

Name: SOLUTIONS

/45

Multiple Choice (10 x 2 = 20 Marks)**Question 1.**For the complex number $z = -2\sqrt{2} - 2\sqrt{2}i$, $|z|$ and $\text{Arg } z$ respectively are:

A. $4, \frac{5\pi}{4}$

$$|z| = \sqrt{(2\sqrt{2})^2 + (2\sqrt{2})^2}$$

$$= 4$$

B. $4, \frac{-3\pi}{4}$

C. $8, \frac{5\pi}{4}$

$$\text{Arg}(z) = \tan^{-1}\left(\frac{-2\sqrt{2}}{-2\sqrt{2}}\right) = \frac{-3\pi}{4}$$

D. $8, \frac{-3\pi}{4}$

E. $4, \frac{\pi}{4}$

Question 2.The solution to the equation $(2+i)z = 7+i$ is:

A. $\frac{13}{5} - i$

B. $5 - i$

C. $\frac{15}{3} - \frac{5}{3}i$

D. $\frac{13}{3} - \frac{5}{3}i$

$$z = \frac{7+i}{2+i} \times \frac{2-i}{2-i} = \frac{14+2i-7i-i^2}{4+1}$$

$$= \frac{15-5i}{5}$$

$$= 3-i$$

E. $3-i$

Question 3.If $z_1 = 5-3i$ and $z_2 = 4+2i$ then $z_1 \times z_2$ equals:

A. $26-2i$

B. $14-2i$

C. $14+2i$

D. $26-22i$

E. $14-22i$

$$(5-3i)(4+2i) = 20 - 12i - 6i^2 + 10i$$

$$= 26 - 2i$$

Question 4.

The solution to the equation $\frac{(8-2i)z}{1+i} + 3 - 4i = 4 - 17i$ is:

- A. $3+2i$
- B. $3-2i$
- C. $2+i$
- ☒ D. $2-i$
- E. $3+i$

$$\frac{8-2i}{1+i} z = 1-13i$$

$$(8-2i)z = 1-13i + i - 13i^2$$

$$= 14-12i$$

$$z = \frac{14-12i}{8-2i} \times \frac{8+2i}{8+2i} = \frac{136-68i}{68}$$

$$= 2-i$$

Question 5.

If $z = 3-4i$ then $z \times \bar{z}$ equals:

- ☒ A. 25
- B. 5
- C. 7
- D. 12
- E. $9-16i$

$$(3-4i)(3+4i)$$

$$= 9+16$$

$$= 25$$

Question 6.

The solutions to the quadratic equation $z^2 + 4z + 9 = 0$ are:

- A. $z = -2 \pm 2\sqrt{5}i$
- B. $z = 2 \pm 2\sqrt{5}i$
- C. $z = 2 \pm \sqrt{5}i$
- ☒ D. $z = -2 \pm \sqrt{5}i$
- E. $z = -2 \pm 2\sqrt{13}i$

$$z = \frac{-4 \pm \sqrt{4^2 - 4 \times 1 \times 9}}{2}$$

$$= \frac{-4 \pm 2\sqrt{5}i}{2}$$

$$= -2 \pm \sqrt{5}i$$

Question 7.

If $z = 2\text{cis}(-\frac{2\pi}{3})$, then z^5 is:

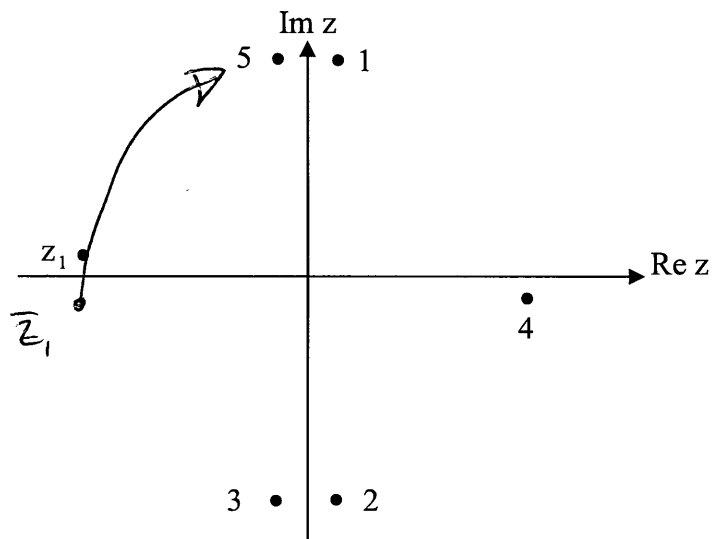
- ☒ A. $32\text{cis}(\frac{2\pi}{3})$
- B. $32\text{cis}(-\frac{2\pi}{3})$
- C. $32\text{cis}(-\frac{4\pi}{3})$
- D. $32\text{cis}(-\frac{10\pi}{3})$
- E. $32\text{cis}(\frac{4\pi}{3})$

$$z^5 = 32 \text{cis}(-\frac{10\pi}{3})$$

$$= 32 \text{cis}(\frac{2\pi}{3})$$

Question 8.

The position of a complex number z_1 is marked on the Argand Diagram below.



The position of $-i\overline{z_1}$ is given by the point:

- A. 1
- B. 2
- C. 3
- D. 4
- E. 5**

Question 9.

If $z = \sqrt{3}\text{cis}(\frac{2\pi}{3})$, then z in cartesian form is:

- A. $-\frac{3}{2} + \frac{3}{2}i$
- B. $-\frac{3}{2} - \frac{3}{2}i$
- C. $-\frac{3}{2} - \frac{\sqrt{3}}{2}i$
- D. $-\frac{3}{2} + \frac{\sqrt{3}}{2}i$
- E. $-\frac{\sqrt{3}}{2} + \frac{3}{2}i$**

$$\begin{aligned}
 z &= \sqrt{3} \left(\cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3} \right) \\
 &= \sqrt{3} \left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i \right) \\
 &= -\frac{\sqrt{3}}{2} + \frac{3}{2}i
 \end{aligned}$$

Question 10.

If $z_1 = \sqrt{2}\text{cis}(\frac{\pi}{3})$ and $z_2 = 2\text{cis}(-\frac{\pi}{3})$ then $iz_1 \times \bar{z}_2$ is:

A. $2\sqrt{2}\text{cis}(\frac{2\pi}{3})$

B. $2\sqrt{2}\text{cis}(\frac{5\pi}{6})$

C. $2\sqrt{2}\text{cis}(-\frac{5\pi}{6})$

D. $2\sqrt{2}\text{cis}(-\frac{2\pi}{3})$

E. $2\sqrt{2}\text{cis}(\frac{\pi}{2})$

$$\begin{aligned} iz_1 \times \bar{z}_2 &= \text{cis}(\pi/2) \times \sqrt{2} \text{cis}(\pi/3) \times 2 \text{cis}(\pi/3) \\ &= 2\sqrt{2} \text{cis}(2\pi/3 + \pi/2) \\ &= 2\sqrt{2} \text{cis}(7\pi/6) \\ &= 2\sqrt{2} \text{cis}(-5\pi/6) \end{aligned}$$

Short Answer (25 marks)

Question 1.

If $z_1 = -4 + 3i$ and $z_2 = 2 + 4i$ then $z_1 - 2\bar{z}_2$ is:

$$z_1 - 2\bar{z}_2 = -4 + 3i - 2(2 - 4i) \quad (1)$$

$$= -4 + 3i - 4 + 8i$$

$$= -8 + 11i \quad (1)$$

(2 Marks)

Question 2.

If $(3-i)z + 4i = 9 + 11i$ then z is:

$$(3-i)z = 9 + 7i \quad (1/2)$$

$$z = \frac{9+7i}{3-i} \times \frac{3+i}{3+i} \quad (1/2)$$

$$= \frac{27 + 21i + 9i + 7i^2}{9+1}$$

$$= \frac{20 + 30i}{10} \quad (1/2)$$

$$= 2 + 3i \quad (1/2)$$

(2 Marks)

Question 3.Solve the following equations over $\mathbb{C}$:

a) $5z^2 + 60 = 0$

$$5(z^2 + 12) = 0 \quad (1/2)$$

$$5(z + 2\sqrt{3}i)(z - 2\sqrt{3}i) = 0 \quad (1/2)$$

$$z = \pm 2\sqrt{3}i \quad (1)$$

b) $z^2 - 3z + 3 = 0$

$$(1/2) \quad z = \frac{3 \pm \sqrt{9 - 4 \times 1 \times 3}}{2} \quad \text{OR} \quad (1/2) \quad (z^2 - 3z + 9/4) - 9/4 + 3 = 0$$

$$(1/2) \quad z = \frac{3 \pm \sqrt{-3}}{2}$$

$$(1/2) \quad (z - 3/2)^2 - (\frac{\sqrt{3}}{2}i)^2 = 0$$

$$(1) \quad z = 3/2 \pm \sqrt{3}/2 i$$

$$(1/2) \quad (z - 3/2 - \frac{\sqrt{3}}{2}i)(z - 3/2 + \frac{\sqrt{3}}{2}i) = 0$$

$$(1/2) \quad z = 3/2 \pm \sqrt{3}/2 i$$

c) $4z^2 - 7z + 19 = 0$

$$z = \frac{7 \pm \sqrt{49 - 4 \times 4 \times 19}}{8} \quad (1)$$

$$= \frac{7 \pm \sqrt{-255}}{8} \quad (1)$$

$$= \frac{7 \pm \sqrt{255}i}{8} \quad (1)$$

(2 + 2 + 3 = 7 marks)

Question 4.If $z_1 = 1 + \sqrt{3}i$ and $z_2 = 2 + 2i$ a) Find z_1 and z_2 in polar form

$$z_1 = 2 \cos \pi/3 \quad (1)$$

$$z_2 = 2\sqrt{2} \cos \pi/4 \quad (1)$$

b) Find $z_1 \times z_2$ in polar form.

$$z_1 \times z_2 = 4\sqrt{2} \cos(\pi/3 + \pi/4) \quad (1/2)$$

$$= 4\sqrt{2} \cos(7\pi/12) \quad (1/2) \quad (2 + 1 = 3 \text{ marks})$$

Question 5.

If $z_1 = 3\text{cis}(\frac{\pi}{3})$ and $z_2 = 2\text{cis}(-\frac{\pi}{6})$

a) Find $z_1 \times z_2$ in polar form.

$$z_1 \times z_2 = 6 \text{cis} \left(\frac{\pi}{3} - \frac{\pi}{6} \right) \quad (1/2)$$

$$= 6 \text{cis} \left(\frac{\pi}{6} \right) \quad (1/2)$$

b) Find $z_1 \times z_2$ in cartesian form.

$$z_1 \times z_2 = 6 \left(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6} \right) \quad (1/2)$$

$$= 6 \left(\frac{\sqrt{3}}{2} + i \frac{1}{2} \right) \quad (1/2)$$

$$= 3\sqrt{3} + 3i \quad (1)$$

(1 + 2 = 3 marks)

Question 6.

Find all solutions in the complex field for the following equation:

$$z^4 + 9z^2 + 8 = 0$$

$$\text{Let } a = z^2 \quad a^2 + 9a + 8 = 0$$

$$(a + 8)(a + 1) = 0 \quad (1)$$

$$a = -8, -1$$

$$z^2 = -8, -1 \quad (1)$$

$$z = \pm 2\sqrt{2}i, \pm i \quad (1)$$

(3 marks)

Question 7.

a) Find all solutions (in polar form) for the equation $z^3 - 8 = 0$

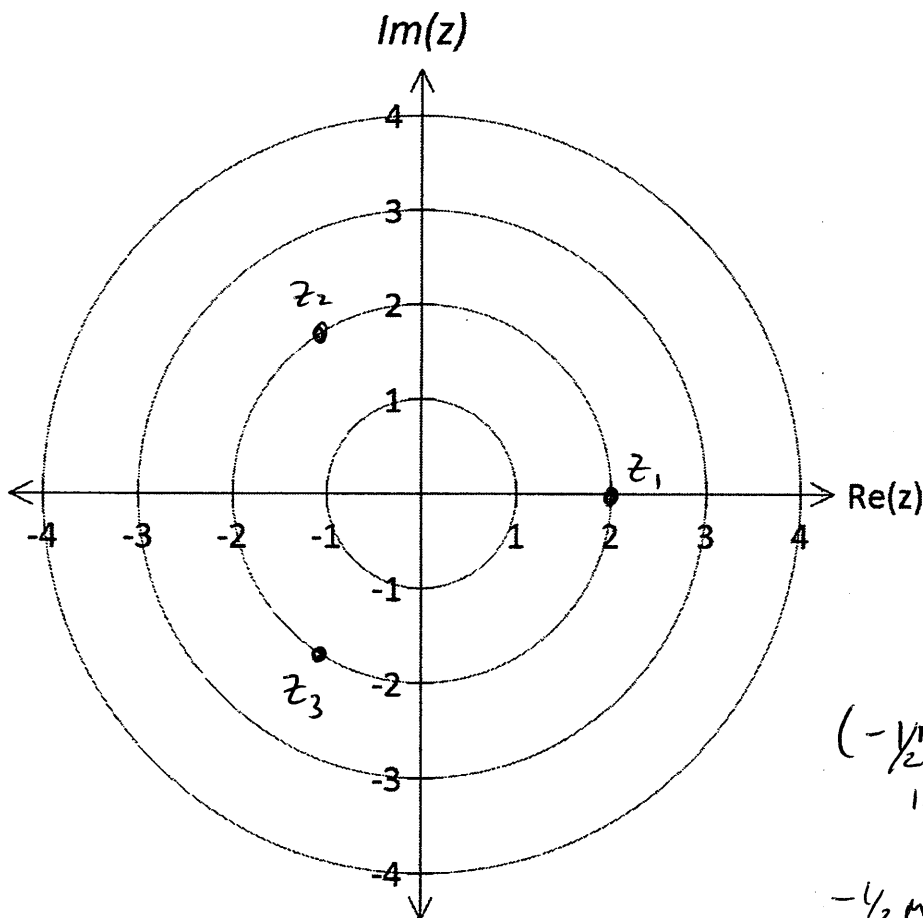
$$z^3 = 8$$

$$r^3 \cos 3\theta = 8 \cos(0) \quad (1)$$

$$r = 2 \quad \theta = -2\pi/3, 0, 2\pi/3 \quad (1)$$

$$z = 2 \cos(-2\pi/3), 2 \cos(0), 2 \cos(2\pi/3) \quad (1)$$

b) Represent these solutions on the argand diagram below:



(-1/2 MARK FOR EACH
IN CORRECT POINT)

-1/2 MARK IF NOT
LABELLED

(3 + 2 = 5 marks)

