AX + B = \begin{bmatrix} -2 & -2 \\ 1 & 14 \end{bmatrix}$ then X is: A. $\begin{bmatrix} -5 & -3 \\ -2 & 12 \end{bmatrix}$ B. $\begin{bmatrix} -8 & -18 \\ -23 & 39 \end{bmatrix}$ C. $\begin{bmatrix} -1 & -1 \\ -4 & 2 \end{bmatrix}$ D. $\begin{bmatrix} -2 & 0 \\ 1 & 3 \end{bmatrix}$ E. $\begin{bmatrix} -19 & -7 \\ 32 & 50 \end{bmatrix}$	
Question 2	
A transformation is given by: $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 3 \\ 1 \end{bmatrix}$ The point (3,-1) is mapped to: A. (9,-3) B. (6,-4) C. (5,5) D. (7,3) E. (1,13)	
Question 3	
The unit square is mapped to a parallelogram of area 6 units by the matrix: $\begin{bmatrix} n & 4 \\ -n & 2n \end{bmatrix}$ Possible values for n are: A. $-1 \pm \sqrt{2}$ B. -3,1 C. $1 \pm \sqrt{2}$ D. -1,3 E. -1,1	

Question 4	
The acceleration of an object is given by: $a(t) = 6t^2 - 2t, t \geq 0$ Initially the object is at rest at the origin. The position of the object is given by: A. $x(t) = \frac{1}{2}t^4 - \frac{1}{3}t^3, t \geq 0$ B. $x(t) = 2t^4 - 3t^3, t \geq 0$ C. $x(t) = t^4 - t^3, t \geq 0$ D. $x(t) = \frac{1}{2}t^4 - t^3, t \geq 0$ E. $x(t) = t^4 - \frac{1}{3}t^3, t \geq 0$	
Question 5	
The expression $\sin(\theta) - \sqrt{3}\cos(\theta)$ can be expressed as: A. $2\sin\left(\theta + \frac{\pi}{3}\right)$ B. $2\sin\left(\theta + \frac{\pi}{6}\right)$ C. $2\cos\left(\theta + \frac{\pi}{3}\right)$ D. $2\cos\left(\theta + \frac{\pi}{6}\right)$ E. $2\sin\left(\theta - \frac{\pi}{3}\right)$	

Question 6	
If $\cos(\theta) = -\frac{5}{7}$ and $\pi \leq \theta \leq \frac{3\pi}{2}$, then $\cot(\theta)$ is: A. $-\frac{7}{5}$ B. $-\frac{\sqrt{24}}{5}$ C. $\frac{\sqrt{24}}{5}$ D. $-\frac{5}{\sqrt{24}}$ E. $\frac{5}{\sqrt{24}}$	
Question 7	
A motorcycle accelerates uniformly from 20 m/s to 40 m/s over a distance of 120 metres. The acceleration is closest to: A. 0.167 m/s² B. 1.2 m/s² C. 5 m/s² D. 12 m/s² E. 10 m/s²	
Question 8	
In Australia the proportion of adults with red hair is 12%. In a randomly selected sample of eight people, the probability that none of them have red hair is closest to: A. 0.3923 B. 0.3454 C. 0.8163 D. 0.9600 E. 0.3596	

Question 9	
A chocolate box contains five soft centres and three hard centres. Five chocolates are randomly drawn without replacement. The probability that the proportion of soft centre chocolates is greater than 0.75 is closest to: A. 0.2679 B. 0.2857 C. 0.0179 D. 0.7143 E. 0.9821	
Question 10	
A boat has an initial velocity of 4 m/s and accelerates uniformly for three seconds and covers a distance of 80 metres. How far did the boat travel in the third second? A. $\frac{308}{9}$ m B. $\frac{80}{3}$ m C. $\frac{376}{9}$ m D. $\frac{148}{3}$ m E. $\frac{448}{9}$ m	

Question 11	
The partial fraction decomposition of $\frac{3x+7}{(x-2)(x+2)^2}$ has the form: A. $\frac{A}{x-2} + \frac{B}{x+2} + \frac{C}{x+2}$ B. $\frac{A}{x-2} + \frac{Bx+C}{(x+2)}$ C. $\frac{A}{x-2} + \frac{B}{x+2} + \frac{C}{(x+2)^2}$ D. $\frac{A}{x-2} + \frac{B}{(x+2)^2} + \frac{C}{(x-2)^2}$ E. $\frac{Ax+B}{(x-2)(x+2)} + \frac{C}{x+2}$	
Question 12	
The third term of an arithmetic sequence is 10 and the eighth term is 30. The first term and common difference are, respectively: A. 2 and 4 B. 10 and 30 C. 10 and 3 D. 3 and 5 E. 2 and 3	
Question 13	
If $z = 2 - 2i$, then z^3 and $\text{Arg}(z^3)$ are, respectively: A. 16 and $\frac{5\pi}{4}$ B. 16 and $\frac{-3\pi}{4}$ C. $16\sqrt{2}$ and $\frac{5\pi}{4}$ D. $16\sqrt{2}$ and $\frac{-3\pi}{4}$ E. 16 and $\frac{-\pi}{4}$	

Question 14	
The value of x, to 2 decimal places, in the diagram shown is: A. 14.25 cm B. 10 cm C. 6.55 cm D. 15.26 cm E. 23.04 cm	
Question 15	
The equation of the graph shown is: A. $\frac{x^2}{3} + \frac{y^2}{2} = 1$ B. $\frac{x^2}{9} + \frac{y^2}{4} = 1$ C. $\frac{x^2}{3} - \frac{y^2}{2} = 1$ D. $\frac{x^2}{9} - \frac{y^2}{4} = 1$ E. $\frac{x^2}{4} + \frac{y^2}{9} = 1$	
Question 16	
The surface areas of two similar shapes are in the ratio 2:3. The ratio of their volumes is: A. $\frac{3}{2}:1$ B. 4:9 C. 1:4.5 D. $2\sqrt{2}:3\sqrt{3}$ E. $\sqrt[3]{2}:\sqrt[3]{3}$	

Question 17	
A man who is 1.8 m tall casts a shadow of 0.76 m in length. If at the same time a telephone pole casts a 3 m shadow. The height of the pole is closest to: A. 4.10m B. 2.19m C. 1.27m D. 7.11m E. 3.00m	
Question 18	
If a vector $\underline{v}$ of magnitude 6 is parallel to the vector $2\underline{i} + 2\underline{j} - \underline{k}$ then $\underline{v}$ is represented by the vector: A. $12\underline{i} + 12\underline{j} - 6\underline{k}$ B. $\left(\frac{1}{3}\right)\underline{i} + \left(\frac{1}{3}\right)\underline{j} - \left(\frac{1}{6}\right)\underline{k}$ C. $4\underline{i} + 4\underline{j} - 2\underline{k}$ D. $-2\underline{i} - 2\underline{j} + 2\underline{k}$ E. $2\underline{i} + 4\underline{j} - 4\underline{k}$	
Question 19	
The vector expression $2\overrightarrow{AB} + \overrightarrow{CD} + \overrightarrow{BA} + \overrightarrow{BC}$ can be simplified to A. $3\overrightarrow{AB} + \overrightarrow{CD} + \overrightarrow{BC}$ B. $2\overrightarrow{AB} + \overrightarrow{CD} + \overrightarrow{AC}$ C. $\overrightarrow{AD}$ D. $3\overrightarrow{AB} + 2\overrightarrow{AC}$ E. $\underline{0}$	

Question 20	
The vector component of $\underline{a}$ in the direction of $\underline{b}$ is $5\underline{i} + 2\underline{j}$. If $\underline{a} = 7\underline{i} - 3\underline{j}$ then the vector component of $\underline{a}$ perpendicular to the direction of $\underline{b}$ is A. $-2\underline{i} + 5\underline{j}$ B. $2\underline{i} - 5\underline{j}$ C. $-5\underline{i} + 2\underline{j}$ D. $7\underline{i} - \underline{j}$ E. $12\underline{i} - \underline{j}$	

Multiple Choice Answer Table									
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20

Question 1

a) What is the determinant of the matrix T ?

c) Find the inverse of the matrix (T^{-1}).

e) Find the image of the point (2,-1) which is first reflected in the line $y = x$ and then undergoes transformation by the matrix T. Find the image of the point.

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Question 2

a) If $\cos(2A) = \frac{7}{9}$, $0 \leq A \leq \frac{\pi}{2}$ find the exact value for $\tan(A)$.

b) If $\sin(B) = \frac{2}{3}$, $\frac{\pi}{2} \leq B \leq \pi$ find the exact value for $\cos(A-B)$.

c) Prove the identity: $\sec(x) + \tan(x) = \frac{\cos(x)}{1 - \sin(x)}$

(3 + 2 + 3 = 8 marks)

Question 3

Two rockets are launched from the ground. Both rockets start from rest at the origin. Rocket A has an acceleration of 45 m/s^2 up. Rocket B is launched 22 seconds after Rocket A and has an acceleration of 60 m/s^2 up. **(Express all answers in this question to two decimal places)**

- a) Set up two equations and use them to show that 164.21 seconds after Rocket A is launched, Rocket B will pass Rocket A.
- b) At what height will the rockets be when Rocket B passes Rocket A?
- c) At the instant Rocket B passes Rocket A, it turns off its engine. What is its speed at this time?
- d) How long after its engine turns off does Rocket B take to hit the ground?

(2 + 2 + 2 + 2 = 8 marks)

Question 4

The following table gives the sample proportion of defectives in a random sample of five washing machines:

$\hat{p}$	0	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{3}{5}$	$\frac{4}{5}$	1
$\Pr(\hat{P} = \hat{p})$	0.62	0.24	0.07	0.04	0.02	0.01

- a) Find the probability that three or more washing machines are defective.
- b) In a particular batch it is known that between one and four (inclusive) washing machines are defective. Find the probability that three or more washing machines are defective.
- c) At the warehouse there are 5000 washing machines of which 45 are defective. Use a binomial approximation to determine the probability that a sample of six washing machines will contain exactly one defective washing machine. (Express your answer to four decimal places)
- d) Explain why using a binomial approximation is suitable in this calculation.

(1 + 2 + 2 + 1 = 6 marks)

Question 5

Express $12x^2 + 4x + 4$ in the form $a(2x+1)^2 + b(2x+1) + c$

(3 marks)

Question 6

A sequence is given by: $t_1 = 1$, $t_{n+1} = t_n(p + t_n)$ where p is a constant and $p \neq 0$

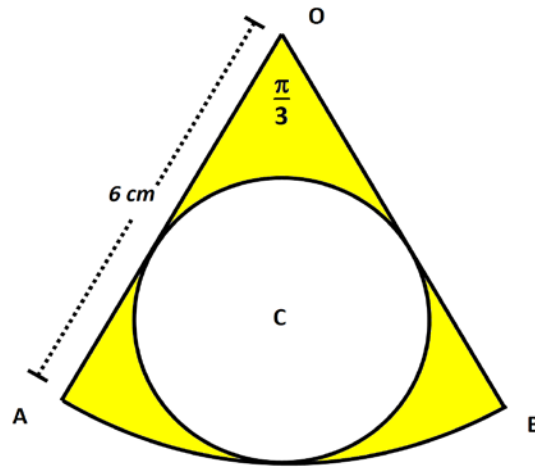
a) Find t_2 in terms of p .

b) Given that $t_3 = 1$, find the value of p .

(1 + 2 = 3 marks)

Question 7

The shape shown in the diagram is a pattern for a pendant. It consists of a sector OAB of a circle centre O , of radius 6 cm , and angle $AOB = \frac{\pi}{3}$. The circle, C , inside the sector, touches the two straight edges, OA and OB , and the arc AB as shown. Leave your answers in exact form.



a) Find the area of the sector OAB in terms of π .

b) Show that the radius of the circle C is 2 cm .

(1 + 2 = 3 marks)

Question 8

Given that $z = x + yi$, find the value of x and the value of y such that $z + 3i\bar{z} = -1 + 13i$, where $\bar{z}$ is the conjugate of z .

(3 marks)

Question 9

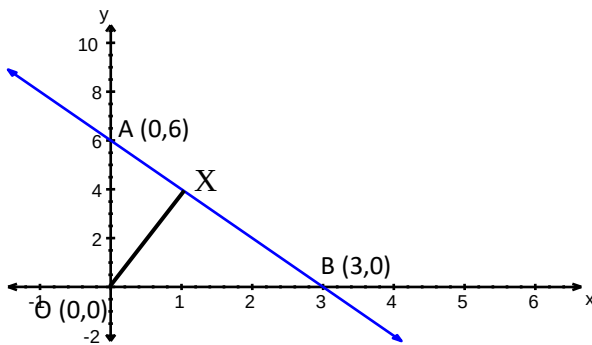
An ellipse is expressed in parametric equations as: $x = 4 \cos t + 1$ and $y = 2 \sin t - 1$. Rewrite it in the form

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

(3 marks)**Question 10**

A **vector method** is to be used to determine the minimum distance between the line $y = 6 - 2x$ and the origin.

Let $\underline{i}$ and $\underline{j}$ be unit vectors directed along the x and y axes respectively and **X** be the point on the line segment BA closest to **O** (0, 0).



- a) Show that the vector $\overrightarrow{BA}$ can be expressed in component form as $-3\underline{i} + 6\underline{j}$.

Let $\overrightarrow{BX} = m \overrightarrow{BA}$.

- b) Determine an expression for $\overrightarrow{BX}$ in terms of m , $\underline{i}$ and $\underline{j}$.

- c) Determine an expression for $\overrightarrow{OX}$ in terms of m , $\underline{i}$ and $\underline{j}$.

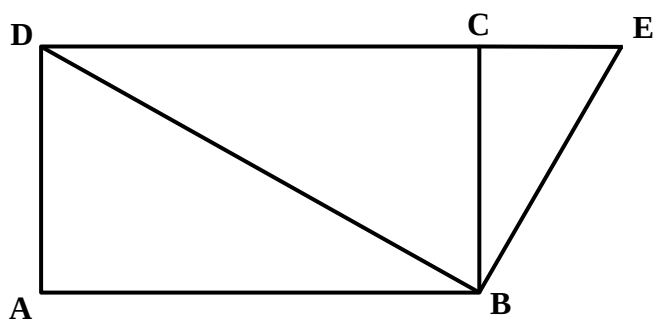
- d)** Determine the value for m , using the fact that if X is the closest point on $\overrightarrow{BA}$ to O , then $\angle OXB$ will be a right angle.

- e)** Substitute the value of m calculated above into the expression for $\overrightarrow{OX}$ and hence calculate the minimum distance between the origin and the line that passes through A and B . Give your answer to 1 decimal place.

(1 + 1 + 1 + 3 + 2 = 8 marks)

Question 11

$ABCD$ is a rectangle. $AB = 4$ cm and $AD = 3$ cm. DCE is a straight line. Angle DBE is 90° .



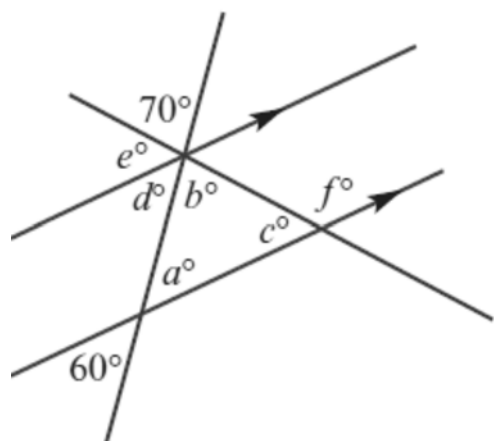
- a)** Explain why triangles ADB and BDE are similar.

b) Find the length of BE .

(2 + 2 = 4 marks)

Question 12

Calculate the values of the unknowns in the diagram below.



(3 marks)

END OF EXAM