

Answers

Chapter 1

Exercise 1A

- 1 **a** 3 **b** 9 **c** $\frac{1}{5}$ **d** $-\frac{8}{2}$
e 5 **f** 2 **g** $\frac{5}{3}$ **h** $-\frac{7}{2}$
i $\frac{7}{3}$ **j** $\frac{20}{3}$ **k** $-\frac{10}{3}$ **l** $\frac{14}{5}$
2 **a** $a + b$ **b** $a - b$ **c** $\frac{b}{a}$ **d** ab
e $\frac{bc}{a}$
3 **a** $y = 5$ **b** $t = 5$ **c** $y = -\frac{3}{2}$ **d** $x = 2$
e $t = \frac{11}{2}$ **f** $y = \frac{8}{3}$ **g** $y = 136$ **h** $t = 1$
i $x = 12$ **j** $y = -\frac{9}{5}$ **k** $x = -7$ **l** $y = 2$
4 **a** $\frac{4}{3}$ **b** -5 **c** 2
5 **a** -1 **b** 18 **c** $\frac{6}{5}$ **d** 23
e 0 **f** 10 **g** 12 **h** 8
i $-\frac{14}{5}$ **j** $\frac{12}{5}$ **k** $\frac{7}{2}$
6 **a** $-\frac{b}{a}$ **b** $\frac{e-d}{c}$ **c** $\frac{c}{a} - b$ **d** $\frac{b}{c-a}$
e $\frac{ab}{b+a}$ **f** $a + b$ **g** $\frac{b-d}{a-c}$ **h** $\frac{bd-c}{a}$
7 **a** -18 **b** -78.2 **c** 16.75 **d** 28
e 34 **f** $\frac{3}{26}$
8 $x = \frac{a^2 + b^2 + 2ab}{ac + bc} = \frac{a+b}{c}$
9 $x = \frac{ab}{a-b-c}$

Exercise 1B

- 1 **a** $x + 2 = 6$, 4 **b** $3x = 10$, $\frac{10}{3}$
c $3x + 6 = 22$, $\frac{16}{3}$ **d** $3x - 5 = 15$, $\frac{20}{3}$
e $6(x + 3) = 56$, $\frac{19}{3}$ **f** $\frac{x+5}{4} = 23$, 87
2 $A = \$8$, $B = \$24$, $C = \$16$ 3 14 and 28
4 8 kg 5 1.3775 m² 6 49, 50, 51
7 17, 19, 21, 23 8 4200 L 9 21
10 3 km 11 9 and 12 dozen
12 7.5 km/h 13 3.6 km 14 30, 6

Exercise 1C

- 1 **a** $x = -1$, $y = -1$ **b** $x = 5$, $y = 21$
c $x = -1$, $y = 5$ **d** $x = 5$, $y = 19$
e $x = -4$, $y = -13$ **f** $x = -\frac{8}{5}$, $y = -\frac{2}{5}$
2 **a** $x = 8$, $y = -2$ **b** $x = -1$, $y = 4$
c $x = 7$, $y = \frac{1}{2}$
3 **a** $x = 2$, $y = -1$ **b** $x = 2.5$, $y = -1$
c $m = 2$, $n = 3$ **d** $x = 2$, $y = -1$
e $s = 2$, $t = 5$ **f** $x = 10$, $y = 13$
g $x = \frac{4}{3}$, $y = \frac{7}{2}$ **h** $p = 1$, $q = -1$
i $x = -1$, $y = \frac{5}{2}$
4 **a** No solutions **b** Infinitely many solutions
c One solution **d** One solution

Exercise 1D

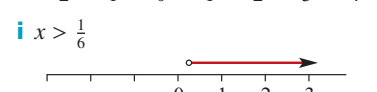
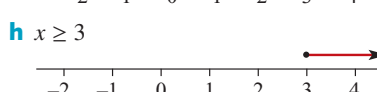
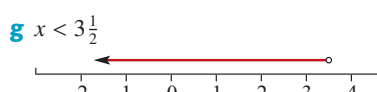
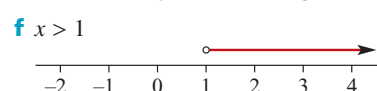
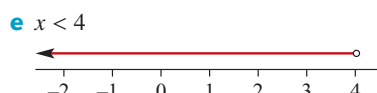
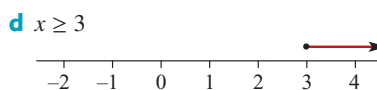
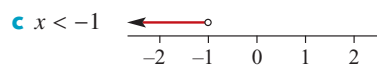
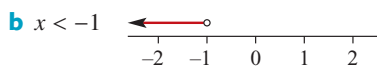
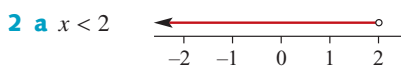
- 1 25, 113 2 22.5, 13.5
3 **a** \$70 **b** \$12 **c** \$3
4 **a** \$168 **b** \$45 **c** \$15

- 5 17 and 28 6 44 and 12
 7 5 pizzas, 25 hamburgers
 8 Started with 60 and 50; finished with 30 each
 9 134 adults, 16 children 10 $\frac{7}{10}$
 11 26 12 420 adults, 540 children
 13 \$17 000 14 120 shirts, 300 ties
 15 360 Outbacks, 300 Bush Walkers
 16 2800 in Mydney, 3200 in Selbourne
 17 20 kg at \$10, 40 kg at \$11 and 40 kg at \$12

Exercise 1E

- 1 a $x < 1$ b $x > 13$ c $x \geq 3$ d $x \leq 12$
 e $x \leq -6$ f $x > 3$ g $x < -2$ h $x \geq -8$

i $x \leq \frac{3}{2}$



- 3 a $x > \frac{-1}{2}$ b $x < 2$ c $x > -5$

- 4 $3x < 20$, $x < \frac{20}{3}$, 6 pages 5 87

Exercise 1F

- 1 a 18 b 9 c 3 d -18
 e 3 f 81 g 5 h 20
 2 a $S = a + b + c$ b $P = xy$ c $C = 5p$
 d $T = dp + cq$ e $T = 60a + b$
 3 a 15 b 31.4 c 1000 d 12
 e 314 f 720

4 a $V = \frac{c}{p}$ b $a = \frac{F}{m}$ c $P = \frac{I}{rt}$

d $r = \frac{w - H}{C}$ e $t = \frac{S - P}{Pr}$ f $r = \frac{R(V - 2)}{V}$

5 a $T = 48$ b $b = 8$ c $h = 3.82$ d $b = 10$

6 a $(4a + 3w)m$ b $(h + 2b)m$
 c $3wh \text{ m}^2$ d $(4ah + 8ab + 6wb) \text{ m}^2$

7 a i $T = 2\pi(p + q) + 4h$ ii $88\pi + 112$

b $p = \frac{A}{\pi h} - q$

8 a $D = \frac{2}{3}$ b $b = 2$ c $n = \frac{60}{29}$ d $r = 4.8$

9 a $D = \frac{1}{2}bc(1 - k^2)$ b $k = \sqrt{1 - \frac{2D}{bc}}$

c $k = \sqrt{\frac{2}{3}} = \frac{\sqrt{6}}{3}$

10 a $P = 4b$ b $A = 2bc - c^2$ c $b = \frac{A + c^2}{2c}$

11 a $b = \frac{a^2 - a}{2}$ b $x = \frac{-ay}{b}$

c $r = \pm \sqrt{3q - p^2x^2}$ d $v = \pm \sqrt{u^2(1 - \frac{x^2}{y^2})}$

Chapter 1 review

Technology-free questions

1 a 1 b $\frac{-3}{2}$ c $\frac{2}{3}$ d -27

e 12 f $\frac{44}{13}$ g $\frac{1}{8}$ h 31

2 a $t = a - b$ b $\frac{cd - b}{a}$ c $\frac{d}{a} + c$

d $\frac{cb - a}{c - 1}$ e $\frac{2b}{c - a}$ f $\frac{1 - cd}{ad}$

3 a $x < \frac{2}{3}$ b $x \leq -148\frac{1}{2}$ c $x < \frac{22}{29}$ d $x \geq \frac{-7}{17}$

4 $x = 2(z + 3t)$, -10

5 a $d = e^2 + 2f$ b $f = \frac{d - e^2}{2}$ c $f = \frac{1}{2}$

6 $400\pi \text{ cm}^2$

7 a 196π b $\frac{975\pi}{2}$

8 a $r = \frac{A}{\pi s}$ b $w = \frac{T - P}{Pr}$

c $r = \frac{n - p}{v^2}$ d $x = \frac{ac - b^2}{b}$

9 a $s = 75$ b $t = 8$

10 $5\sqrt{2} \text{ cm}$

11 12 m and 17 m

12 $m = 2$ and $n = 15$

13 Mr Apollo earns \$100 000, Mr Adonis earns \$107 200 and Ms Aphrodite earns \$96 000

14 a $a = \frac{28}{11}$, $b = -\frac{9}{11}$ b $a = -\frac{11}{5}$, $b = -\frac{33}{5}$

15 5 hours travelling on highways

Multiple-choice questions

- 1 D 2 D 3 C 4 A 5 C 6 C
7 B 8 B 9 A 10 B 11 E 12 B

Extended-response questions

- 1 a $C = \frac{-10}{9}$ b $F = 86$ c $x = -40$
d $x = -62.5$ e $x = \frac{-160}{13}$ f $k = 5$

- 2 a $r = \frac{2uv}{u+v}$ b $m = \frac{v}{u}$

- 3 a $T = 6w + 6\ell$
b i $T = 8w$ ii $\ell = \frac{25}{6}, w = 12\frac{1}{2}$

- c i $y = \frac{L-6x}{8}$ ii $y = 22$

- d $x = 10, y = 5$

- 4 a Distance Tom travelled = ut km
Distance Julie travelled = vt km

- b i $t = \frac{d}{u+v}$ h

- ii Distance from A = $\frac{ud}{u+v}$ km

- c $t = 1.25$ h, distance from A = 37.5 km

- 5 a Average speed = $\frac{2uv}{u+v}$

- b i $\frac{uT}{v}$ ii $\frac{vT+uT}{v}$

- 6 a $\frac{3}{a} + \frac{3}{b}$ c i $c = \frac{2ab}{a+b}$ ii $\frac{40}{3}$

- 7 a $\frac{x}{8}, \frac{y}{10}$ b $\frac{80(x+y)}{10x+8y}$

- c $x = \frac{320}{9}, y = \frac{310}{9}$

- 8 The three lines intersect at the point (4, 3)

Chapter 2

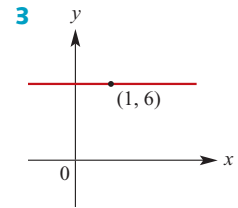
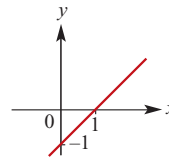
Exercise 2A

- 1 a (5, 8) b $(\frac{1}{2}, \frac{1}{2})$ c (1.6, 0.7)
d (-0.7, 0.85)
2 $M_{AB}(3, 3), M_{BC}(8, 3\frac{1}{2}), M_{AC}(6, 1\frac{1}{2})$
3 Coordinates of C are (6, 8.8)
4 a (4, 4) b (2, -0.2) c (-2, 5) d (-4, -3)
5 $(\frac{1+a}{2}, \frac{4+b}{2})$, $a = 9, b = -6$
6 a $5\sqrt{2} \approx 7.07$ b $\sqrt{17} \approx 4.12$
c $\sqrt{34} \approx 5.83$ d 13
7 $\sqrt{97} + \sqrt{85} + \sqrt{104} \approx 29.27$
8 $PM = \sqrt{145} \approx 12.04$ 9 DN

Exercise 2B

- 1 a 4 b 2 c $\frac{1}{4}$ d -4 e 1 f -1
g $\frac{5}{4}$ h -2 i $-\frac{5}{4}$ j $\frac{4}{3}$ k 0

- 2 Any line parallel to the one shown
 $y = x - 1$



- 4 a $-\frac{1}{4}$ b $-\frac{5}{2}$ c -2 d -8 e 0 f -1
g 7 h 11 i -13 j 11 k 111 l 61

- 5 a -2 b $\frac{2}{5}$

- 6 a 54 b $\frac{5}{6}$

- 7 a 45° b 45° c 26.57° d 135°

- 8 a 45° b 26.57° c 161.57° d 49.4°
e 161.57° f 135°

- 9 a 1 b -1 c $\sqrt{3}$ d $-\sqrt{3}$

Exercise 2C

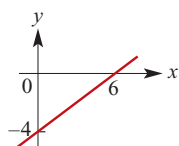
- 1 a $m = 3, c = 6$ b $m = -6, c = 7$
c $m = 3, c = -6$ d $m = -1, c = -4$
2 a $y = 3x + 5$ b $y = -4x + 6$ c $y = 3x - 4$
3 a $m = 3, c = -6$ b $m = 2, c = -4$
c $m = \frac{1}{2}, c = -2$ d $m = \frac{1}{3}, c = -\frac{5}{3}$
4 a $m = 2, c = -9$ b $m = -\frac{3}{4}, c = \frac{5}{2}$
c $m = -\frac{1}{3}, c = -2$ d $m = \frac{5}{2}, c = -2$
5 a $y = 3x - 11$ b $y = -2x + 9$
6 a $y = -\frac{1}{3}x + \frac{11}{3}$ b $y = -\frac{7}{5}x + 4$
c $y = -2x + 4$ d $y = \frac{11}{3}x - \frac{61}{3}$
7 a 2 b $y = 2x + 6$
8 a $y = 2x + 4$ b $y = -2x + 8$
9 a $y = 2x + 6$ b $y = -2x + 4$
c $y = -5x + 15$
10 a $y = -\frac{2}{3}x + 4$ b $y = -2x - 6$
c $y = -x + 4$ d $y = -\frac{3x}{2} + 3$
11 a $y = \frac{2}{3}x + 4$ b $y = \frac{2}{3}x - \frac{2}{3}$
c $y = \frac{1}{2}x + 1\frac{1}{2}$ d $y = -\frac{1}{2}x + 2$
e $y = x + 3.5$ f $y = -0.5x + 0.25$
12 a $y = 4x + 4$ b $y = -\frac{2}{3}x$ c $y = -x - 2$
d $y = \frac{1}{2}x - 1$ e $y = 3\frac{1}{2}$ f $x = -2$
13 Yes 14 Only c
15 a $x = 4$ b $y = 11$ c $x = 11$ d $y = -1$

Exercise 2D

1 a $(0, 4), (4, 0)$

c $(0, -6), (-6, 0)$

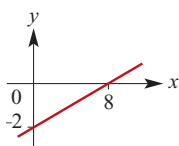
2 a



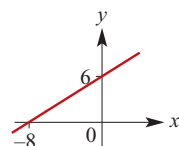
b $(0, -4), (4, 0)$

d $(0, 8), (-8, 0)$

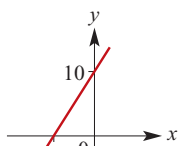
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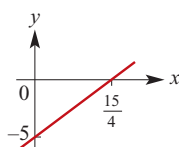
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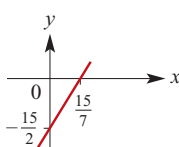
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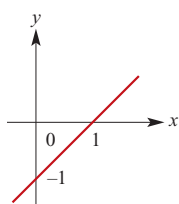
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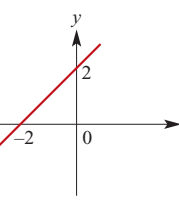
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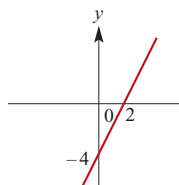
3 a



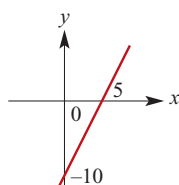
b



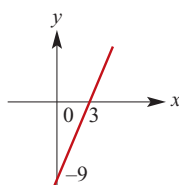
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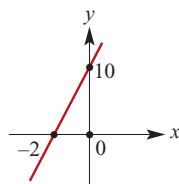
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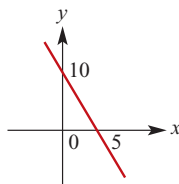
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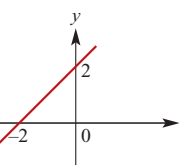
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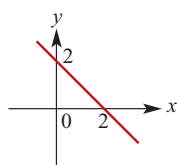
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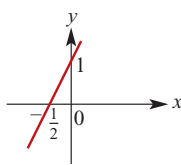
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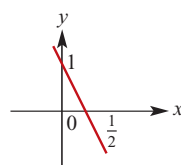
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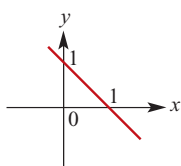
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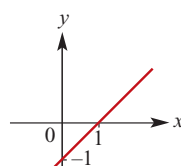
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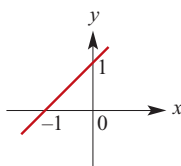
6 a



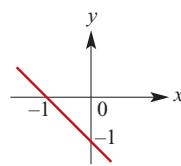
b



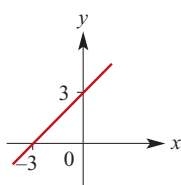
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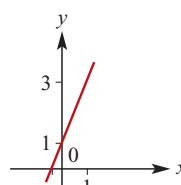
d



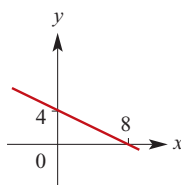
7 a



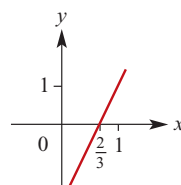
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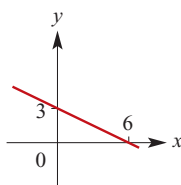
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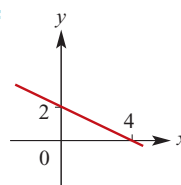
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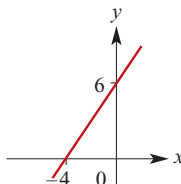
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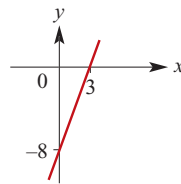
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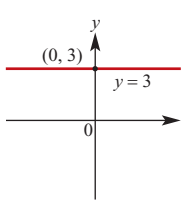
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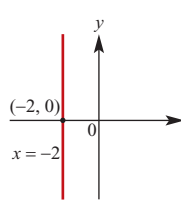
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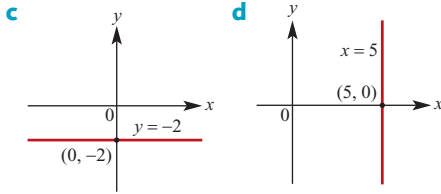


8 a



b





- 9 a** 45° **b** 135° **c** 45° **d** 135°
e 63.43° (to 2 d.p.) **f** 116.57° (to 2 d.p.)
10 a 71.57° **b** 135° **c** 45° **d** 161.57°
11 a -4 , $b = \frac{4}{3}$, $d = -1$, $e = \frac{14}{3}$

Exercise 2E

- 1 a** $y = 2x - 10$ **b** $y = -\frac{1}{2}x$
c $y = -2x + 6$ **d** $y = \frac{1}{2}x - 4$
e $y = \frac{2}{3}x - \frac{14}{3}$ **f** $y = -\frac{3}{2}x + 4$
g $y = -\frac{1}{3}x - \frac{2}{3}$ **h** $y = 3x - 14$
2 Parallel lines: a, b, c; non-parallel lines: d
3 a $y = 4$ **b** $x = 2$ **c** $y = 4$ **d** $x = 3$
4 $y = 2x + 2$
5 Midpoint of AB is $(-1, 6)$; $y = 2x + 8$
6 $m_{BC} = -\frac{3}{5}$, $m_{AB} = \frac{5}{3}$
 $\therefore m_{BC} \times m_{AB} = -\frac{3}{5} \times \frac{5}{3} = -1$
 $\therefore \triangle ABC$ is a right-angled triangle
7 $m_{AB} = -2$, $m_{BC} = \frac{1}{2}$
8 $m_{RS} = -\frac{1}{2}$, $m_{ST} = 2 \therefore RS \perp ST$
 $m_{UT} = -\frac{1}{2}$, $m_{ST} = 2 \therefore UT \perp ST$
 (Also need to show $SR = UT$.)
 $\therefore RSTU$ is a rectangle
9 $\ell = -\frac{16}{3}$, $m = \frac{80}{3}$
10 a $y = -\frac{1}{2}x + \frac{11}{2}$ **b** $B(1, 5)$ **c** $C(2, 7)$

Exercise 2F

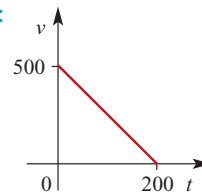
- 1** $m = 5$
2 $c = 5$
3 a $y = -\frac{1}{m}x + 3$ **b** $m = \frac{1}{7}$
4 $m = 2$
5 a $x = \frac{3}{m}$ **b** $m = \frac{9}{5}$ **c** $m \geq 3$
d $y = -\frac{x}{m} - 3$
6 a $x = -\frac{c}{2}$ **b** $c = -4$ **c** $c \geq -2$
d $y = -\frac{1}{2}x + c$

- 7 a** $x = 4a$ **b** $m = \frac{12}{a}$ **c i** $a = 6$ **ii** $a = -6$
8 a $x = \frac{c}{2}$ **b** $c = 9$ **c** $c \leq 2$ **d** $y = \frac{1}{2}x + c$
e i $c = 12$ **ii** $c = 4$ **iii** $c = 8$
9 a $\frac{12}{b}$ **b** $-\frac{3}{b}$ **c i** $b = -3$ **ii** $b = \frac{3}{2}$
d $y = \frac{b}{3}x - \frac{4b}{3}$

Exercise 2G

- 1** $w = 20n + 350$ for $n \in \mathbb{N} \cup \{0\}$
2 a $d_1 = 50t$ **b** $d_2 = 80 - 40t$
c Gradient = 50 Gradient = -40

3 a $V = 5t$ **b** $V = 10 + 5t$
4 a $v = 500 - 2.5t$
b Domain $0 \leq t \leq 200$; Range $0 \leq v \leq 500$
c

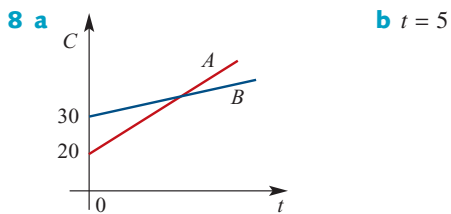


- 5** $C = 1.5n + 2.6$
6 a $C = 0.24x + 85$ **b** \$145
7 $d = 200 - 5t$
8 a

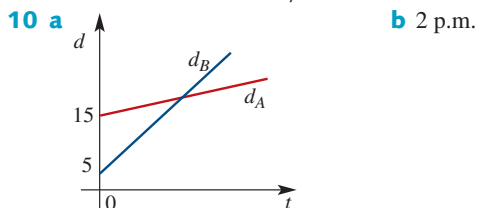
b $w = 0.2x + 50$ **c** $x = 12.5$ cm
9 a $C = 0.06n - 1$ **b** \$59
10 a $C = 5n + 175$ **b** Yes **c** \$175

Exercise 2H

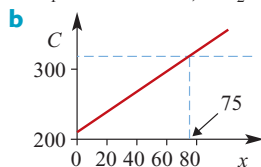
- 1** Both lines have gradient -1 , but their y -axis intercepts are 6 and $\frac{13}{2}$
2 $(t, 6 - t)$ where t is a real number
3 a $m = 4$ **b** $m \neq 4$ **c** $m = \frac{9}{5}$
4 $k = 2$, $m = 5$
5 $k = 24$, $m = 0$
6 $m = -3$
7 a $m = -5$ **b** $m = 3$



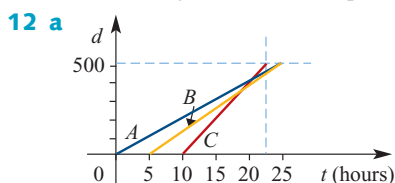
9 $b = 0.28$ and $a = 0.3$, $\frac{25}{7}$ m/s



11 a $C_1 = 210 + 1.6x$, $C_2 = 330$



c Fixed-charge method is cheaper for $x > 75$



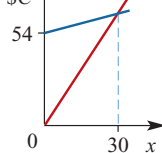
b C wins the race

d C leaves 5 hours after B, and overtakes B $13\frac{1}{2}$ hours after B had started, then overtakes A 20 hours after A had started. C wins with a total time of $22\frac{1}{2}$ hours ($12\frac{1}{2}$ hours for journey + 10 hours handicap), with A and B deadheating for second place, each with a total time of 25 hours.

13 Both craft will pass over the point $(5\frac{1}{3}, -4)$

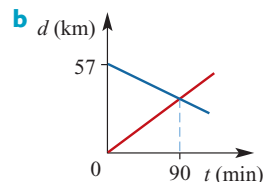
14 a $C_T = 2.8x$, $C_B = 54 + x$

b \$C **c** More than 30



15 a $d_A = \frac{1}{3}t$

$d_M = 57 - \frac{3}{10}t$



c 10:30 a.m. **d** Anne 30 km, Maureen 27 km

Chapter 2 review

Technology-free questions

1 a Midpoint = $(3, 2)$; Length = 4

b Midpoint = $(-\frac{1}{2}, -\frac{9}{2})$; Length = $\sqrt{74}$

c Midpoint = $(5, \frac{5}{2})$; Length = 5

2 a $\frac{9}{4}$ **b** $-\frac{10}{11}$ **c** Undefined

d -1 **e** $\frac{b}{a}$ **f** $\frac{-b}{a}$

3 a $y = 4x$ **b** $y = 4x + 5$

c $y = 4x + 2$ **d** $y = 4x - 5$

4 a $a = -2$ **b** $\frac{20}{3}$

5 $4y + 3x = -7$

6 $3y + 2x = -5$

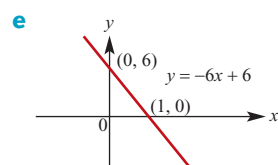
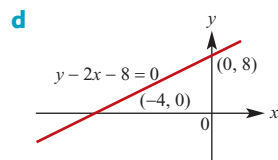
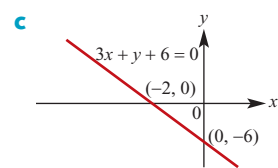
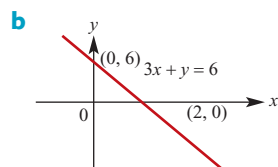
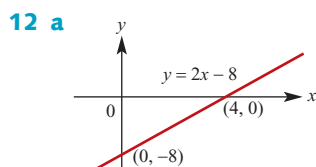
7 a $y = 11$ **b** $y = 6x - 10$ **c** $3y + 2x = -3$

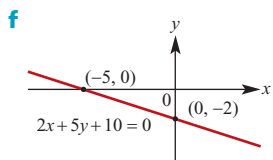
8 $\sqrt{3}y - x = 3\sqrt{3} - 2$

9 $y + x = 1$

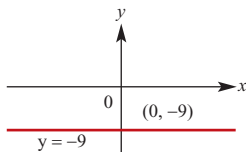
10 $y = \frac{1}{3}x + \frac{20}{3}$

11 $a = 1$, $b = -\frac{1}{2}$, $d = 5$, $e = 3$

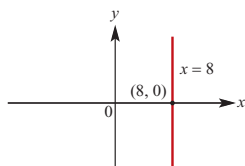




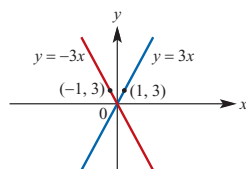
13 a $y = -9$



b $x = 8$



c i $y = 3x$ **ii** $y = -3x$



14 a $d = 60t$ **b** $m = 60$

15 $S = 800 + 500n$

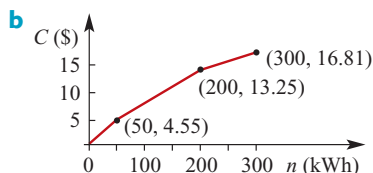
16 a $y = 2x + 2$ **b i** $\frac{-2}{a}$ **ii** $-2 < a < 0$
c $\left(\frac{1}{a-1}, \frac{1}{a-1} + 3\right)$

Multiple-choice questions

- 1** D **2** E **3** A **4** E **5** C **6** D
7 B **8** E **9** C **10** E **11** C

Extended-response questions

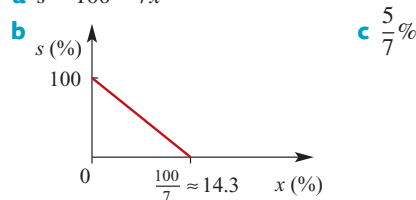
- 1 a** $C = 550 + 190n$ **b** 12 days
c Fewer than 5 days
2 a Cost of the plug
b Cost per metre of cable **c** 1.8 **d** $11\frac{1}{9}$ m
3 a The maximum profit (when $x = 0$)
b 43 seats
c The profit reduces by \$24 for each empty seat
4 a i $C = 0.091n$
ii $C = 1.65 + 0.058n$
iii $C = 6.13 + 0.0356n$



- i** For 30 kWh, $C = 2.73$
ii For 90 kWh, $C = 6.87$
iii For 300 kWh, $C = 16.81$
c 389.61 kWh

5 a $y = -\frac{7}{3}x + 14\frac{2}{3}$ **b** $20\frac{1}{3}$ km south

6 a $s = 100 - 7x$



d $14\frac{2}{7}\%$

e Probably not a realistic model when $s = 0$

f $0 \leq x \leq 14\frac{2}{7}$

7 a $AB, y = x + 2; CD, y = 2x - 6$

b Intersection is at (8, 10), on the near bank

8 a $\frac{128}{19}$

b $y = -\frac{199}{190}x + \frac{128}{19}$

c No, since gradient of AB is $\frac{20}{19} \approx 1.053$, whereas the gradient of VC is -1.047

9 a No **b** $1\frac{41}{71}$ km to the east of H

10 a $y = x - 38$

b $B(56, 18)$

c $y = -2x + 166$

d (78, 10)

11 a $y = 3x + 2$

b (0, 2)

c $y = 3x - 8$

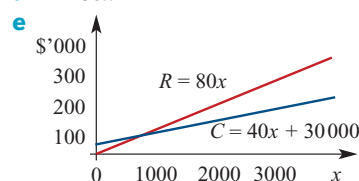
d (2, -2)

e Area = 10 square units

f Area = 40 square units

12 a $C = 40x + 30\,000$ **b** \$45 **c** 5000

d $R = 80x$



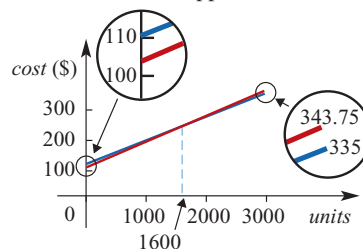
f 751 **g** $P = 40x - 30\,000$

13 a Cost with method 1 is \$226.75; cost with method 2 is \$227; so method 1 is cheaper

b

	0	1000	2000	3000
Method 1	100	181.25	262.50	343.75
Method 2	110	185	260	335

c Cost the same for approx. 1600 units



d $C_1 = 0.08125x + 100$, $C_2 = 0.075x + 110$, $x = 1600$

- 14 a** (17, 12) **b** $3y = 2x + 2$
15 a $PD: y = \frac{2}{3}x + 120$; $DC: y = \frac{2}{5}x + 136$;
 $CB: y = -\frac{5}{2}x + 600$; $AB: y = \frac{2}{5}x + 20$;
 $AP: y = -\frac{3}{5}x + 120$
b At B and C, since product of gradients is -1
 E.g. $m_{DC} = \frac{2}{5}$, $m_{CB} = -\frac{5}{2}$, $m_{DC} \times m_{CB} = -1$

Chapter 3

Exercise 3A

- 1 a** $2x - 8$ **b** $-2x + 8$ **c** $6x - 12$
d $-12 + 6x$ **e** $x^2 - x$ **f** $2x^2 - 10x$
2 a $6x + 1$ **b** $3x - 6$ **c** $x + 1$ **d** $5x - 3$
3 a $14x - 32$ **b** $2x^2 - 11x$
c $32 - 16x$ **d** $6x - 11$
4 a $2x^2 - 11x$ **b** $3x^2 - 15x$
c $-20x - 6x^2$ **d** $6x - 9x^2 + 6x^3$
e $2x^2 - x$ **f** $6x - 6$
5 a $6x^2 - 2x - 28$ **b** $x^2 - 22x + 120$
c $36x^2 - 4$ **d** $8x^2 - 22x + 15$
e $x^2 - (\sqrt{3} + 2)x + 2\sqrt{3}$
f $2x^2 + \sqrt{5}x - 5$ **g** $3x^2 + \sqrt{7}x - 14$
h $5x^2 + (10\sqrt{2} - 3)x - 6\sqrt{2}$
i $5x^2 - (3\sqrt{5} + 32\sqrt{10})x + 96\sqrt{2}$
6 a $6x^3 - 5x^2 - 14x + 12$ **b** $x^3 - 1$
c $24 - 20x - 8x^2 + 6x^3$ **d** $3x^2 + 4x + 3$
e $-10x^2 + 5x - 2$
7 a $x^2 - 8x + 16$ **b** $4x^2 - 12x + 9$
c $36 - 24x + 4x^2$ **d** $x^2 - x + \frac{1}{4}$
e $x^2 - 2\sqrt{5}x + 5$ **f** $x^2 - 4\sqrt{3}x + 12$
8 a $x^2 - 9$ **b** $4x^2 - 16$ **c** $81x^2 - 121$
d $4x^2 - 9$ **e** $4x^2 - 25$ **f** $x^2 - 5$
g $4x^2 - 27$ **h** $3x^2 - 7$
9 a $x^2 + y^2 - z^2 - 2xy$ **b** $4a^2 - 4ab + b^2 - c^2$
c $9w^2 + 8uz - 16z^2 - u^2$
d $4a^2 - 5b^2 + 4ac + c^2$
10 a i $x^2 + 2x + 1$ **ii** $(x + 1)^2$
b i $(x - 1)^2 + 2(x - 1) + 1$ **ii** x^2

Exercise 3B

- 1 a** $2(x + 2)$ **b** $4(a - 2)$ **c** $3(2 - x)$
d $2(x - 5)$ **e** $6(3x + 2)$ **f** $8(3 - 2x)$
2 a $2x(2x - y)$ **b** $8x(a + 4y)$ **c** $6b(a - 2)$
d $2xy(3 + 7x)$ **e** $x(x + 2)$ **f** $5x(x - 3)$
g $-4x(x + 4)$ **h** $7x(1 + 7x)$ **i** $x(2 - x)$
3 a $6x^2y^2(x + 2)$ **b** $xy(7x - 6y)$
c $2xy^2(4x + 3)$
4 a $(x^2 + 1)(x + 5)$ **b** $(x + 3)(y + 2)$

- c** $(x - 1)(x + 1)(y - 1)(y + 1)$
d $(a + b)(x + y)$ **e** $(a^2 + 1)(a - 3)$
f $(2a - 5)(b - 6)$ **g** $(2x + 5)(x - 1)$
h $(x + 2)(x - 2)(x + 2)$
i $(x - a)(x + a)(x - b)$
5 a $(x - 6)(x + 6)$ **b** $(x - 9)(x + 9)$
c $(x - a)(x + a)$ **d** $(2x - 9)(2x + 9)$
e $(3x - 4)(3x + 4)$ **f** $(5x - y)(5x + y)$
g $3(x - 4)(x + 4)$ **h** $2(x - 7)(x + 7)$
i $3a(x - 3)(x + 3)$ **j** $(a - \sqrt{7})(a + \sqrt{7})$
k $(\sqrt{2}a - \sqrt{5})(\sqrt{2}a + \sqrt{5})$
l $(x - 2\sqrt{3})(x + 2\sqrt{3})$

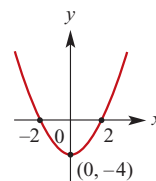
- 6 a** $(x - 6)(x + 2)$ **b** $(7 + x)(3 - x)$
c $3(x - 1)(x + 3)$ **d** $-5(2x + 1)$
e $-24x$ **f** $-5(x + 7)(x + 1)$
7 a $(x - 9)(x + 2)$ **b** $(y - 16)(y - 3)$
c $(a - 2)(a - 12)$ **d** $(a + 9)^2$
e $(x - 8)(x + 3)$ **f** $(x - 12)(x + 10)$
8 a $(3x - 1)(x - 2)$ **b** $(2x + 1)(3x + 2)$
c $(5x + 3)(x + 4)$ **d** $(2x + 1)(x + 4)$
e $(3x - 2)(2x - 5)$ **f** $(3x + 1)(2x - 3)$
g $(3x - 2)(4x - 3)$ **h** $(x - 2)(5x + 6)$
i $x(5x - 6)(x - 2)$
9 a $3(y - 6)(y + 2)$ **b** $2(x - 7)(x - 2)$
c $4(x - 3)(x - 6)$ **d** $3(x + 2)(x + 3)$
e $a(x + 3)(x + 4)$ **f** $3x(4 - x)^2$
10 a $x(x + 2)$ **b** $(2x - 3)(x + 2)$
c $2(2x + 5)(x + 2)$

Exercise 3C

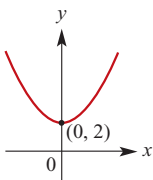
- 1 a** 2 or 3 **b** 0 or 2 **c** 4 or 3 **d** 4 or 3
e 3 or -4 **f** 0 or 1 **g** $\frac{5}{2}$ or 6 **h** -4 or 4
2 a -0.65, 4.65 **b** -0.58, 2.58 **c** -2.58, 0.58
3 a 9, -8 **b** 4, 2 **c** 11, -3 **d** 4, -16
e 2, -7 **f** -3, 8
4 a $-\frac{3}{2}, -1$ **b** $\frac{1}{2}, \frac{3}{2}$ **c** $-\frac{2}{3}, -\frac{3}{2}$ **d** $-\frac{3}{2}, 2$
e $\frac{5}{6}, 3$ **f** $-\frac{3}{2}, 3$ **g** $\frac{1}{2}, \frac{3}{5}$ **h** $-\frac{3}{4}, \frac{2}{3}$
i $\frac{1}{2}$ **j** -5, 1 **k** 0, 3 **l** -5, -3
m $\frac{1}{5}, 2$
5 3 **6** 4 or 9 **7** $2, 2\frac{3}{8}$
8 13 **9** 50 **10** 6 cm, 2 cm
11 5 **12** \$90, \$60 **13** 42

Exercise 3D

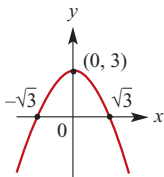
- 1 a i** (0, -4)
ii $x = 0$
iii (-2, 0), (2, 0)



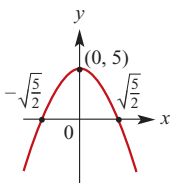
- b** i (0, 2)
 ii $x = 0$
 iii None



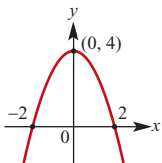
- c** i (0, 3)
 ii $x = 0$
 iii $(-\sqrt{3}, 0), (\sqrt{3}, 0)$



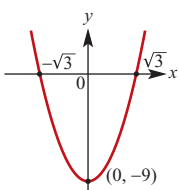
- d** i (0, 5)
 ii $x = 0$
 iii $(-\sqrt{\frac{5}{2}}, 0), (\sqrt{\frac{5}{2}}, 0)$



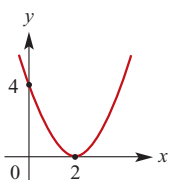
- e** i (0, 4)
 ii $x = 0$
 iii $(-2, 0), (2, 0)$



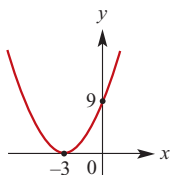
- f** i (0, -9)
 ii $x = 0$
 iii $(-\sqrt{3}, 0), (\sqrt{3}, 0)$



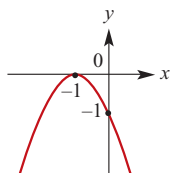
- 2 a** i (2, 0)
 ii $x = 2$
 iii (2, 0)



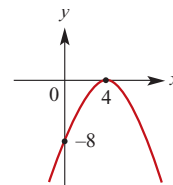
- b** i (-3, 0)
 ii $x = -3$
 iii (-3, 0)



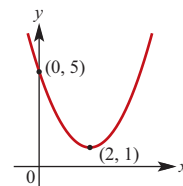
- c** i (-1, 0)
 ii $x = -1$
 iii (-1, 0)



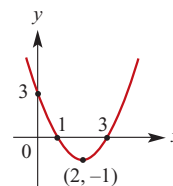
- d** i (4, 0)
 ii $x = 4$
 iii (4, 0)



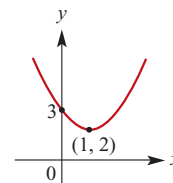
- 3 a** i (2, 1)
 ii $x = 2$
 iii None



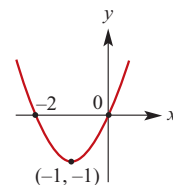
- b** i (2, -1)
 ii $x = 2$
 iii (1, 0), (3, 0)



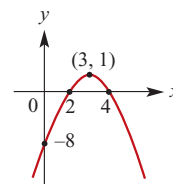
- c** i (1, 2)
 ii $x = 1$
 iii None



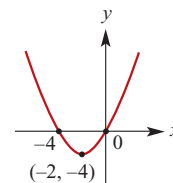
- d** i (-1, -1)
 ii $x = -1$
 iii $(-2, 0), (0, 0)$



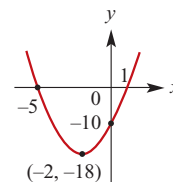
- e** i (3, 1)
 ii $x = 3$
 iii (2, 0), (4, 0)



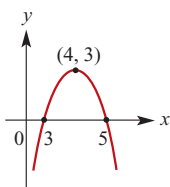
- f** i (-2, -4)
 ii $x = -2$
 iii $(-4, 0), (0, 0)$



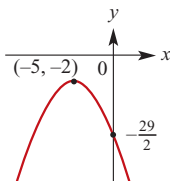
- g** i (-2, -18)
 ii $x = -2$
 iii $(-5, 0), (1, 0)$



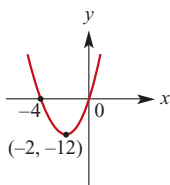
- h** i (4, 3)
 ii $x = 4$
 iii (3, 0), (5, 0)



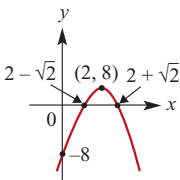
- i** i (-5, -2)
 ii $x = -5$
 iii None



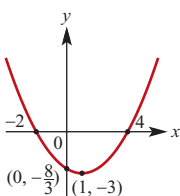
- j** i (-2, -12)
 ii $x = -2$
 iii (0, 0), (-4, 0)



- k** i (2, 8)
 ii $x = 2$
 iii $(2 - \sqrt{2}, 0)$,
 $(2 + \sqrt{2}, 0)$



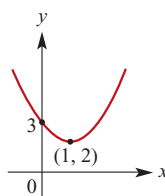
- l** i (1, -3)
 ii $x = 1$
 iii (-2, 0), (4, 0)



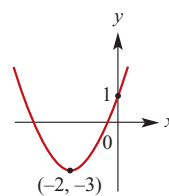
Exercise 3E

- 1** a $x^2 - 2x + 1$ b $x^2 + 4x + 4$
 c $x^2 - 6x + 9$ d $x^2 - 6x + 9$
 e $x^2 + 4x + 4$ f $x^2 - 10x + 25$
 g $x^2 - x + \frac{1}{4}$ h $x^2 - 3x + \frac{9}{4}$
- 2** a $(x - 2)^2$ b $(x - 6)^2$ c $-(x - 2)^2$
 d $2(x - 2)^2$ e $-2(x - 3)^2$ f $(x - \frac{1}{2})^2$
 g $(x - \frac{3}{2})^2$ h $(x + \frac{5}{2})^2$
- 3** a $1 \pm \sqrt{2}$ b $2 \pm \sqrt{6}$ c $3 \pm \sqrt{7}$
 d $\frac{5 \pm \sqrt{17}}{2}$ e $\frac{2 \pm \sqrt{2}}{2}$ f $-\frac{1}{3}, 2$
 g $-1 \pm \sqrt{1 - k}$ h $\frac{-1 \pm \sqrt{1 - k^2}}{k}$
 i $\frac{3k \pm \sqrt{9k^2 - 4}}{2}$

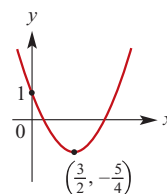
4 a $y = (x - 1)^2 + 2$
 t. pt (1, 2)



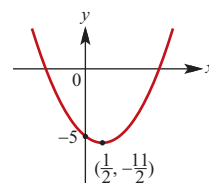
b $y = (x + 2)^2 - 3$
 t. pt (-2, -3)



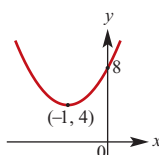
c $y = (x - \frac{3}{2})^2 - \frac{5}{4}$
 t. pt $(\frac{3}{2}, -\frac{5}{4})$



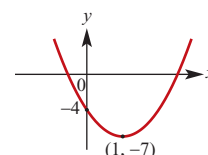
5 a $y = 2(x - \frac{1}{2})^2 - \frac{11}{2}$
 t. pt $(\frac{1}{2}, -\frac{11}{2})$



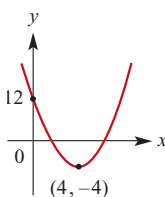
b $y = 4(x + 1)^2 + 4$
 t. pt (-1, 4)



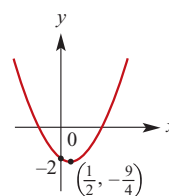
c $y = 3(x - 1)^2 - 7$
 t. pt (1, -7)



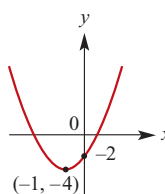
6 a $y = (x - 4)^2 - 4$
 t. pt (4, -4)



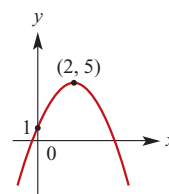
b $y = (x - \frac{1}{2})^2 - \frac{9}{4}$
 t. pt $(\frac{1}{2}, -\frac{9}{4})$



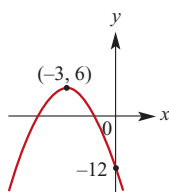
c $y = 2(x + 1)^2 - 4$
 t. pt (-1, -4)



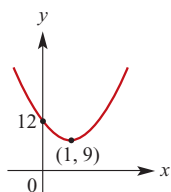
d $y = -(x - 2)^2 + 5$
 t. pt (2, 5)



e $y = -2(x+3)^2 + 6$
t. pt $(-3, 6)$



f $y = 3(x-1)^2 + 9$
t. pt $(1, 9)$

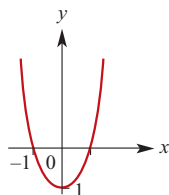


Exercise 3F

1 a 7 **b** 7

2 a -2 **b** 8

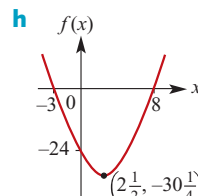
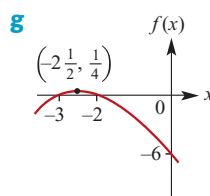
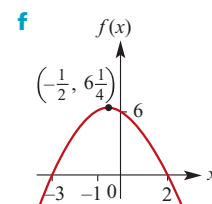
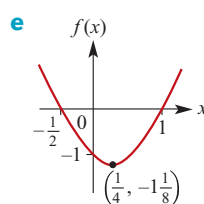
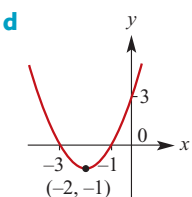
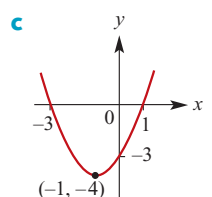
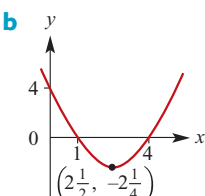
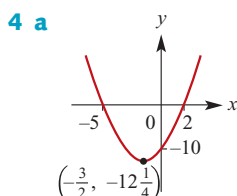
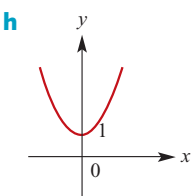
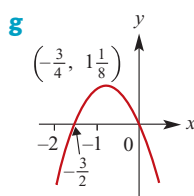
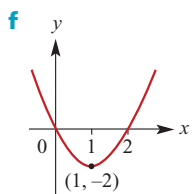
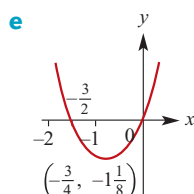
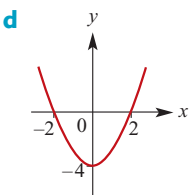
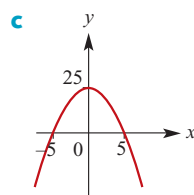
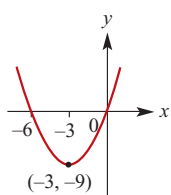
3 a



c 1

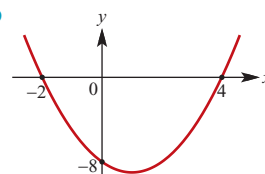
c 4

b



Exercise 3G

1 a -2, 4 **b**



c $-2 \leq x \leq 4$

d $x < -2$ or $x > 4$

2 a $x \leq -2$ or $x \geq 3$

b $-4 < x < -3$

c $-4 \leq x \leq \frac{1}{2}$

d $x < 2$ or $x > 6$

e $2 < x < 3$

f $\frac{3}{2} \leq x \leq \frac{7}{2}$

g $-\frac{7}{2} < x < 2$

h $-2 \leq x \leq \frac{5}{2}$

i $x < -5$ or $x > \frac{5}{2}$

j $-2 \leq x \leq \frac{7}{2}$

k $x < \frac{2}{5}$ or $x > \frac{7}{2}$

l $x \leq \frac{5}{2}$ or $x \geq \frac{11}{2}$

3 a $x < -5$ or $x > 5$

b $-\frac{2}{3} \leq y \leq \frac{2}{3}$

c $y < -4$ or $y > 4$

d $-\frac{6}{5} \leq x \leq \frac{6}{5}$

e $y \leq -\frac{1}{4}$ or $y \geq \frac{1}{4}$

f $y < -\frac{5}{6}$ or $y > \frac{5}{6}$

4 a $x \geq 2$ or $x \leq -4$

b $-3 < x < 8$

c $-2 \leq x \leq 6$

d $x > 3$ or $x < -\frac{3}{2}$

e $-\frac{3}{2} < x < -\frac{2}{3}$

f $-3 \leq x \leq -2$

g $x > \frac{2}{3}$ or $x < -\frac{3}{4}$

h $\frac{1}{2} \leq x \leq \frac{3}{5}$

i $-4 \leq x \leq 5$

j $\frac{1}{2}(5 - \sqrt{41}) \leq p \leq \frac{1}{2}(5 + \sqrt{41})$

k $y < -1$ or $y > 3$

l $x \leq -2$ or $x \geq -1$

5 a $x \leq \frac{-3 - \sqrt{29}}{2}$ or $x \geq \frac{-3 + \sqrt{29}}{2}$

b $\frac{5 - \sqrt{17}}{2} < x < \frac{5 + \sqrt{17}}{2}$

c $\frac{3 - \sqrt{17}}{4} \leq x \leq \frac{3 + \sqrt{17}}{4}$

d $\frac{-3 - \sqrt{41}}{2} < x < \frac{-3 + \sqrt{41}}{2}$

e $\frac{-7 - \sqrt{41}}{4} < x < \frac{-7 + \sqrt{41}}{4}$

f $x \leq \frac{4 - \sqrt{6}}{2}$ or $x \geq \frac{4 + \sqrt{6}}{2}$

6 The square of any number is greater than or equal to zero.

7 The negative of the square of any number is less than or equal to zero.

8 $x^2 + 2x + 7 = (x + 1)^2 + 6$. For all x , we have $(x + 1)^2 \geq 0$ and so $(x + 1)^2 + 6 \geq 6$

9 $-x^2 - 2x - 7 = -(x + 1)^2 - 6$. For all x , we have $-(x + 1)^2 \leq 0$ and so $-(x + 1)^2 - 6 \leq -6$

Exercise 3H

1 a i 40 **ii** $2\sqrt{10}$

b i 28 **ii** $2\sqrt{7}$

c i 172 **ii** $2\sqrt{43}$

d i 96 **ii** $4\sqrt{6}$

e i 189 **ii** $3\sqrt{21}$

2 a $1 + \sqrt{5}$ **b** $\frac{3 - \sqrt{5}}{2}$ **c** $\frac{1 + \sqrt{5}}{2}$ **d** $1 + 2\sqrt{2}$

3 a $-3 \pm \sqrt{13}$ **b** $\frac{7 \pm \sqrt{61}}{2}$ **c** $\frac{1}{2}, 2$

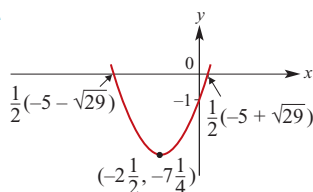
d $-1 \pm \frac{3}{2}\sqrt{2}$ **e** $-2 \pm \frac{3}{2}\sqrt{2}$ **f** $1 \pm \frac{\sqrt{30}}{5}$

g $1 \pm \frac{\sqrt{2}}{2}$ **h** $1, \frac{-3}{2}$ **i** $\frac{-3 \pm \sqrt{6}}{5}$

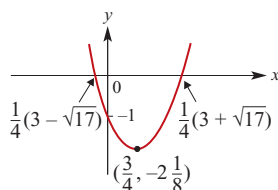
j $\frac{-13 \pm \sqrt{145}}{12}$ **k** $\frac{2 \pm \sqrt{4 - 2k^2}}{2k}$

l $\frac{2k \pm \sqrt{6k^2 - 2k}}{2(1 - k)}$

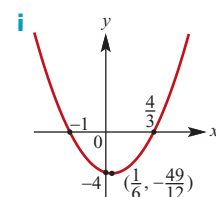
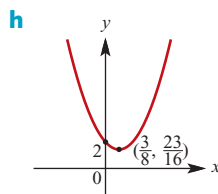
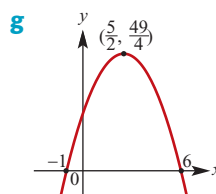
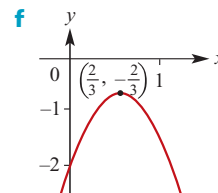
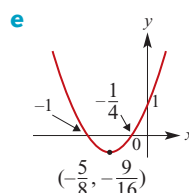
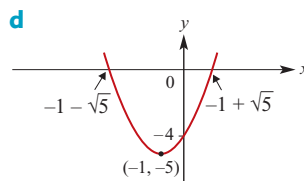
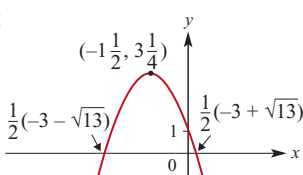
4 a



b



c



Exercise 3I

1 a 20 **b** -12 **c** 25 **d** 41 **e** 41

2 a Crosses the x -axis **b** Does not cross

c Just touches the x -axis

d Crosses the x -axis **e** Does not cross

f Does not cross

3 a Two real solutions **b** No real solutions

c Two real solutions **d** Two real solutions

e Two real solutions **f** No real solutions

4 a $\Delta = 0$, one rational solution

b $\Delta = 1$, two rational solutions

c $\Delta = 17$, two irrational solutions

d $\Delta = 0$, one rational solution

e $\Delta = 57$, two irrational solutions

f $\Delta = 1$, two rational solutions

5 a i $-\sqrt{5} < m < \sqrt{5}$ **ii** $m = \pm\sqrt{5}$

iii $m > \sqrt{5}$ or $m < -\sqrt{5}$

b i $0 < m < \frac{4}{3}$ **ii** $m = \frac{4}{3}$

iii $m > \frac{4}{3}$ or $m < 0$

c i $-\frac{4}{5} < m < 0$ **ii** $m = 0$ or $m = -\frac{4}{5}$

iii $m < -\frac{4}{5}$ or $m > 0$

d i $-2 < m < 1$ **ii** $m = -2$ or $m = 1$

iii $m > 1$ or $m < -2$

$$6 \Delta = (2m - n)^2, \text{ a perfect square} \quad 7 p > \frac{4}{3}$$

$$8 p = \frac{-1}{2}$$

$$9 \text{ a } p = \pm 3 \quad \text{b } p > 1 \quad \text{c } p > \frac{2}{3} \quad \text{d } p > 1$$

$$10 -2 < p < 8$$

$$11 \Delta = -4q^2 < 0 \text{ for all values of } q$$

$$12 \text{ a } \Delta = 16m^2 - 96m + 176 = 16(m - 3)^2 + 32$$

$$\text{b } \Delta \geq 32; \text{ therefore two solutions}$$

$$13 \text{ a } \Delta = 16$$

$$\text{b } \Delta > 0; \text{ therefore two solutions}$$

$$14 \Delta = (m + 4)^2, \text{ a perfect square; therefore rational solutions}$$

$$15 \Delta = (m - 2n)^2, \text{ a perfect square; therefore rational solutions}$$

$$16 \text{ The graph will cross the } x\text{-axis twice}$$

$$17 \text{ The graph will cross the } x\text{-axis twice}$$

Exercise 3J

$$1 \text{ a } (1 - \sqrt{5}, -1 - \sqrt{5}), (1 + \sqrt{5}, -1 + \sqrt{5})$$

$$\text{b } (-3, 9), (2, 4) \quad \text{c } (-3, 9), \left(\frac{7}{4}, \frac{49}{16}\right)$$

$$\text{d } (1, 3), (2, 5)$$

$$2 \text{ a } (2, 0), (-5, 7) \quad \text{b } (1, -3), (4, 9)$$

$$\text{c } (1, -3), (-3, 1) \quad \text{d } (-1, 1), (-3, -3)$$

$$\text{e } \left(\frac{1 + \sqrt{33}}{2}, -3 - \sqrt{33}\right), \left(\frac{1 - \sqrt{33}}{2}, -3 + \sqrt{33}\right)$$

$$\text{f } \left(\frac{5 + \sqrt{33}}{2}, 23 + 3\sqrt{33}\right), \left(\frac{5 - \sqrt{33}}{2}, 23 - 3\sqrt{33}\right)$$

$$3 \text{ a } \text{Touch at } (2, 0) \quad \text{b } \text{Touch at } (3, 9)$$

$$\text{c } \text{Touch at } (-2, -4) \quad \text{d } \text{Touch at } (-4, -8)$$

$$4 \text{ a } x = 8, y = 16 \text{ and } x = -1, y = 7$$

$$\text{b } x = -\frac{16}{3}, y = 37\frac{1}{3} \text{ and } x = 2, y = 30$$

$$\text{c } x = \frac{4}{5}, y = 10\frac{2}{5} \text{ and } x = -3, y = 18$$

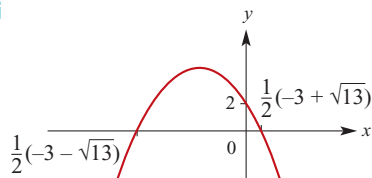
$$\text{d } x = 10\frac{2}{3}, y = 0 \text{ and } x = 1, y = 29$$

$$\text{e } x = 0, y = -12 \text{ and } x = \frac{3}{2}, y = -7\frac{1}{2}$$

$$\text{f } x = 1.14, y = 14.19 \text{ and } x = -1.68, y = 31.09$$

$$5 \text{ a } -13$$

$$\text{b } i$$



$$\text{ii } m = -6 \pm \sqrt{32} = -6 \pm 4\sqrt{2}$$

$$6 \text{ a } c = \frac{-1}{4} \quad \text{b } c > \frac{-1}{4}$$

$$7 \text{ a } 3 \text{ or } a = -1$$

$$8 \text{ b } 1$$

$$9 \text{ y } = (2 + 2\sqrt{3})x - 4 - 2\sqrt{3} \text{ and } y = (2 - 2\sqrt{3})x - 4 + 2\sqrt{3}$$

Exercise 3K

$$1 \text{ a } = -4, c = 6$$

$$2 \text{ a } \Delta = b^2 - 16a \quad \text{b } a = \frac{b^2}{16} \quad \text{c } a = \frac{1}{4}, b = 2$$

$$3 \text{ a } y = 2(x + 2)(x - 6) \quad \text{b } y = -2(x + 2)^2 + 4$$

$$\text{c } y = -x^2 + 2x - 3$$

$$4 \text{ 2}$$

$$5 \text{ a } = \frac{4}{7}, b = \frac{-24}{7}$$

$$6 \text{ a } = -2, b = 1, c = 6$$

$$7 \text{ a } y = -\frac{5}{16}x^2 + 5 \quad \text{b } y = x^2$$

$$\text{c } y = \frac{1}{11}x^2 + \frac{7}{11}x \quad \text{d } y = x^2 - 4x + 3$$

$$\text{e } y = -\frac{5}{4}x^2 - \frac{5}{2}x + \frac{15}{4} \quad \text{f } y = x^2 - 4x + 6$$

$$8 \text{ y } = \frac{5}{16}(x + 1)^2 + 3 \quad 9 \text{ y } = -\frac{1}{2}(x^2 - 3x - 18)$$

$$10 \text{ y } = (x + 1)^2 + 3 \quad 11 \text{ y } = \frac{1}{180}x^2 - x + 75$$

$$12 \text{ y } = 2x^2 - 4x \quad 13 \text{ y } = x^2 - 2x - 1$$

$$14 \text{ a } C \quad \text{b } B \quad \text{c } D \quad \text{d } A$$

$$15 \text{ a } y = a\left(x + \frac{1}{a}\right)^2 + a - \frac{1}{a} \quad \text{b } \left(-\frac{1}{a}, a - \frac{1}{a}\right)$$

$$\text{c } a = \pm 1 \quad \text{d } -1 < a < 1$$

$$16 \text{ y } = -2x^2 + 8x - 6$$

$$17 \text{ a } y = ax(x - 10), a > 0$$

$$\text{b } y = a(x + 4)(x - 10), a < 0$$

$$\text{c } y = \frac{1}{18}(x - 6)^2 + 6 \quad \text{d } y = a(x - 8)^2, a < 0$$

$$18 \text{ a } y = -\frac{1}{4}x^2 + x + 2 \quad \text{b } y = x^2 + x - 5$$

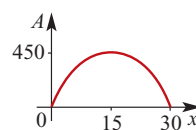
$$19 \text{ r } = -\frac{1}{8}t^2 + 2\frac{1}{2}t - 6\frac{3}{8} \quad 20 \text{ a } B \quad \text{b } D$$

$$21 \text{ a } y = -2x^2 - x + 5 \quad \text{b } y = 2x^2 - x - 5$$

$$\text{c } y = 2x^2 + \frac{5}{2}x - \frac{11}{2}$$

Exercise 3L

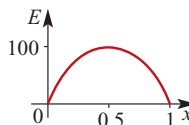
$$1 \text{ a } A = 60x - 2x^2 \quad \text{b } A$$



$$\text{c } \text{Maximum area} = 450 \text{ m}^2$$

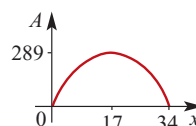
$$2 \text{ A } = x(10 - x); \text{ Maximum area} = 25 \text{ m}^2$$

$$3 \text{ a } E \quad \text{b } 0 \text{ and } 1 \quad \text{c } 0.5$$



$$\text{d } 0.23 \text{ and } 0.77$$

$$4 \text{ a } A = 34x - x^2 \quad \text{b } A$$

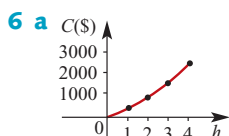


$$\text{c } 289 \text{ cm}^2$$

5 a $4x + 10y = 80$

b i $A = 1.64x^2 - 25.6x + 256$

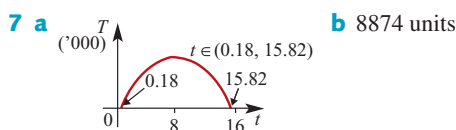
ii 31.22 and 48.78



The domain depends on the height of the alpine area. In Victoria, the highest mountain is approx. 2 km high and the minimum alpine height would be approx. 1 km. Thus, for Victoria, domain = $[1, 2]$.

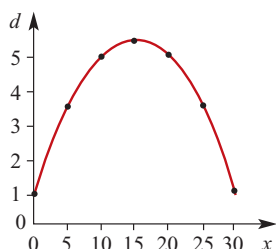
b Theoretically, no. But of course there is a practical maximum.

c \$1225



8 a

x	0	5	10	15	20	25	30
d	1	3.5	5	5.5	5	3.5	1



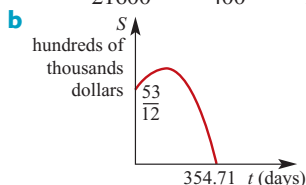
b i 5.5 m

ii $15 - 5\sqrt{7}$ m or $15 + 5\sqrt{7}$ m from the bat

iii 1 m above the ground

9 $a = -\frac{16}{15}$, $b = \frac{8}{5}$, $c = 0$

10 a $a = -\frac{7}{21600}$, $b = \frac{41}{400}$, $c = \frac{53}{12}$



c i $S = \$1\,236\,666$ ii $S = \$59\,259$

Chapter 3 review

Technology-free questions

1 a $\left(x + \frac{9}{2}\right)^2$ b $(x + 9)^2$ c $\left(x - \frac{2}{5}\right)^2$

d $(x + b)^2$ e $(3x - 1)^2$ f $(5x + 2)^2$

2 a $-3x + 6$ b $-ax + a^2$

c $49a^2 - b^2$ d $x^2 - x - 12$

e $2x^2 - 5x - 12$ f $x^2 - y^2$

g $a^3 - b^3$

i $3a^2 - 5a - 2$

k $2u + 2v - uv$

3 a $4(x - 2)$

c $3x(8a - 1)$

e $a(u + 2v + 3w)$

f $a^2(2b - 3a)(2b + 3a)$

g $(1 - 6ax)(1 + 6ax)$

i $(x + 2)(x - 1)$

k $(3x + 2)(2x + 1)$

m $(3x - 2)(x + 1)$

o $(3x - 2)(2x - 1)$

4 a $x = 5$ or $x = -3$

c $x = 2$ or $x = 3$

e $x = -3$ or $x = -2$

g $x = -\frac{1}{2}$ or $x = 3$

i $x = -\frac{12}{5}$ or $x = 1$

h $6x^2 + 8xy + 2y^2$

j $4xy$

l $-3x^2 + 15x - 12$

b $x(3x + 8)$

d $(2 - x)(2 + x)$

h $(x + 4)(x - 3)$

j $(2x - 1)(x + 2)$

l $(3x + 1)(x - 3)$

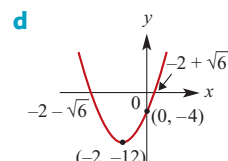
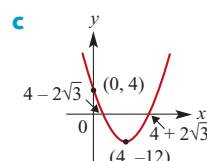
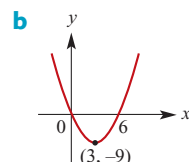
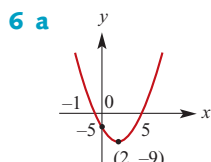
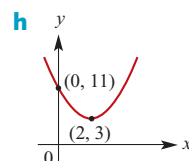
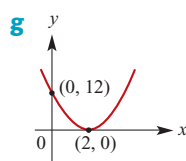
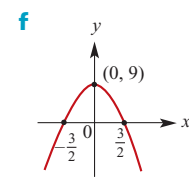
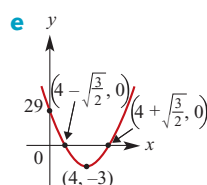
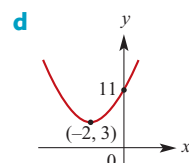
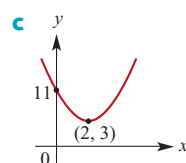
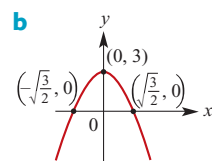
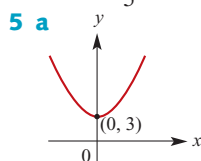
n $(3a - 2)(2a + 1)$

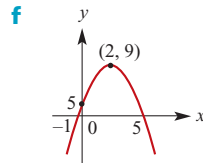
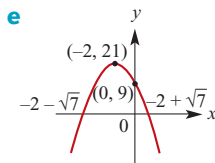
b $x = 9$ or $x = 0$

d $x = -1$ or $x = 25$

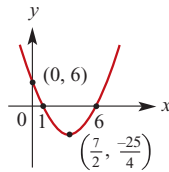
f $x = 6$

h $x = -\frac{5}{6}$ or $x = \frac{3}{2}$

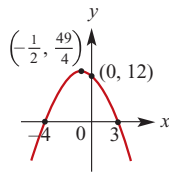




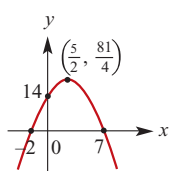
7 a $x = \frac{7}{2}$



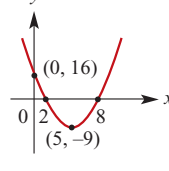
b $x = -\frac{1}{2}$



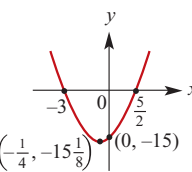
c $x = \frac{5}{2}$



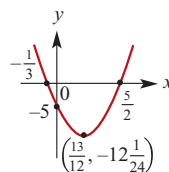
d $x = 5$



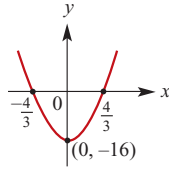
e $x = -\frac{1}{4}$



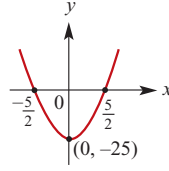
f $x = \frac{13}{12}$



g $x = 0$



h $x = 0$



8 $p = 1$ or $p = -\frac{3}{10}$

9 a $x < 0$ or $x > 1$

b $-2 - \sqrt{34} \leq x \leq -2 + \sqrt{34}$

c $-2 \leq x \leq \frac{1}{3}$

d $\frac{3}{2} \leq x \leq 5$

10 a $-3 + \sqrt{6}, -3 - \sqrt{6}$
b $\frac{-9 + \sqrt{33}}{2}, \frac{-9 - \sqrt{33}}{2}$

c $2 - \sqrt{2}, 2 + \sqrt{2}$

d $\frac{-7 - \sqrt{33}}{4}, \frac{-7 + \sqrt{33}}{4}$

e $\frac{-7 - \sqrt{17}}{4}, \frac{-7 + \sqrt{17}}{4}$

f $\frac{-9 - \sqrt{93}}{6}, \frac{-9 + \sqrt{93}}{6}$

11 $y = \frac{5}{3}x(x-5)$ **12** $y = 3(x-5)^2 + 2$

13 $m < -21 - 4\sqrt{29}$ or $m > -21 + 4\sqrt{29}$

14 225

15 $y = 5(x-1)^2 + 5$

16 a $(3, 9), (-1, 1)$

b $\left(\frac{4 - \sqrt{38}}{2}, 27 - 4\sqrt{38}\right), \left(\frac{4 + \sqrt{38}}{2}, 27 + 4\sqrt{38}\right)$

c $\left(\frac{-7 - \sqrt{73}}{6}, 2\right), \left(\frac{-7 + \sqrt{73}}{6}, 2\right)$

d $\left(\frac{1}{2}, \frac{1}{2}\right), (-2, 8)$

17 a $y = 2(x+4)(x-1)$ **b** $y = -2(x+1)^2 + 3$

c $y = 2x^2 - 2x - 3$

18 2.16 m

19 a $m = \pm\sqrt{8} = \pm 2\sqrt{2}$ **b** $m \leq -\sqrt{5}$ or $m \geq \sqrt{5}$

20 a $x = 0$ and $x = -b$ **b** $\left(-\frac{b}{2}, -\frac{b^2}{4}\right)$

c i $(0, 0), (1-b, 1-b)$ **ii** $b = 1$ **iii** $b \neq 1$

Multiple-choice questions

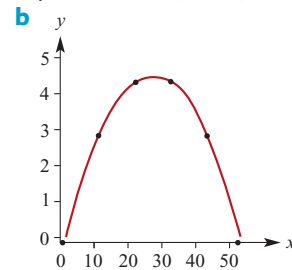
1 A **2** C **3** C **4** E **5** B

6 C **7** E **8** E **9** D **10** A

11 B **12** E **13** D

Extended-response questions

1 a $y = -0.0072x(x-50)$



c 10.57 m and 39.43 m $\left(25 \pm \frac{25\sqrt{3}}{3} \text{ m}\right)$

d 3.2832 m **e** 3.736 m (correct to 3 d.p.)

2 a Width of rectangle = $\frac{12-4x}{6}$ cm

Length of rectangle = $\frac{12-4x}{3}$ cm

b $A = \frac{17}{9}x^2 - \frac{16}{3}x + 8$

c Length for square = $\frac{96}{17} \approx 5.65$ cm and

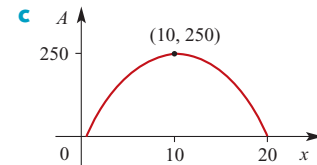
length for rectangle = $\frac{108}{17} \approx 6.35$ cm

3 a $V = 0.72x^2 - 1.2x$ **b** 22 hours

4 a $V = 10\,800x + 120x^2$

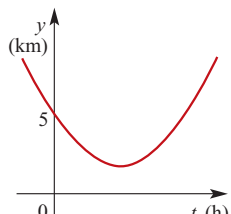
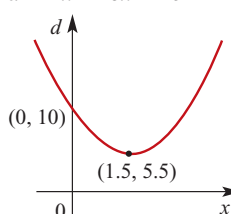
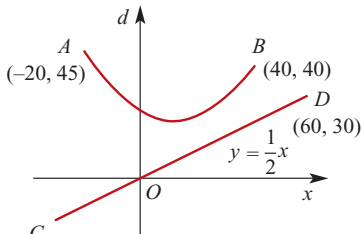
b $V = 46.6x^2 + 5000x$ **c** $\ell = 55.18$ m

5 a $\ell = 50 - \frac{5x}{2}$ **b** $A = 50x - \frac{5}{2}x^2$



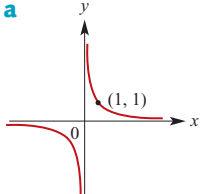
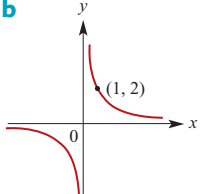
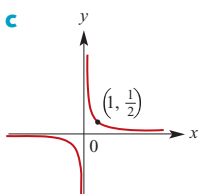
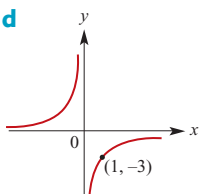
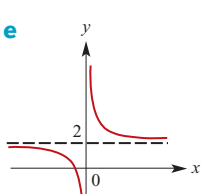
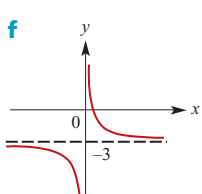
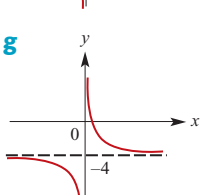
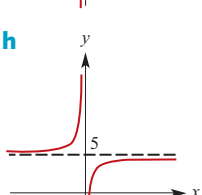
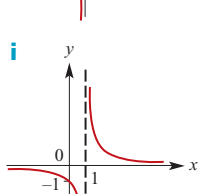
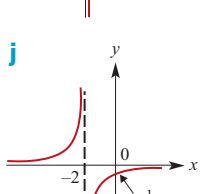
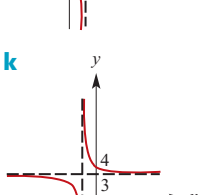
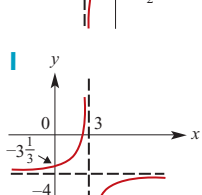
d Maximum area = 250 m² when $x = 10$ m

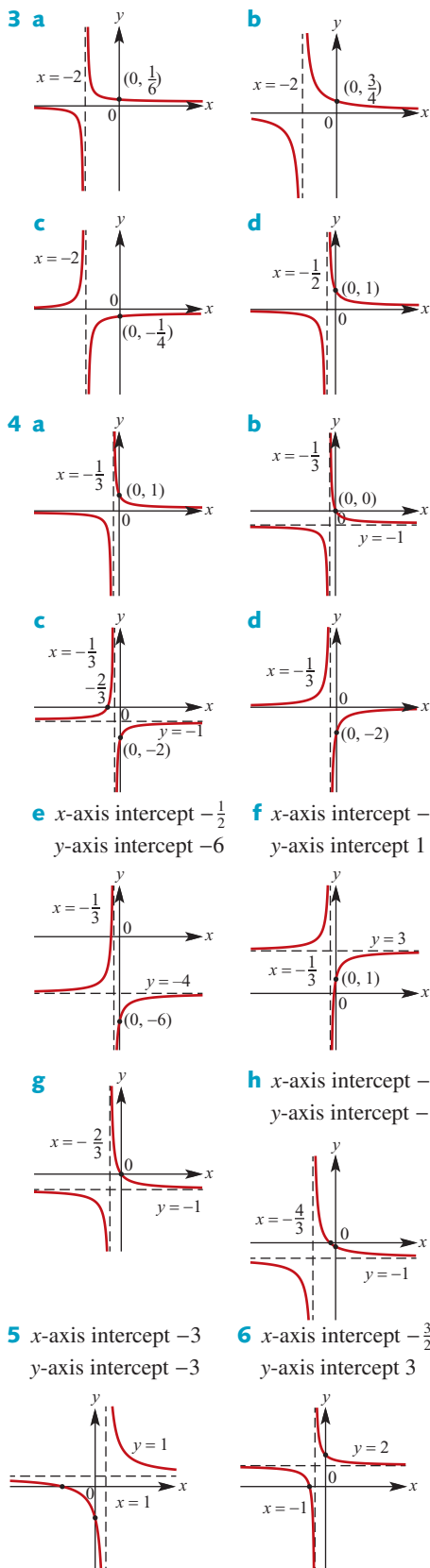
6 $x = \frac{-1 + \sqrt{5}}{2}$

- 7 a $\sqrt{25 + x^2}$
 b i $16 - x$ ii $\sqrt{x^2 - 32x + 265}$
 c 7.5 d 10.840 e 12.615
- 8 a i $y = \sqrt{64t^2 + 100(t - 0.5)^2}$
 $= \sqrt{164t^2 - 100t + 25}$
 ii 
 iii $t = \frac{1}{2}$, 1:30 p.m.; $t = \frac{9}{82}$, 1:07 p.m.
 iv 0.305; 1:18 p.m.; distance 3.123 km
- b i 0, $\frac{25}{41}$ ii $\frac{25 \pm 2\sqrt{269}}{82}$
- 9 b $2x + 2y = b$
 c $8x^2 - 4bx + b^2 - 16a^2 = 0$
 e i $x = 6 \pm \sqrt{14}$, $y = 6 \mp \sqrt{14}$
 ii $x = y = \sqrt{2a}$
 f $x = \frac{(5 \pm \sqrt{7})a}{4}$, $y = \frac{(5 \mp \sqrt{7})a}{4}$
- 10 a $b = -2$, $c = 4$, $h = 1$
 b i $(x, -6 + 4x - x^2)$ ii $(x, x - 1)$
 iii $(0, -1)$, $(1, 0)$, $(2, 1)$, $(3, 2)$, $(4, 3)$
 iv $y = x - 1$
 c i $d = 2x^2 - 6x + 10$
 ii 
 iii Min value of $d = 5.5$ when $x = 1.5$
- 11 a $45\sqrt{5}$
 b i $y = \frac{1}{600}(7x^2 - 190x + 20\,400)$
 ii $\left(\frac{190}{14}, \frac{5351}{168}\right)$
 c 
 d i The distance (measured parallel to the y-axis) between path and pond
 ii Minimum value = $\frac{473}{24}$ when $x = 35$

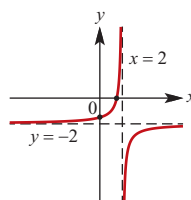
Chapter 4

Exercise 4A

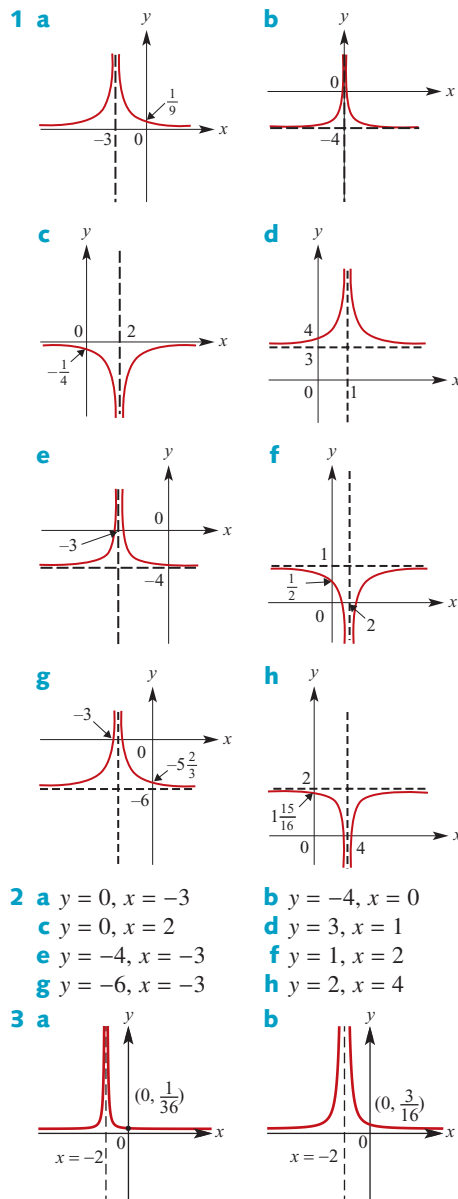
- 1 a  b 
- c  d 
- e  f 
- g  h 
- i  j 
- k  l 
- 2 a $y = 0$, $x = 0$ b $y = 0$, $x = 0$
 c $y = 0$, $x = 0$ d $y = 0$, $x = 0$
 e $y = 2$, $x = 0$ f $y = -3$, $x = 0$
 g $y = -4$, $x = 0$ h $y = 5$, $x = 0$
 i $y = 0$, $x = 1$ j $y = 0$, $x = -2$
 k $y = 3$, $x = -1$ l $y = -4$, $x = 3$

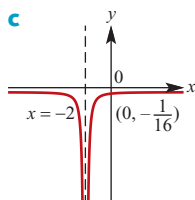


7 x -axis intercept $\frac{3}{2}$
 y -axis intercept $-\frac{3}{2}$



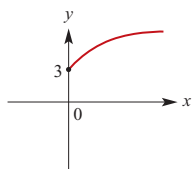
Exercise 4B



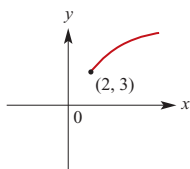


Exercise 4C

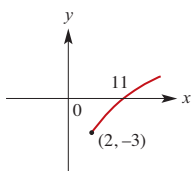
1 a $x \geq 0$ and $y \geq 3$



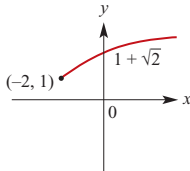
b $x \geq 2$ and $y \geq 3$



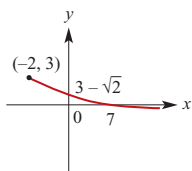
c $x \geq 2$ and $y \geq -3$



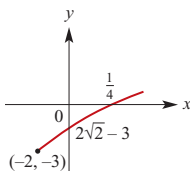
d $x \geq -2$ and $y \geq 1$



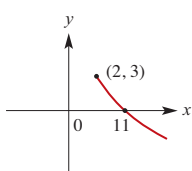
e $x \geq -2$ and $y \leq 3$



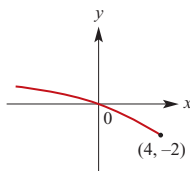
f $x \geq -2$ and $y \geq -3$



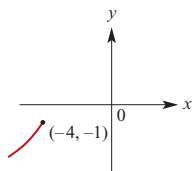
2 a $x \geq 2$ and $y \leq 3$



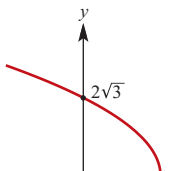
b $x \leq 4$ and $y \geq -2$



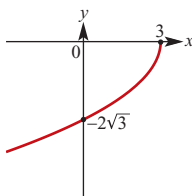
c $x \leq -4$ and $y \leq -1$



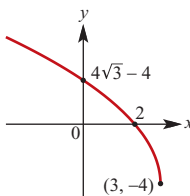
d $x \leq 3$ and $y \geq 0$



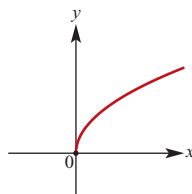
e $x \leq 3$ and $y \leq 0$



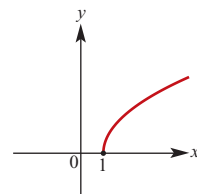
f $x \leq 3$ and $y \geq -4$



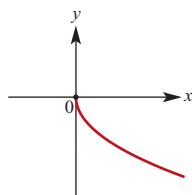
3 a $x \geq 0$ and $y \geq 0$



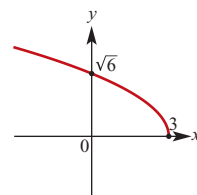
b $x \geq 1$ and $y \geq 0$



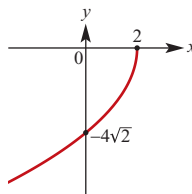
c $x \geq 0$ and $y \leq 0$



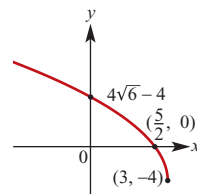
d $x \leq 3$ and $y \geq 0$



e $x \leq 2$ and $y \leq 0$



f $x \leq 3$ and $y \geq -4$



Exercise 4D

1 a $x^2 + y^2 = 9$

b $x^2 + y^2 = 16$

c $(x-1)^2 + (y-3)^2 = 25$

d $(x-2)^2 + (y+4)^2 = 9$

e $(x+3)^2 + (y-4)^2 = \frac{25}{4}$

f $(x+5)^2 + (y+6)^2 = (4.6)^2$

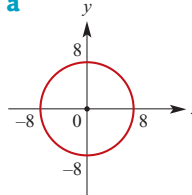
2 a $C(1, 3), r = 2$

b $C(2, -4), r = \sqrt{5}$

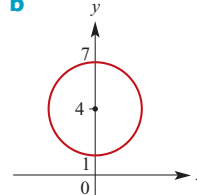
c $C(-3, 2), r = 3$

d $C(-5, 4), r = \sqrt{8}$

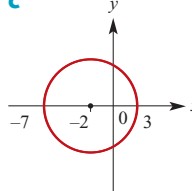
3 a



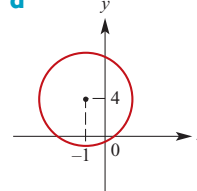
b



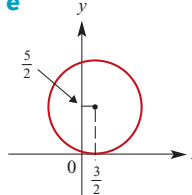
c



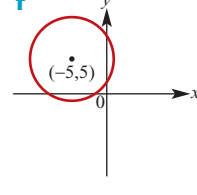
d



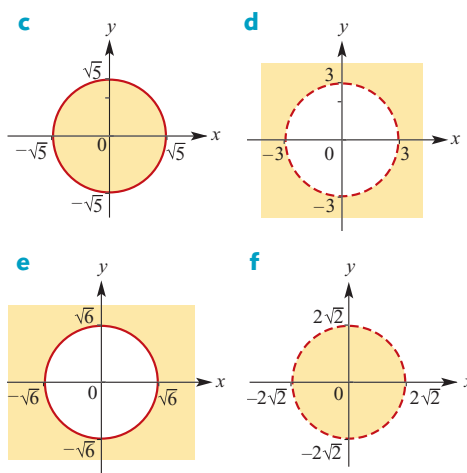
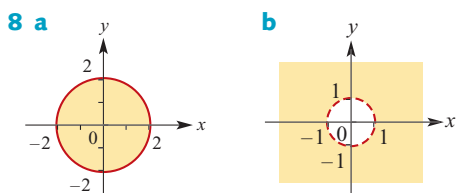
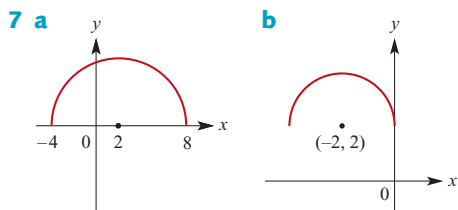
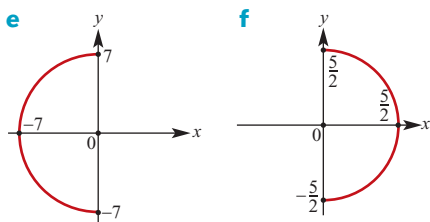
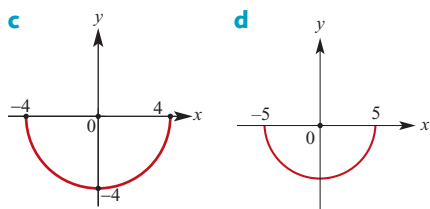
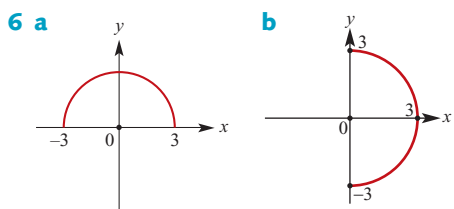
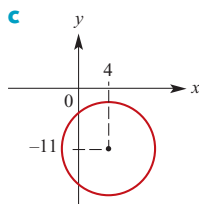
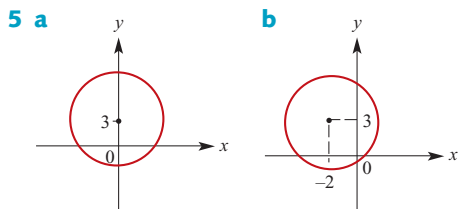
e



f



- 4 a** $C(0, 3), r = 5$ **b** $C(4, -6), r = \sqrt{42}$
c $C(3, -2), r = 2$ **d** $C(-2, 3), r = 5$
e $C(4, -2), r = \sqrt{19}$ **f** $C(\frac{1}{2}, -2), r = \frac{3}{2}$

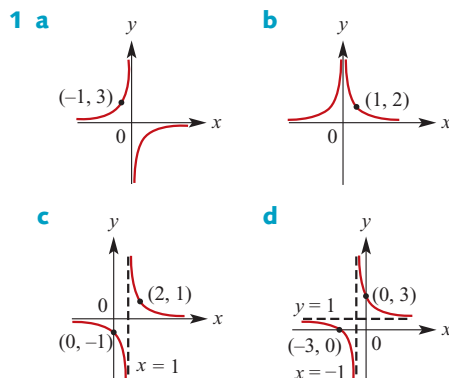


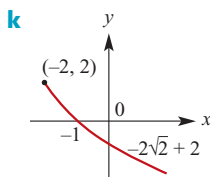
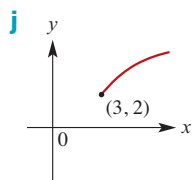
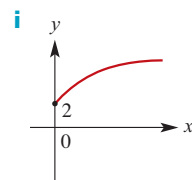
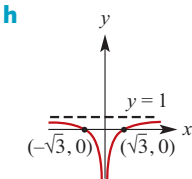
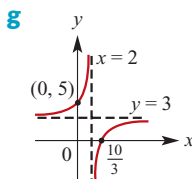
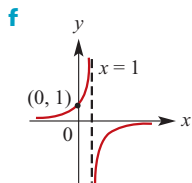
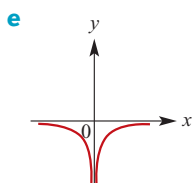
Exercise 4E

- 1** $a = 5$ **2** $a = -6, h = 3, k = 4$
3 $a = \frac{1}{2}, k = \frac{15}{2}$ **4** $a = -16, h = 2, k = -4$
5 $a = 4\sqrt{2}$ **6** $a = \frac{2\sqrt{3}}{3}, h = -2$
7 $(x - 2)^2 + (y - 1)^2 = 20$
8 $(x + 2)^2 + (y - 3)^2 = 1$
9 $(x + 2)^2 + (y - 3)^2 = 16$
10 $(x - 2)^2 + (y + 3)^2 = 9$
11 $(x - 4)^2 + (y - 4)^2 = 20$
12 $(x - 4)^2 + (y - 5)^2 = 25$ and $(x + 4)^2 + (y - 5)^2 = 25$
13 $(x + 1)^2 + (y + 1)^2 = 10$
14 a $(x - 2)^2 + (y + 2)^2 = 49$
b $y = 3\sqrt{x - 1} - 2$ **c** $y = \frac{1}{x - 2} + 2$
d $y = -\frac{2}{x - 1} - 2$ **e** $y = \sqrt{2 - x} + 1$
f $y = \frac{1}{(x - 2)^2} - 3$

Chapter 4 review

Technology-free questions





2 a $(x-3)^2 + (y+2)^2 = 25$

b $\left(x - \frac{3}{2}\right)^2 + \left(y + \frac{5}{2}\right)^2 = \frac{50}{4}$

c $\left(x - \frac{1}{4}\right)^2 + \left(y + \frac{1}{4}\right)^2 = \frac{17}{8}$

d $(x+2)^2 + (y-3)^2 = 13$

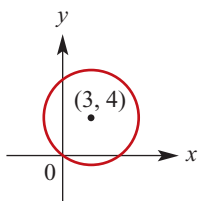
e $(x-3)^2 + (y-3)^2 = 18$

f $(x-2)^2 + (y+3)^2 = 13$

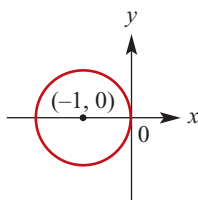
3 $2y + 3x = 0$

4 $2x + 2y = 1$ or $y = x - \frac{5}{2}$

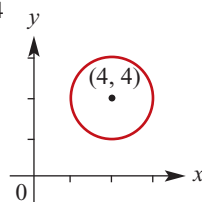
5 a $(x-3)^2 + (y-4)^2 = 25$



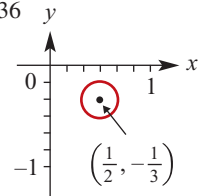
b $(x+1)^2 + y^2 = 1$



c $(x-4)^2 + (y-4)^2 = 4$



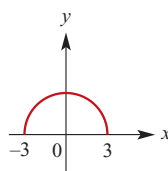
d $\left(x - \frac{1}{2}\right)^2 + \left(y + \frac{1}{3}\right)^2 = \frac{1}{36}$



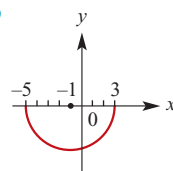
6 $C(-2, 3), r = 6$

7 y-axis: $4\sqrt{6}$; x-axis: $2\sqrt{21}$

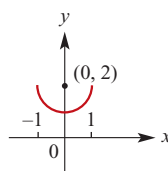
8 a



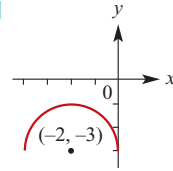
b



c



d



Multiple-choice questions

1 E **2** B **3** E **4** A **5** A

6 D **7** D **8** C **9** E **10** B

Extended-response questions

1 a $(x-10)^2 + y^2 = 25$ **c** $m = \pm \frac{\sqrt{3}}{3}$

d $P\left(\frac{15}{2}, \pm 5\sqrt{3}\right)$ **e** $5\sqrt{3}$

2 a $x^2 + y^2 = 16$

b ii $m = \pm \frac{\sqrt{3}}{3}; \quad y = \frac{\sqrt{3}}{3}x - \frac{8\sqrt{3}}{3}$
 $y = -\frac{\sqrt{3}}{3}x + \frac{8\sqrt{3}}{3}$

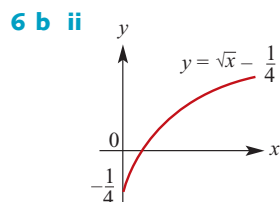
3 a $\frac{4}{3}$ **b** $-\frac{3}{4}$ **c** $4y + 3x = 25$ **d** $\frac{125}{12}$

4 a i $\frac{y_1}{x_1}$ **ii** $\frac{-x_1}{y_1}$

c $\sqrt{2}x + \sqrt{2}y = 8$ or $\sqrt{2}x + \sqrt{2}y = -8$

5 a $y = \frac{-\sqrt{3}}{3}x + \frac{2\sqrt{3}}{3}a$, $y = \frac{\sqrt{3}}{3}x - \frac{2\sqrt{3}}{3}a$

b $x^2 + y^2 = 4a^2$



c i $-\frac{1}{4} < k < 0$ ii $k = 0$ or $k < -\frac{1}{4}$

iii $k > 0$

7 a $0 < k < \frac{1}{4}$ b $k = \frac{1}{4}$ or $k \leq 0$

Chapter 5

Exercise 5A

1 a {7, 11} b {7, 11}

c {1, 2, 3, 5, 7, 11, 25, 30}

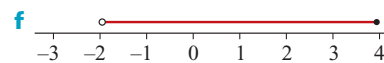
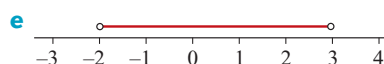
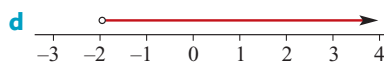
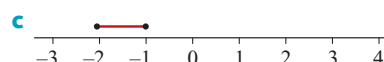
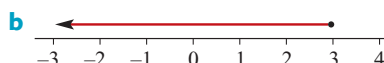
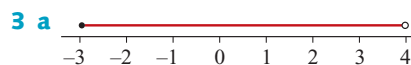
d {1, 2, 3, 5, 7, 11, 15, 25, 30, 32}

e {1, 2, 3, 5, 7, 11, 15, 25, 30, 32}

f {1, 7, 11, 25, 30}

2 a {1, 2, 3, 5, 15} b {25, 30, 32}

c {2, 3, 5, 15} d {25, 30}



4 a (-2, 1] b [-3, 3] c [-3, 2) d (-1, 2)

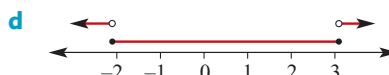
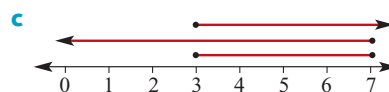
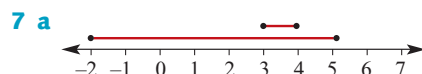
5 a [-1, 2] b (-4, 2] c (0, $\sqrt{2}$)

d $(-\frac{\sqrt{3}}{2}, \frac{1}{\sqrt{2}}]$ e (-1, ∞) f $(-\infty, -2]$

g $(-\infty, \infty)$ h [0, ∞) i $(-\infty, 0]$

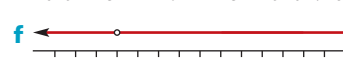
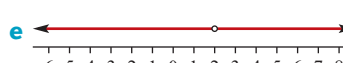
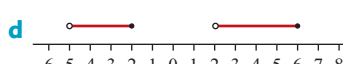
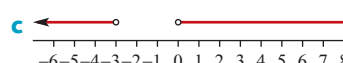
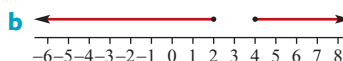
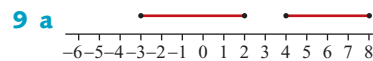
6 a {7} b B, i.e. {7, 11, 25, 30, 32}

c (2, ∞) d {30, 32}



8 a $(-\infty, -2) \cup (-2, \infty)$ b $(-\infty, 3) \cup (3, \infty)$

c $(-\infty, 4) \cup (4, \infty)$

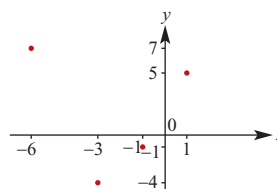


10 a (-6, -3) b $\emptyset$ c [-6, 0]

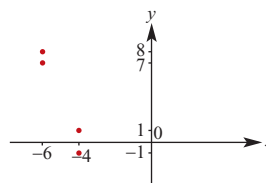
d [-1, 2] e {1} f (-10, -1]

Exercise 5B

1 a Domain = {-3, -1, -6, 1};
Range = {-4, -1, 7, 5}

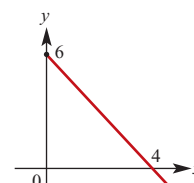
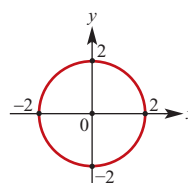


b Domain = {-4, -6}; Range = {-1, 1, 7, 8}

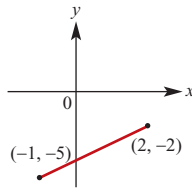


c Domain = [-2, 2] Range = [-2, 2]

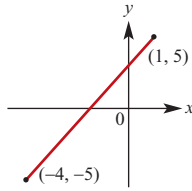
d Domain = [0, ∞) Range = $(-\infty, 6]$



e Domain = $[-1, 2]$
Range = $[-5, -2]$



f Domain = $[-4, 1]$
Range = $[-5, 5]$



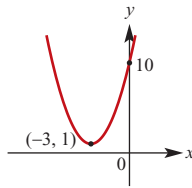
2 a Domain = $[-2, 2]$; Range = $[-1, 2]$

b Domain = $[-2, 2]$; Range = $[-2, 2]$

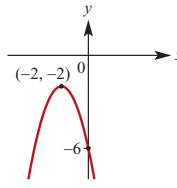
c Domain = $\mathbb{R}$; Range = $[-1, \infty)$

d Domain = $\mathbb{R}$; Range = $(-\infty, 4]$

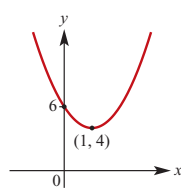
3 a $y = (x + 3)^2 + 1$
Range = $[1, \infty)$



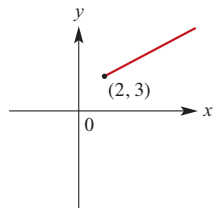
b $y = -(x + 2)^2 - 2$
Range = $(-\infty, -2]$



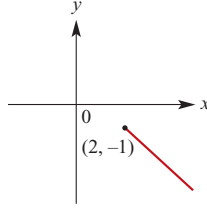
c $y = 2(x - 1)^2 + 4$
Range = $[4, \infty)$



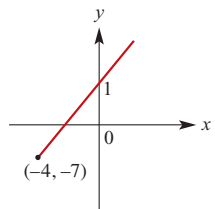
4 a Range = $[3, \infty)$



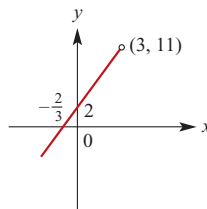
b Range = $(-\infty, -1]$



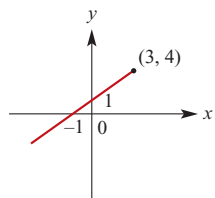
c Range = $[-7, \infty)$



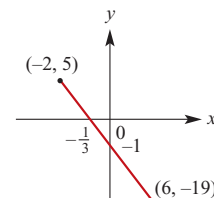
d Range = $(-\infty, 11]$



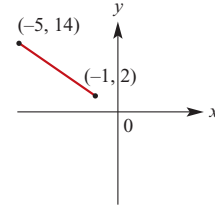
e Range = $(-\infty, 4]$



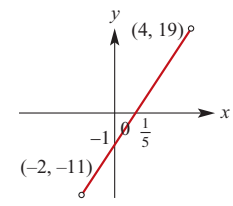
f Range = $[-19, 5]$



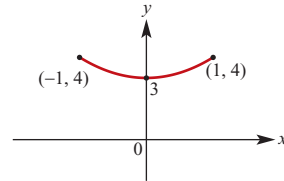
g Range = $[2, 14]$



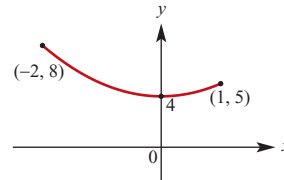
h Range = $(-11, 19)$



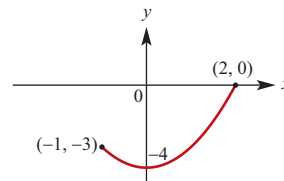
5 a Range = $[3, 4]$



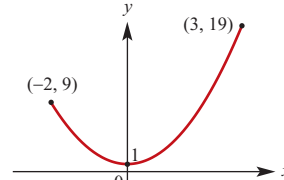
b Range = $[4, 8]$



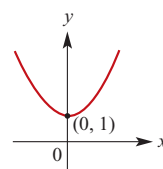
c Range = $[-4, 0]$



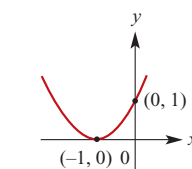
d Range = $[1, 19]$



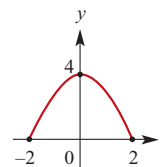
6 a Range = $[1, \infty)$



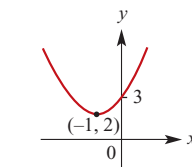
b Range = $[0, \infty)$



c Range = $[0, 4]$

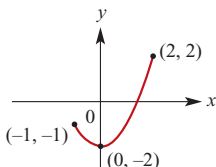
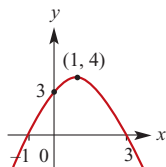


d Range = $[2, \infty)$



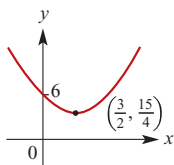
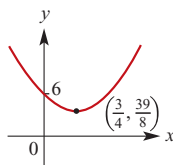
e Range = $(-\infty, 4]$

f Range = $[-2, 2]$

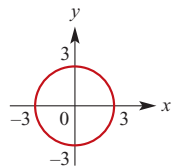


g Range = $\left[\frac{39}{8}, \infty\right)$

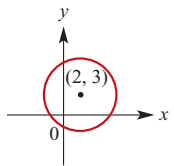
h Range = $\left[\frac{15}{4}, \infty\right)$



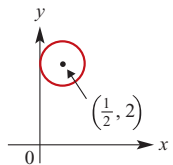
7 a Domain $-3 \leq x \leq 3$ Range $-3 \leq y \leq 3$



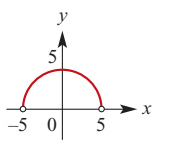
b Domain $-2 \leq x \leq 6$ Range $-1 \leq y \leq 7$



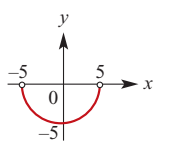
c Domain $0 \leq x \leq 1$ Range $1\frac{1}{2} \leq y \leq 2\frac{1}{2}$



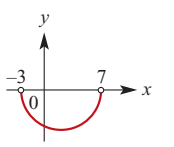
d Domain $-5 \leq x \leq 5$ Range $0 \leq y \leq 5$



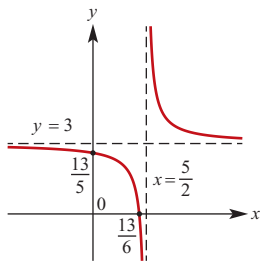
e Domain $-5 \leq x \leq 5$ Range $-5 \leq y \leq 0$



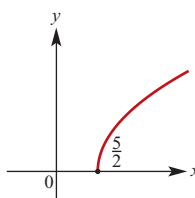
f Domain $-3 \leq x \leq 7$ Range $-5 \leq y \leq 0$



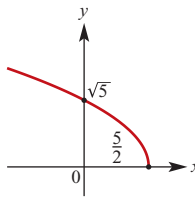
8 a Domain = $\mathbb{R} \setminus \left\{\frac{5}{2}\right\}$; Range = $\mathbb{R} \setminus \{3\}$



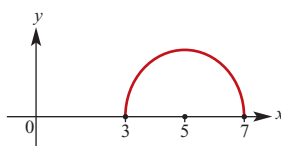
b Domain = $\left[\frac{5}{2}, \infty\right)$ Range = $\mathbb{R}^+ \cup \{0\}$



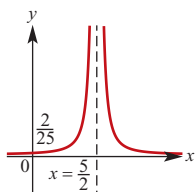
c Domain = $\left(-\infty, \frac{5}{2}\right]$ Range = $\mathbb{R}^+ \cup \{0\}$



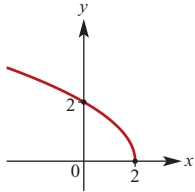
d Domain = $[3, 7]$; Range = $[0, 2]$



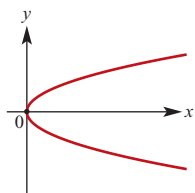
e Domain = $\mathbb{R} \setminus \left\{\frac{5}{2}\right\}$ Range = $(0, \infty)$



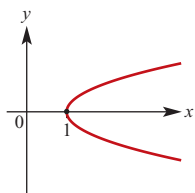
f Domain = $(-\infty, 2]$ Range = $\mathbb{R}^+ \cup \{0\}$



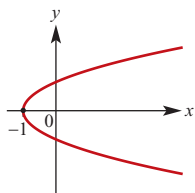
9 a Domain = $[0, \infty)$ Range = $\mathbb{R}$



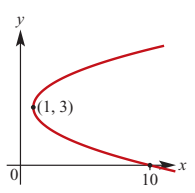
b Domain = $[1, \infty)$ Range = $\mathbb{R}$



c Domain = $[-1, \infty)$ Range = $\mathbb{R}$



d Domain = $[1, \infty)$ Range = $\mathbb{R}$



Exercise 5C

1 a Not a function; Domain = $\{0, 1, 2, 3\}$; Range = $\{1, 2, 3, 4\}$

b A function; Domain = $\{-2, -1, 0, 1, 2\}$; Range = $\{-5, -2, -1, 2, 4\}$

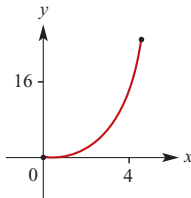
c Not a function; Domain = $\{-1, 0, 3, 5\}$; Range = $\{1, 2, 4, 6\}$

d A function; Domain = $\{1, 2, 4, 5, 6\}$; Range = $\{3\}$

2 a A function

Domain = $[0, 4]$

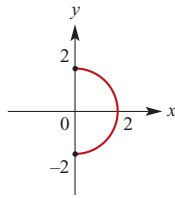
Range = $[0, 16]$



b Not a function

Domain = $[0, 2]$

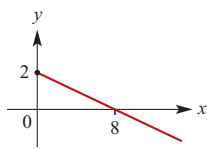
Range = $[-2, 2]$



c A function

Domain = $[0, \infty)$

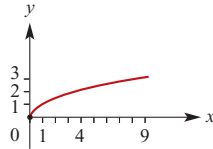
Range = $(-\infty, 2]$



d A function

Domain = $(0, \infty)$

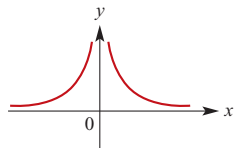
Range = $(0, \infty)$



e A function

Domain = $\mathbb{R} \setminus \{0\}$

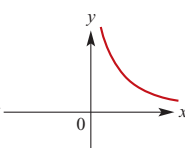
Range = $\mathbb{R}^+$



f A function

Domain = $\mathbb{R}^+$

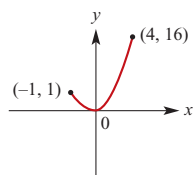
Range = $\mathbb{R}^+$



g A function

Domain = $[-1, 4]$

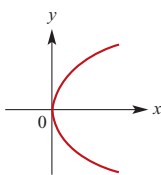
Range = $[0, 16]$



h Not a function

Domain = $[0, \infty)$

Range = $\mathbb{R}$



3 a $f: \mathbb{R} \rightarrow \mathbb{R}, f(x) = 3x + 2$

b $f: \mathbb{R} \rightarrow \mathbb{R}, f(x) = -\frac{3}{2}x + 6$

c $f: [0, \infty) \rightarrow \mathbb{R}, f(x) = 2x + 3$

d $f: [-1, 2] \rightarrow \mathbb{R}, f(x) = 5x + 6$

e $f: [-5, 5] \rightarrow \mathbb{R}, f(x) = -x^2 + 25$

f $f: [0, 1] \rightarrow \mathbb{R}, f(x) = 5x - 7$

4 a A function; Domain = $\mathbb{R}$; Range = $\{-2\}$

b Not a function; Domain = $\{3\}$; Range = $\mathbb{Z}$

c Function; Domain = $\mathbb{R}$; Range = $\mathbb{R}$

d A function; Domain = $\mathbb{R}$; Range = $[5, \infty)$

e Not a function; Domain = $[-3, 3]$; Range = $[-3, 3]$

5 a i -3 **ii** 5 **iii** -5 **iv** 9

v $2x - 5$ **vi** $\frac{2}{a} - 3$

b i 4 **ii** -4 **iii** $\frac{4}{3}$ **iv** 2

c i 4 **ii** 36 **iii** 36 **iv** $(a - 2)^2$

d i 0 **ii** $\frac{a}{1 + a}$ **iii** $\frac{-a}{1 - a}$ **iv** $1 - a$

6 a $5, 2t + 1$ **b** $x = \frac{5}{2}$ **c** $x = -\frac{1}{2}$

d $t = -1$ **e** $x \geq -1$ **f** $x \geq 1$

7 a 1 **b** $\frac{1}{6}$ **c** ± 3 **d** -1, 4 **e** -1, 3 **f** -2, 3

8 a $g(-1) = -1, g(2) = 8, g(-2) = 0$

b $h(-1) = 3, h(2) = 18, h(-2) = -14$

c i $g(-3x) = 9x^2 - 6x$

ii $g(x - 5) = x^2 - 8x + 15$

iii $h(-2x) = -16x^3 - 4x^2 + 6$

iv $g(x + 2) = x^2 + 6x + 8$

v $h(x^2) = 2x^6 - x^4 + 6$

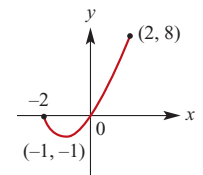
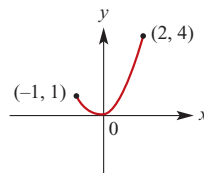
9 a $f(2) = 5, f(-4) = 29$ **b** Range = $[-3, \infty)$

10 a $f(2) = 7$ **b** $x = 2$ **c** $x = -1$

11 a 2 **b** ± 1 **c** $x = \pm\sqrt{3}$

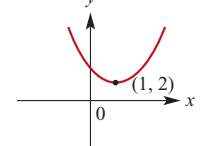
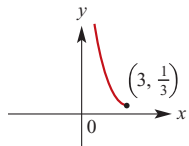
12 a $x = -1$ **b** $x > -1$ **c** $x = -\frac{6}{7}$

13 a Range = $[0, 4]$ **b** Range = $[-1, 8]$



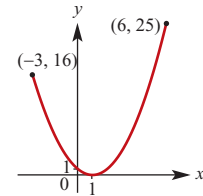
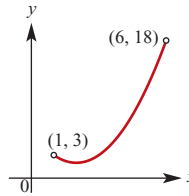
c Range = $[\frac{1}{3}, \infty)$

d Range = $[2, \infty)$



e Range = $[2, 18]$

f Range = $[0, 25]$



Exercise 5D

1 One-to-one functions: b, d, e, g

2 i Functions: a, c, d, f, g

ii One-to-one functions: c, g

3 a Domain = $\mathbb{R}$; Range = $\mathbb{R}$

b Domain = $\mathbb{R}^+ \cup \{0\}$; Range = $\mathbb{R}^+ \cup \{0\}$

c Domain = $\mathbb{R}$; Range = $[1, \infty)$

d Domain = $[-3, 3]$; Range = $[-3, 0]$

e Domain = $\mathbb{R}^+$; Range = $\mathbb{R}^+$

f Domain = $\mathbb{R}$; Range = $(-\infty, 3]$

g Domain = $[2, \infty)$; Range = $\mathbb{R}^+ \cup \{0\}$

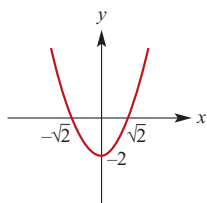
h Domain = $[\frac{1}{2}, \infty)$; Range = $[0, \infty)$

i Domain = $(-\infty, \frac{3}{2}]$; Range = $[0, \infty)$

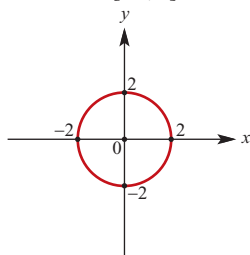
j Domain = $\mathbb{R} \setminus \{\frac{1}{2}\}$; Range = $\mathbb{R} \setminus \{0\}$

- k** Domain = $\mathbb{R} \setminus \{\frac{1}{2}\}$; Range = $(-3, \infty)$
l Domain = $\mathbb{R} \setminus \{\frac{1}{2}\}$; Range = $\mathbb{R} \setminus \{2\}$
4 a Domain = $[4, \infty)$; Range = $[0, \infty)$
b Domain = $(-\infty, 4]$; Range = $[0, \infty)$
c Domain = $[2, \infty)$; Range = $[3, \infty)$
d Domain = $\mathbb{R} \setminus \{4\}$; Range = $\mathbb{R} \setminus \{0\}$
e Domain = $\mathbb{R} \setminus \{4\}$; Range = $\mathbb{R} \setminus \{3\}$
f Domain = $\mathbb{R} \setminus \{-2\}$; Range = $\mathbb{R} \setminus \{-3\}$
5 a Domain = $\mathbb{R}$; Range = $\mathbb{R}$
b Domain = $\mathbb{R}$; Range = $[2, \infty)$
c Domain = $[-4, 4]$; Range = $[-4, 0]$
d Domain = $\mathbb{R} \setminus \{-2\}$; Range = $\mathbb{R} \setminus \{0\}$
6 $y = \sqrt{2-x}$, Domain $(-\infty, 2]$, Range $[0, \infty)$
 $y = -\sqrt{2-x}$, Domain $(-\infty, 2]$, Range $(-\infty, 0]$

7 a



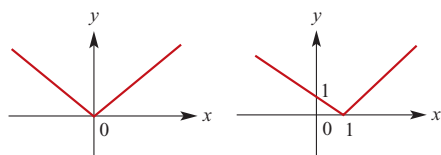
- b** $f_1: [0, \infty) \rightarrow \mathbb{R}, f_1(x) = x^2 - 2$
 $f_2: (-\infty, 0] \rightarrow \mathbb{R}, f_2(x) = x^2 - 2$
8 a $f_1: [1, \infty) \rightarrow \mathbb{R}, f_1(x) = x^2 - 2x + 4$
 $f_2: (-\infty, 1] \rightarrow \mathbb{R}, f_2(x) = x^2 - 2x + 4$
9 a $f_1: (2, \infty) \rightarrow \mathbb{R}, f_1(x) = \frac{1}{(x-2)^2}$
 $f_2: (-\infty, 2) \rightarrow \mathbb{R}, f_2(x) = \frac{1}{(x-2)^2}$
10 a Domain = $[-2, 2]$



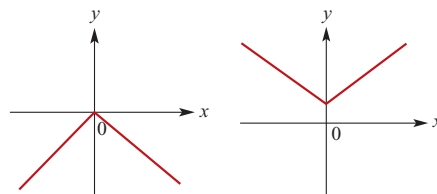
- b** $f_1: [0, 2] \rightarrow \mathbb{R}, f_1(x) = \sqrt{4-x^2}$
 $f_2: [0, 2] \rightarrow \mathbb{R}, f_2(x) = -\sqrt{4-x^2}$
c $f_1: [-2, 0] \rightarrow \mathbb{R}, f_1(x) = \sqrt{4-x^2}$
 $f_2: [-2, 0] \rightarrow \mathbb{R}, f_2(x) = -\sqrt{4-x^2}$

Exercise 5E

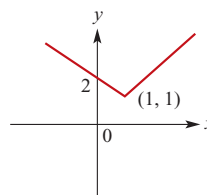
- 1 a** Range = $[0, \infty)$ **b** Range = $[0, \infty)$



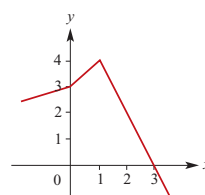
- c** Range = $(-\infty, 0]$ **d** Range = $[1, \infty)$



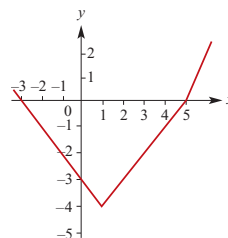
- e** Range = $[1, \infty)$



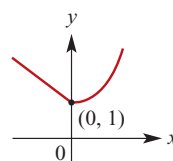
- 2** Range = $(-\infty, 4]$



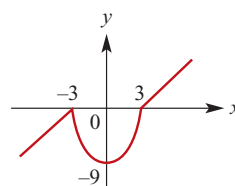
3



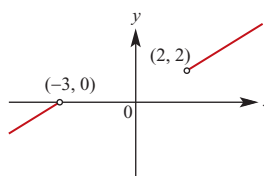
- 4** Range $[1, \infty)$



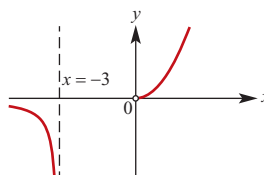
- 5** Range = $\mathbb{R}$



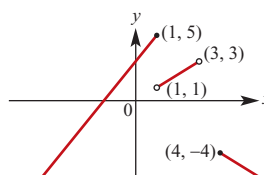
- 6 a** Range = $(-\infty, 0) \cup (2, \infty)$



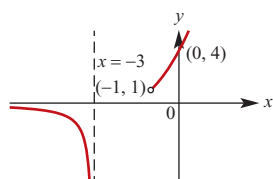
- b** Range = $\mathbb{R} \setminus \{0\}$



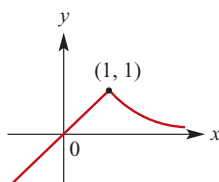
- c** Range = $(-\infty, 5]$



d Range = $\mathbb{R} \setminus [0, 1]$



7 Range = $(-\infty, 1]$



8
$$f(x) = \begin{cases} x + 3, & -3 \leq x \leq -1 \\ -x + 1, & -1 < x \leq 2 \\ -\frac{1}{2}x, & 2 \leq x \leq 4 \end{cases}$$

Exercise 5F

1 b i 25.06 **ii** 25.032 **iii** 25.2 **iv** 26

2 a $a = -3, b = \frac{1}{2}$ **b** 6

3 $f(x) = 7 - 5x$

4 a i $f(0) = -\frac{9}{2}$ **ii** $f(1) = -3$ **b** 3

5 $f(x) = -7(x - 2)(x - 4)$

6 $f(x) = (x - 3)^2 + 7$, Range = $[7, \infty)$

7 $a = \frac{1}{10}, b = -\frac{9}{10}, c = 2$

8 $f(x) = -2(x - 1)(x + 5)$
 $g(x) = -50(x - 1)\left(x + \frac{1}{5}\right)$

9 a $k < \frac{-37}{12}$ **b** $k = -\frac{25}{12}$

Exercise 5G

1 a $\{(3, 1), (6, -2), (5, 4), (1, 7)\}$
 Domain = $\{3, 6, 5, 1\}$; Range = $\{1, -2, 4, 7\}$

b $\{(3, 2), (6, -1), (-5, 4), (7, 1), (-4, 6)\}$
 Domain = $\{3, 6, -5, 7, -4\}$
 Range = $\{-1, 1, 2, 4, 6\}$

c $\{(3, 3), (-4, -2), (-1, -1), (1, -8)\}$
 Domain = $\{3, 1, -1, -4\}$
 Range = $\{3, -2, -1, -8\}$

d $\{(3, 1), (-7, -10), (-6, -7), (8, 2), (4, 11)\}$
 Domain = $\{3, -7, -6, 8, 4\}$
 Range = $\{1, -10, -7, 2, 11\}$

2 a $f^{-1}(x) = \frac{6-x}{2}$; Domain = $\mathbb{R}$; Range = $\mathbb{R}$

b $f^{-1}(x) = 3 - x$
 Domain = $[-2, 2]$; Range = $[1, 5]$

c $f^{-1}(x) = x - 4$
 Domain = $(4, \infty)$; Range = $\mathbb{R}^+$

d $f^{-1}(x) = x - 4$
 Domain = $(-\infty, 8]$; Range = $(-\infty, 4]$

e $f^{-1}(x) = 8 - \frac{x}{2}$
 Domain = $[2, 18]$; Range = $[-1, 7]$

3 a $f^{-1}(x) = \sqrt{x}$
 Domain = $\mathbb{R}^+ \cup \{0\}$; Range = $\mathbb{R}^+ \cup \{0\}$

b $f^{-1}(x) = 2 + \sqrt{x - 3}$
 Domain = $[3, \infty)$; Range = $[2, \infty)$

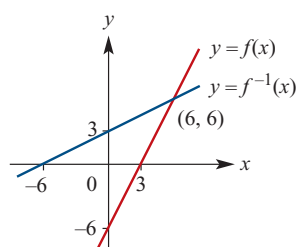
c $f^{-1}(x) = 4 - \sqrt{x - 6}$
 Domain = $[6, \infty)$; Range = $(-\infty, 4]$

d $f^{-1}(x) = 1 - x^2$
 Domain = $[0, 1]$; Range = $[0, 1]$

e $f^{-1}(x) = \sqrt{16 - x^2}$
 Domain = $[0, 4]$; Range = $[0, 4]$

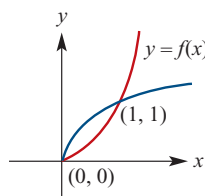
f $f^{-1}(x) = -4 + \sqrt{x - 6}$
 Domain = $[22, \infty)$; Range = $[0, \infty)$

4 a



b (6, 6)

5 a



b (0, 0), (1, 1)

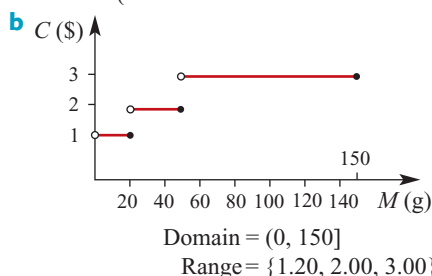
6 $a = \frac{-1}{2}, b = \frac{5}{2}$

7 a $f^{-1}(x) = a - x^2$ **b** $a = 1$ or $a = 2$

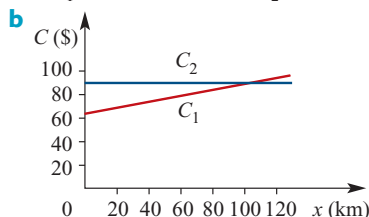
Exercise 5H

1 $C = 0.15n + 45$ where n is the number of calls

2 a
$$C(m) = \begin{cases} 1.20 & \text{for } 0 < m \leq 20 \\ 2.00 & \text{for } 20 < m \leq 50 \\ 3.00 & \text{for } 50 < m \leq 150 \end{cases}$$



3 a i $C_1 = 64 + 0.25x$ ii $C_2 = 89$



c $x > 100$ km

4 a Length = $(50 - x)$ cm

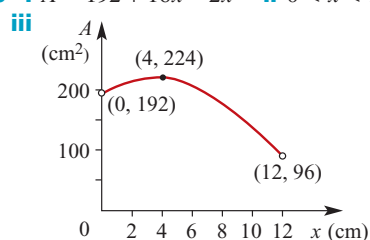
b $A(x) = x(50 - x)$ c $0 \leq x \leq 50$

d Maximum area = 625 cm^2 when $x = 25$

5 a i $A = (8 + x)y - x^2$

ii $P = 2x + 2y + 16$

b i $A = 192 + 16x - 2x^2$ ii $0 < x < 12$



iv 224 cm^2

Chapter 5 review

Technology-free questions

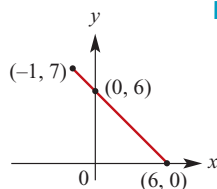
1 a $[-2, 4]$ b $[-2, 4]$ c $[1, 8]$ d $(-1, 6]$

e $(-4, -2] \cup (1, 5]$ f $(-4, -2] \cup (2, \infty)$

g $(-\infty, -3] \cup (1, \infty)$

2 a -16 b 26 c $-\frac{2}{3}$

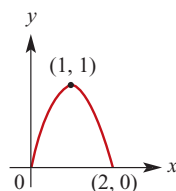
3 a b Range = $[0, 7]$



4 a Range = $\mathbb{R}$ b Range = $[-5, 4]$
c Range = $[0, 4]$ d Range = $(-\infty, 9]$
e Range = $[2, \infty)$ f Range = $\{-6, 2, 4\}$
g Range = $[0, \infty)$ h Range = $\mathbb{R} \setminus \{2\}$
i Range = $[-5, 1]$ j Range = $[-1, 3]$

5 a $a = -15, b = \frac{33}{2}$ b Domain = $\mathbb{R} \setminus \{0\}$

6 a b Range = $[0, 1]$



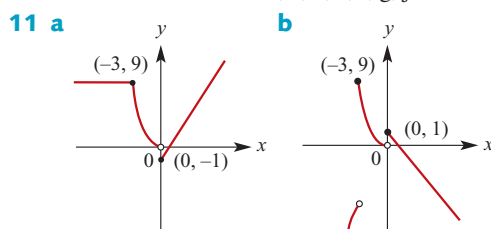
7 a $3, b = -5$

8 $a = -\frac{1}{2}, b = 2, c = 0$

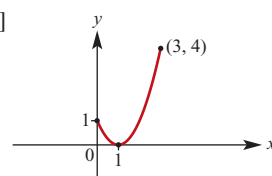
9 a $\mathbb{R} \setminus \{2\}$ b $[2, \infty)$ c $[-5, 5]$ d $\mathbb{R} \setminus \{\frac{1}{2}\}$

e $[-10, 10]$ f $(-\infty, 4]$

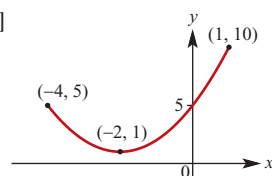
10 One-to-one functions: b, c, d, e, f, g, j



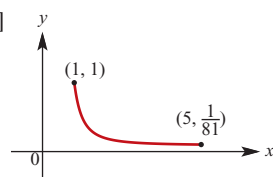
12 a Range = $[0, 4]$



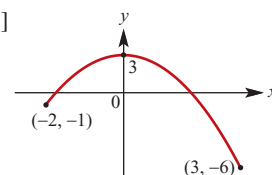
b Range = $[1, 10]$



c Range = $[\frac{1}{81}, 1]$



d Range = $[-6, 3]$



13 a Domain = $[1, \infty)$; Range = $[0, \infty)$

b Domain = $(-\infty, 1]$; Range = $[0, \infty)$

c Domain = $[0, \infty)$; Range = $(-\infty, 1]$

14 a Domain = $\mathbb{R} \setminus \{1\}$; Range = $\mathbb{R} \setminus \{0\}$

b Domain = $\mathbb{R} \setminus \{-1\}$; Range = $\mathbb{R} \setminus \{0\}$

c Domain = $\mathbb{R} \setminus \{1\}$; Range = $\mathbb{R} \setminus \{3\}$

15 a Domain = $[-1, 1]$; Range = $[0, 1]$

b Domain = $[-3, 3]$; Range = $[0, 3]$

c Domain = $[-1, 1]$; Range = $[3, 4]$

16 a $f^{-1}(x) = \frac{x+2}{3}$; Domain = $[-5, 13]$

b $f^{-1}(x) = (x-2)^2 - 2$; Domain = $[2, \infty)$

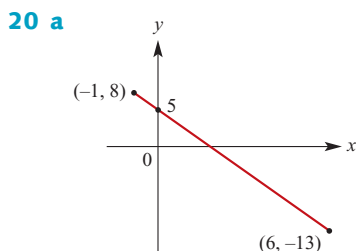
c $f^{-1}(x) = \sqrt{\frac{x}{3}} - 1$; Domain = $[0, \infty)$

d $f^{-1}(x) = -\sqrt{x} + 1$; Domain = $(0, \infty)$

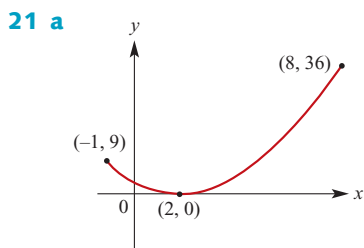
17 a $2p + 5$ b $2(p + h) + 5$ c $2h$ d 2

18 -2

- 19 a $(-\infty, -\frac{15}{8}]$ b $[3\frac{7}{8}, \infty)$
 c $(-\infty, 20]$ d $(-\infty, 3]$



b Range = $[-13, 8]$



b Range = $[0, 36]$

22 a Domain $-3 \leq x \leq 3$; Range $-3 \leq y \leq 3$

b Domain $1 \leq x \leq 3$; Range $-1 \leq y \leq 1$

c Domain $0 \leq x \leq 1$; Range $0 \leq y \leq 1$

d Domain $-1 \leq x \leq 9$; Range $-5 \leq y \leq 5$

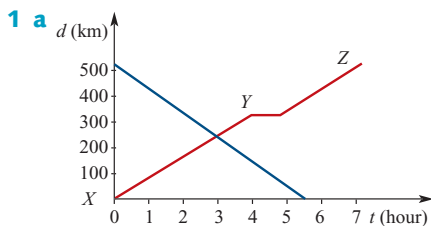
e Domain $-4 \leq x \leq 4$; Range $-2 \leq y \leq 6$

- 23 a $\{2, 4, 6, 8\}$ b $\{4, 3, 2, 1\}$
 c $\{-3, 0, 5, 12\}$ d $\{1, \sqrt{2}, \sqrt{3}, 2\}$

Multiple-choice questions

- 1 B 2 E 3 D 4 B 5 E 6 C
 7 E 8 B 9 D 10 C 11 D

Extended-response questions



Coach starting from X:

$$d = \begin{cases} 80t & \text{for } 0 \leq t \leq 4 \\ 320 & \text{for } 4 < t \leq 4\frac{3}{4} \\ 80t - 60 & \text{for } 4\frac{3}{4} < t \leq 7\frac{1}{4} \end{cases}$$

Range = $[0, 520]$

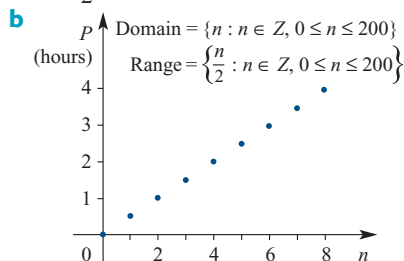
Coach starting from Z:

$$d = 520 - \frac{1040t}{11} \text{ for } 0 \leq t \leq 5\frac{1}{2}$$

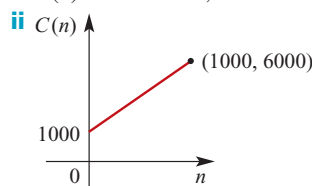
Range = $[0, 520]$

b The coaches pass $238\frac{1}{3}$ km from X

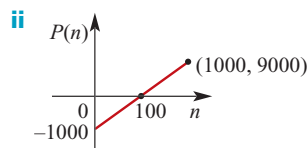
2 a $P = \frac{1}{2}n$



3 a i $C(n) = 1000 + 5n, n > 0$

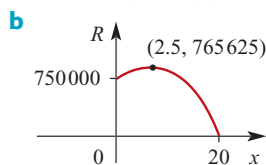


b i $P(n) = 15n - (1000 + 5n)$
 $= 10n - 1000$



4 $V = 8000(1 - 0.05n) = 8000 - 400n$

5 a $R = (50000 - 2500x)(15 + x)$
 $= 2500(x + 15)(20 - x)$

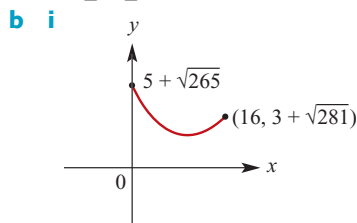


c Price for max revenue = \$17.50

6 a $A(x) = \frac{x}{4}(2a - (6 - \sqrt{3})x)$ b $0 < x < \frac{a}{3}$

7 a i $d(x) = \sqrt{x^2 + 25} + \sqrt{(16 - x)^2 + 9}$

ii $0 \leq x \leq 16$



ii 1.54

iii 3.40 or 15.04

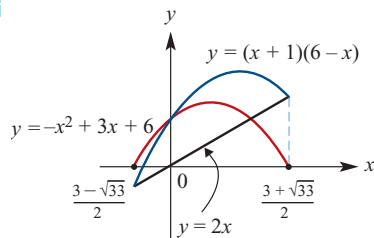
c i Minimum is $8\sqrt{5}$, occurs when $x = 10$

ii Range = $[8\sqrt{5}, 5 + \sqrt{265}]$

8 a $A\left(\frac{3+\sqrt{33}}{2}, 3+\sqrt{33}\right), B\left(\frac{3-\sqrt{33}}{2}, 3-\sqrt{33}\right)$

b i $d(x) = -x^2 + 3x + 6$

ii



c i Maximum value of $d(x)$ is 8.25

ii Range = $[0, 8.25]$

d $A(2.45, 12.25), B(-2.45, -12.25)$

$d(x) = -x^2 + 6$

Maximum value of $d(x)$ is 6; Range = $[0, 6]$

Chapter 6

Exercise 6A

1 a -3 b -1 c -7 d -15

2 a 0 b 0

3 a 6 b 9 c 26 d 11

e $a^3 + 4a^2 - 2a + 6$ f $8a^3 + 16a^2 - 4a + 6$

4 a $a = 4$ b $a = 4$ c $c = 6$
d $a = -33, b = -15$ e $a = -9, b = 23$

5 a $x^3 - 2x^2 - 2x + 2$ b $x^3 - x^2 + 2x$
c $x^3 - 2x^2 + 4x - 2$ d $3x^3 - 6x^2 + 3x$

e $-3x^4 + 8x^3 - 7x^2 + 2x$
f $-3x^3 - x^2 + 2x$ g $x^3 - x^2 - x + 2$

h $x^5 - x^4 - x^3 + x^2$

6 a $x^3 - 4x^2 + 7x - 6$ b $x^3 - 6x^2 + 11x - 12$

c $2x^3 - 5x^2 - x + 4$

d $x^3 + (b-2)x^2 + (c-2b)x - 2c$

e $2x^3 - 7x^2 - 10x - 3$

7 a $x^3 + (b+1)x^2 + (c+b)x + c$

b $b = -8, c = 12$ c $(x+1)(x-6)(x-2)$

8 $b = -3, c = -11$

9 a $a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$

b $a^6 + 6a^5b + 15a^4b^2 + 20a^3b^3 + 15a^2b^4 + 6ab^5 + b^6$

10 a $(x-y)^4 = x^4 - 4x^3y + 6x^2y^2 - 4xy^3 + y^4$

b $(2x+y)^4 = 16x^4 + 32x^3y + 24x^2y^2 + 8xy^3 + y^4$

Exercise 6B

1 a $x^2 + 2x + \frac{3}{x-1}$ b $2x^2 - x - 3 + \frac{6}{x+1}$

c $3x^2 - 10x + 22 - \frac{43}{x+2}$

d $2x^2 + 3x + 10 + \frac{28}{x-3}$

2 a $x^2 - x + 4 - \frac{8}{x+1}$

b $2x^2 - 8x + 49 - \frac{181}{x+4}$

c $x^2 + x - 3 + \frac{11}{x+3}$ d $x^2 - x + 4 + \frac{8}{x-2}$

3 a $x^2 - 2x + 5$ b $2x^2 - 2x - 6$

c $x^2 - 2x - 6$ d $3x^2 - x - 6$

4 a Quotient $x^2 - 3$; Remainder 7

b Quotient $x^2 + 2x + 15$; Remainder 71

c Quotient $2x^2 - 3x$; Remainder -7

d Quotient $5x^2 + 20x + 77$; Remainder 315

5 a $\frac{1}{2}x^2 + \frac{7}{4}x - \frac{3}{8} + \frac{103}{8(2x+5)}$

b $x^2 + 2x - 3 - \frac{2}{2x+1}$ c $x^2 + 2x - 15$

d $\frac{1}{3}x^2 - \frac{8}{9}x - \frac{8}{27} + \frac{19}{27(3x-1)}$

6 a $x^2 + 3x + 8 + \frac{9}{x-1}$

b $x^2 - \frac{x}{2} + \frac{9}{4} + \frac{21}{4(2x-1)}$

7 a Quotient $2x - 6$; Remainder 0

b Quotient $x - 6$; Remainder -2

c Quotient $2x - 6$; Remainder 42

d Quotient $x^2 - 4x + 2$; Remainder $-x + 7$

e Quotient $x^2 - 3x + 7$; Remainder $-10x + 9$

f Quotient $x^2 + x - \frac{3}{2}$; Remainder $\frac{15}{2}x + 16$

Exercise 6C

1 a -2 b -29 c 15 d 4 e 7

f -12 g 0 h -5 i -8

2 a $a = -3$ b $a = 2$ c $a = 4$ d $a = -10$

3 a $P(1) = 0$ b $P(1) = 0$

c $P(-2) = 0$ d $P(\frac{3}{2}) = 0$

4 a 6 b 28 c $-\frac{1}{3}$

5 a $(x-1)(x+1)(2x+1)$ b $(x+1)^3$

c $(x-1)(6x^2 - 7x + 6)$

d $(x-1)(x+5)(x-4)$

e $(x+1)^2(2x-1)$ f $(x+1)(x-1)^2$

g $(x-2)(4x^2 + 8x + 19)$

h $(x+2)(2x+1)(2x-3)$

6 1

7 a $(2x-3)(x^2 - 2x + 5)$

b $(2x+1)(x^2 - 2x + 5)$

c $(2x+1)(x-1-\sqrt{6})(x-1+\sqrt{6})$

d $(2x+3)(x-1-\sqrt{2})(x-1+\sqrt{2})$

8 a $(x-1)(x^2 + x + 1)$

b $(x+4)(x^2 - 4x + 16)$

c $(3x-1)(9x^2 + 3x + 1)$

d $(4x-5)(16x^2 + 20x + 25)$

e $(1-5x)(1+5x+25x^2)$

f $(3x+2)(9x^2 - 6x + 4)$

g $(4m-3n)(16m^2 + 12mn + 9n^2)$

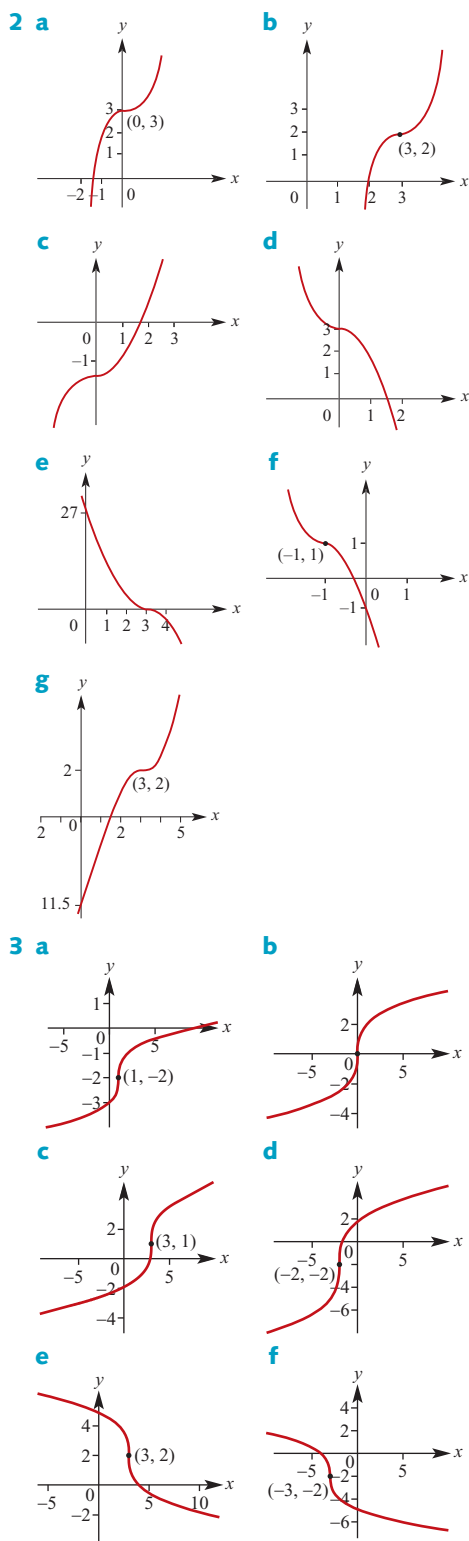
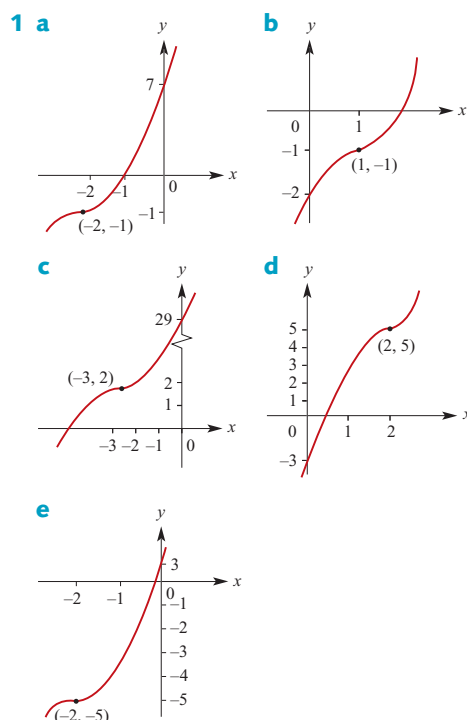
h $(3b+2a)(9b^2 - 6ab + 4a^2)$

- 9 a** $(x+2)(x^2-x+1)$
b $(3x+2)(x-1)(x-2)$
c $(x-3)(x+1)(x-2)$
d $(3x+1)(x+3)(2x-1)$
10 $a=3, b=-3, P(x)=(x-1)(x+3)(x+1)$
11 b i n odd **ii** n even
12 a $a=1, b=1$
b i $P(x)=x^3-2x^2+3$

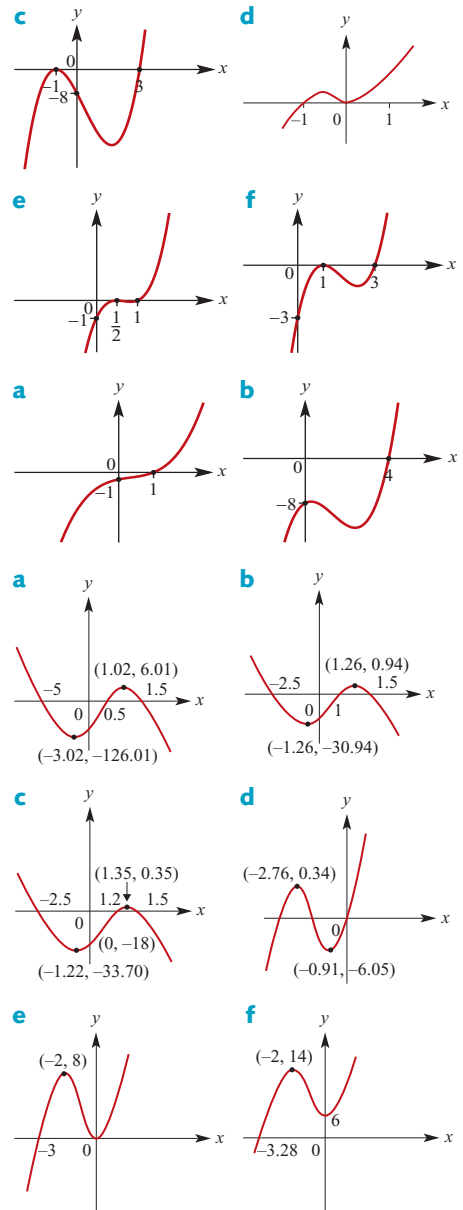
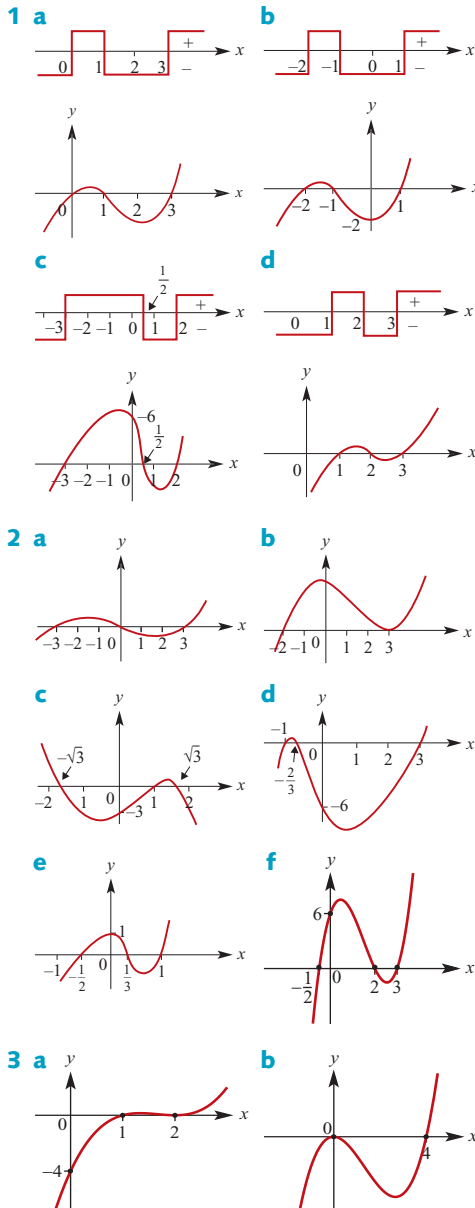
Exercise 6D

- 1 a** 1, -2, 4 **b** 4, 6 **c** $\frac{1}{2}, 3, -\frac{2}{3}$ **d** 0, -3, $\frac{5}{2}$
2 a -2, 0, 4 **b** 0, $-1 \pm 2\sqrt{3}$ **c** -5, 0, 8
d 0, $-1 \pm \sqrt{17}$
3 a 1 **b** -1 **c** $5, \pm\sqrt{10}$ **d** $\pm 4, a$
4 a 2, 3, -5 **b** $-1, -\frac{2}{3}, 3$ **c** $1, -\sqrt{2}, \sqrt{2}$
d $-\frac{2}{5}, -4, 2$ **e** $-\frac{1}{2}, \frac{1}{3}, 1$ **f** $-2, -\frac{3}{2}, 5$
5 a -6, 2, 3 **b** $-2, -\frac{2}{3}, \frac{1}{2}$ **c** 3
d -1 **e** -1, 3 **f** $3, -2 \pm \sqrt{3}$
6 a 0, $\pm 2\sqrt{2}$ **b** $1 + 2\sqrt{2}$ **c** -2
d -5 **e** $\frac{1}{10}$
7 a $2(x-9)(x-13)(x+11)$
b $(x+11)(x+3)(2x-1)$
c $(x+11)(2x-9)(x-11)$
d $(2x-1)(x+11)(x+15)$

Exercise 6E



4 a $f^{-1}(x) = \left(\frac{x-3}{2}\right)^{\frac{1}{3}}$ **b** $f^{-1}(x) = \frac{x^3}{27}$
c $f^{-1}(x) = \left(\frac{x-1}{2}\right)^{\frac{1}{3}} - 1$
d $f^{-1}(x) = \frac{(x+2)^3}{8} - 3$
e $f^{-1}(x) = 1 - \frac{(x-4)^3}{8}$
f $f^{-1}(x) = -\frac{(x+1)^3}{8} - 2$

Exercise 6F


6 $f(x) = (x+1)^2(x-3)$, so graph just touches the x -axis at $x = -1$ and cuts it at $x = 3$

Exercise 6G

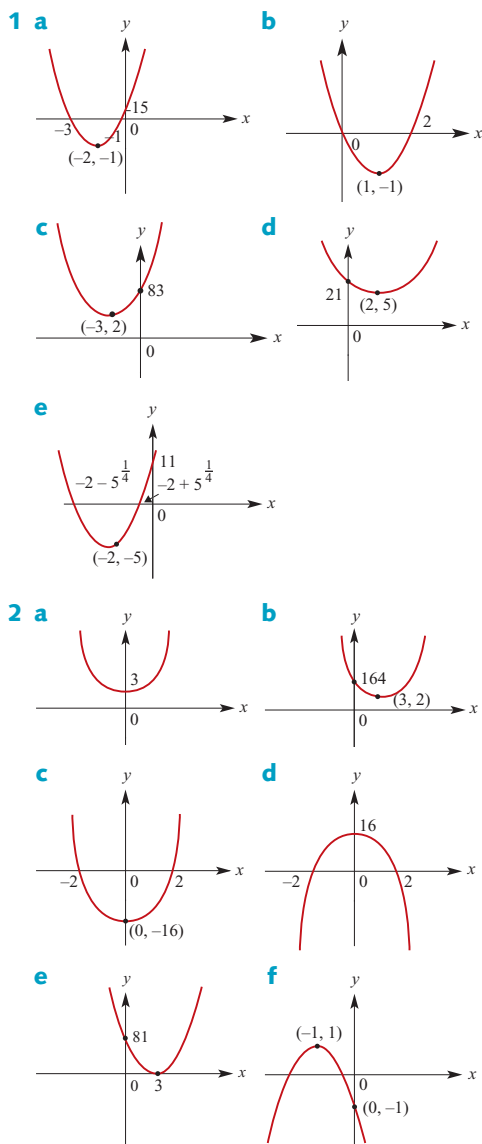
- 1 a** $(-\infty, -2] \cup [1, 3]$ **b** $[-2, -1] \cup [4, \infty)$
c $(-\infty, 1)$ **d** $(-2, 0) \cup (3, \infty)$
e $(-\infty, -1]$ **f** $[1, \infty)$
g $(4, \infty)$ **h** $(-\infty, -3]$
2 a $(-2, 0) \cup (2, \infty)$ **b** $(-\infty, 0) \cup (0, 5)$
c $(-\infty, 0] \cup \{2\}$ **d** $(-3, 0) \cup (3, \infty)$
e $[6, \infty)$ **f** $(-\infty, -\sqrt{2}) \cup (\sqrt{2}, 3)$

Exercise 6H

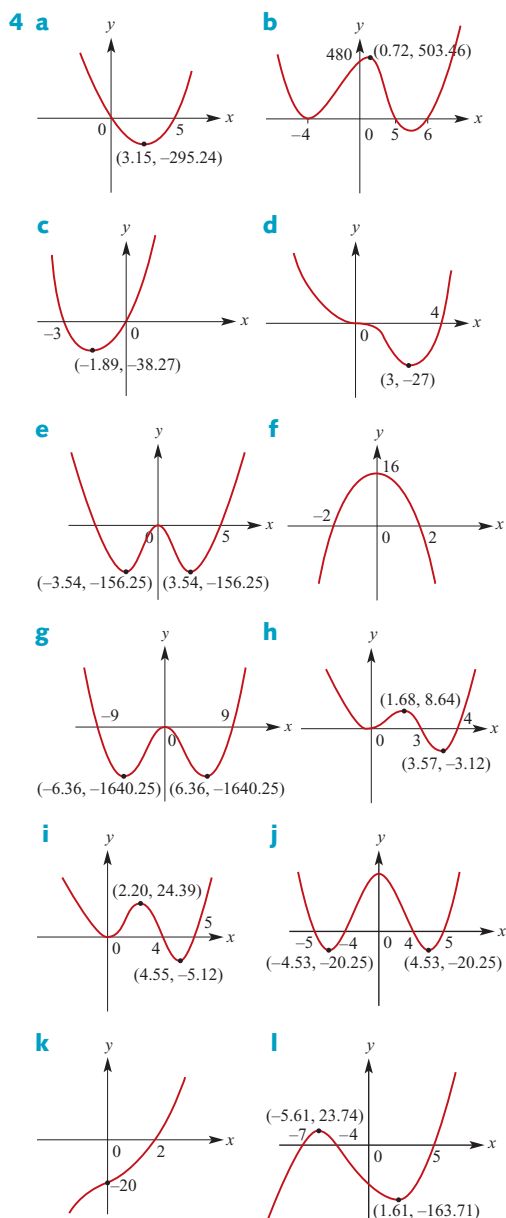
- 1 a** $a = 11$ **b** $a = 2$ **c** $a = \frac{4}{3}, b = \frac{44}{3}$

- 2 a** $y = -\frac{1}{8}(x+2)^3$ **b** $y - 2 = -\frac{1}{4}(x-3)^3$
3 $y = 2x(x-2)^2$ **4** $y = -2x(x+4)^2$
5 $y = -2(x-1)(x-3)(x+1)$ **6** $a = 36$
7 a $y = (x-3)^3 + 2$ **b** $y = \frac{23}{18}x^3 + \frac{67}{18}x^2$
c $y = 5x^3$
8 a $y = -\frac{1}{3}x^3 + \frac{4}{3}x$ **b** $y = \frac{1}{4}x(x^2 + 2)$
9 a $y = -4x^3 - 50x^2 + 96x + 270$
b $y = 4x^3 - 60x^2 + 80x + 26$
c $y = x^3 - 2x^2 + 6x - 4$
d $y = 2x^3 - 3x$
e $y = 2x^3 - 3x^2 - 2x + 1$
f $y = x^3 - 3x^2 - 2x + 1$
g $y = -x^3 - 3x^2 - 2x + 1$

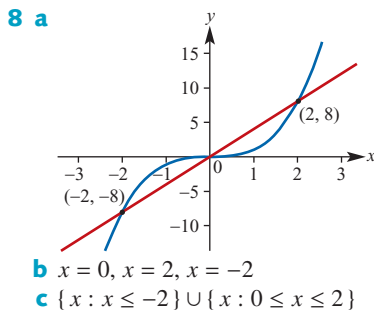
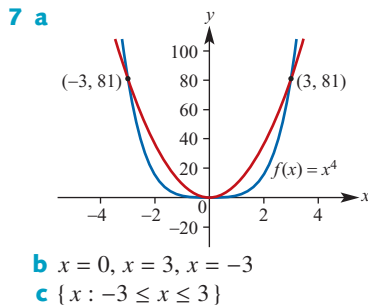
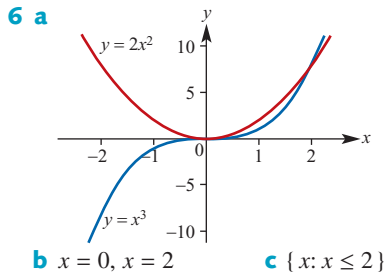
Exercise 61



- 3 a** $x = 0$ or $x = 3$
b $x = 2$ or $x = -1$ or $x = 5$ or $x = -3$
c $x = 0$ or $x = -2$ **d** $x = 0$ or $x = 6$
e $x = 0$ or $x = 3$ or $x = -3$
f $x = 3$ or $x = -3$
g $x = 0$ or $x = 4$ or $x = -4$
h $x = 0$ or $x = 4$ or $x = 3$
i $x = 0$ or $x = 4$ or $x = 5$
j $x = 2$ or $x = -2$ or $x = 3$ or $x = -3$
k $x = 4$ **l** $x = -4$ or $x = 2$



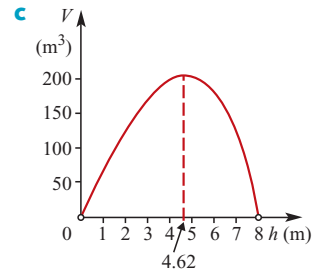
- 5 a** Even **b** Odd **c** Even **d** Odd



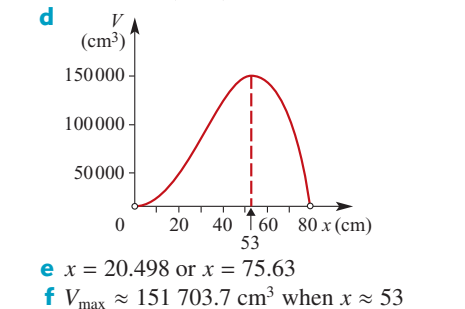
Exercise 6J

- 1 a** Length of each edge = $20 - 2x$
b $V(x) = 4x(10 - x)^2$
c $V(5) = 500$; Volume of box = 500 cm^3
d $x = 5$ or $x = \frac{5}{2}(3 - \sqrt{5})$
- 2 a** $\ell = 12 - 2x, w = 10 - 2x$
b $V = 4x(6 - x)(5 - x)$
c
-
- d** $V = 80$ **e** $x = 3.56$ or $x = 0.51$
f $V_{\max} = 96.8 \text{ cm}^3$ when $x = 1.81$
- 3 a** Surface area = $x^2 + 4xh$
b $h = \frac{75 - x^2}{4x}$ **c** $V = \frac{1}{4}(75x - x^3)$
d i $\frac{71}{2}$ **ii** $\frac{125}{2}$ **iii** 22
e $x = -2 + 3\sqrt{7}$

- 4 a** $h = 60 - 20x$ **b** $V = 600x^2(3 - x)$
c 0 **d** $x = 1$ or $x = 1 + \sqrt{3}$
5 a $x = \sqrt{64 - h^2}$ **b** $V = \frac{\pi h}{3}(64 - h^2)$



- d** Domain = $\{h : 0 < h < 8\}$ **e** 64π
f $h = 2.48$ or $h = 6.47$
g $V_{\max} \approx 206.37 \text{ m}^3$ when $h = 4.62$

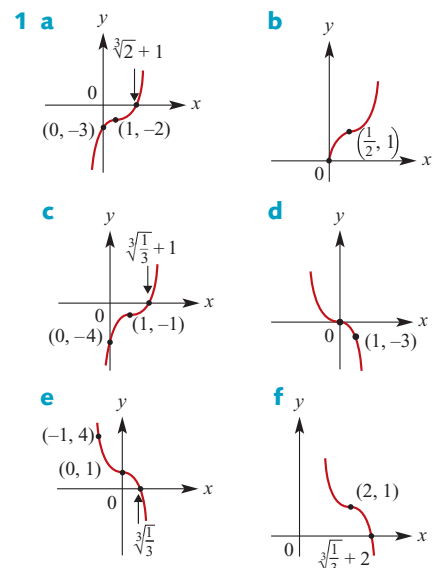


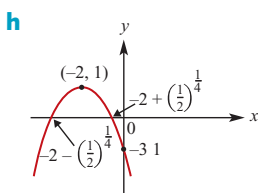
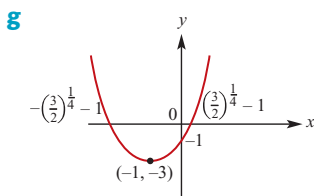
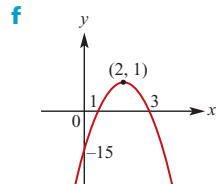
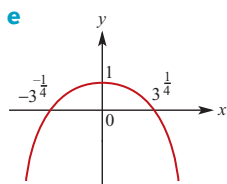
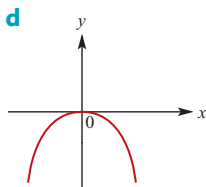
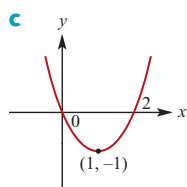
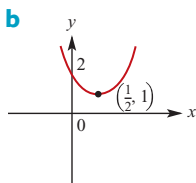
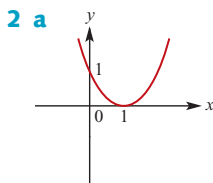
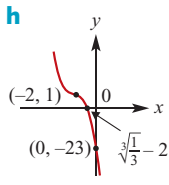
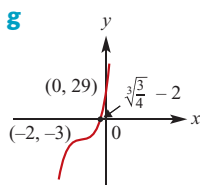
Exercise 6K

- 1 a** 1.32 **b** 1.164 **c** 1.124 or 1.451
d 2.151 **e** -1.75

Chapter 6 review

Technology-free questions





3 a $x = 2, x = -\frac{1}{2}, x = -3$

b $x = 2, x = \frac{\sqrt{17} + 1}{4}, x = \frac{1 - \sqrt{17}}{4}$

c $x = -1, x = 2, x = 6$

4 a $P\left(\frac{3}{2}\right) = 0$ and $P(-2) = 0$; $(3x + 1)$

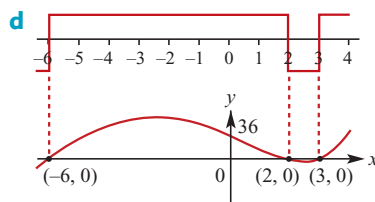
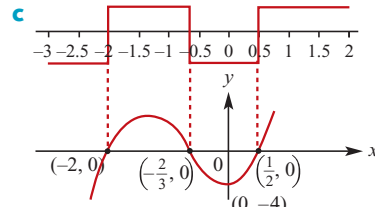
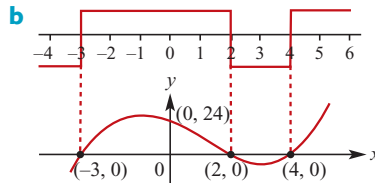
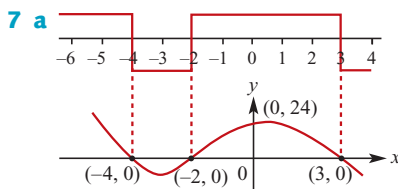
b $x = -2, \frac{1}{2}, 3$

c $x = -1, -\sqrt{11}, +\sqrt{11}$

d i $P\left(\frac{1}{3}\right) = 0$ **ii** $(3x - 1)(x + 3)(x - 2)$

5 a $f(1) = 0$ **b** $(x - 1)(x^2 + (1 - k)x + k + 1)$

6 $a = 3, b = -24$



8 a -41 **b** 12 **c** $\frac{43}{9}$

9 $y = -\frac{2}{5}(x + 2)(x - 1)(x - 5)$

10 $y = \frac{2}{81}x(x + 4)^2$

11 a $a = 3, b = 8$ **b** $(x + 3)(2x - 1)(x - 1)$

12 a $(\infty, -4] \cup \{3\}$ **b** $(-\infty, -4] \cup [-3, 2]$

c $(-\infty, -1) \cup (2, 3)$

13 a Dilation of factor 2 from the x -axis, then translation of 1 unit in the positive direction of the x -axis and 3 units in the positive direction of the y -axis

b Reflection in the x -axis, then translation of 1 unit in the negative direction of the x -axis and 2 units in the positive direction of the y -axis

c Dilation of factor $\frac{1}{2}$ from the y -axis, then translation of $\frac{1}{2}$ unit in the negative direction of the x -axis and 2 units in the negative direction of the y -axis

Multiple-choice questions

- 1** B **2** D **3** A **4** D **5** A
6 C **7** B **8** B **9** D **10** B

Extended-response questions

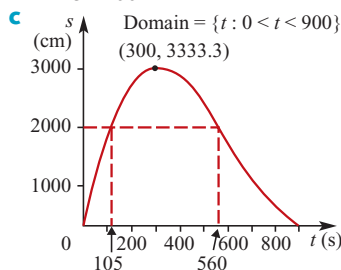
1 a $V = \pi r^2(6 - r)$ b $0 \leq r \leq 6$

c $V(3) = 27\pi$ d $r = 3$ or $r = \frac{3}{2}(1 + \sqrt{5})$

e Maximum ≈ 100.53 (correct to 2 d.p.)

2 a $v = \frac{1}{32400}(t - 900)^2$

b $s = \frac{t}{32400}(t - 900)^2$


 d No, it is not feasible as the maximum range of the taxi is less than 3.5 km (≈ 3.33 km)

e Maximum speed $\approx \frac{2000}{105} = 19$ m/s

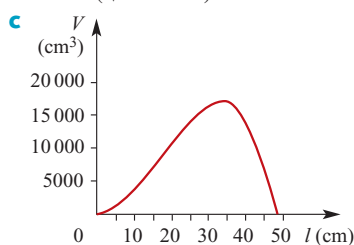
Minimum speed $\approx \frac{2000}{560} = 3.6$ m/s

3 a $R = a(x - 5)^3 + 10$ b $a = \frac{2}{25}$

c $R = \frac{12}{343}(x - 7)^3 + 12$

4 a 4730 cm^2

b $V = \ell^2(\sqrt{2365} - \ell)$



d i $\ell = 23.69$ or $\ell = 39.79$

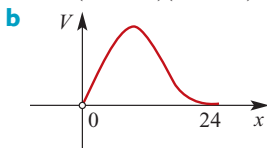
ii $\ell = 18.1$ or $\ell = 43.3$

e $V_{\max} \approx 17\,039 \text{ cm}^3$ when $\ell \approx 32.42$ cm

5 a $a = \frac{-43}{15\,000}$, $b = 0.095$, $c = \frac{-119}{150}$, $d = 15.8$

b i (5.59, 13.83) ii (0, 15.8)

6 a $V = (96 - 4x)(48 - 2x)x = 8x(24 - x)^2$



i $0 < x < 24$

ii $V_{\max} = 16\,384 \text{ cm}^3$ when $x = 8.00$

c $15\,680 \text{ cm}^3$ d $14\,440 \text{ cm}^3$ e 9720 cm^3

Chapter 7

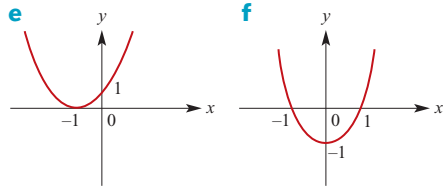
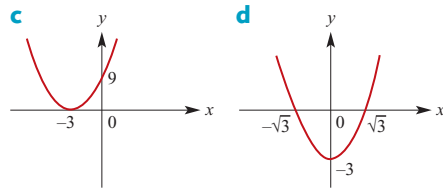
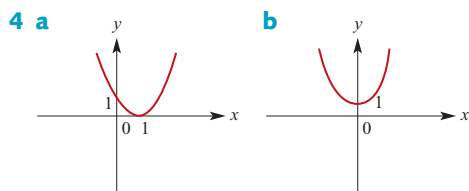
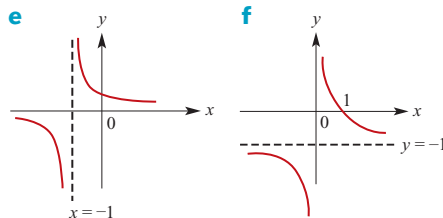
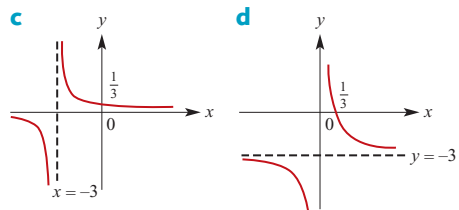
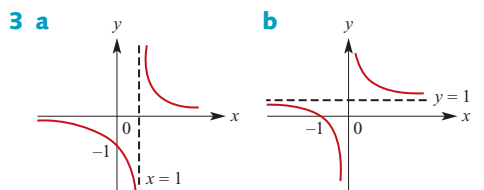
Exercise 7A

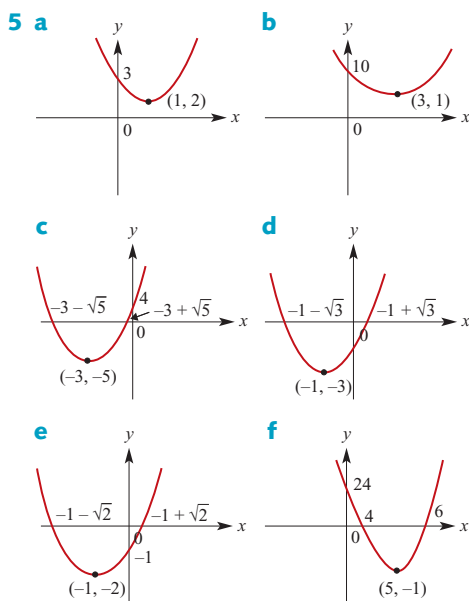
1 a (-1, 1) b (-5, 8) c (-6, 2) d (-7, 9)
e (-5, 3)

2 a $g(x) = \frac{1}{x-2} - 1$ b $g(x) = \frac{1}{(x-4)^2} + 3$

c $g(x) = (x+2)^2 - 3$ d $g(x) = (x-4)^2 - 2$

e $g(x) = \sqrt{x-2} - 1$





Exercise 7B

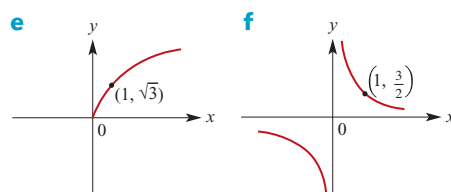
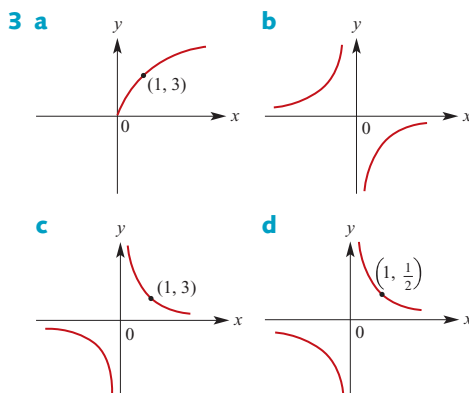
1 a $(-2, 3)$ **b** $(2, -3)$ **c** $(-2, -12)$ **d** $(-8, -3)$

2 a **i** $y = 4x^2$ **ii** $y = \frac{x^2}{25}$ **iii** $y = \frac{2x^2}{3}$
iv $y = 4x^2$ **v** $y = -x^2$ **vi** $y = x^2$

b **i** $y = \frac{1}{4x^2}$ **ii** $y = \frac{25}{x^2}$ **iii** $y = \frac{2}{3x^2}$
iv $y = \frac{4}{x^2}$ **v** $y = \frac{-1}{x^2}$ **vi** $y = \frac{1}{x^2}$

c **i** $y = \frac{1}{2x}$ **ii** $y = \frac{5}{x}$ **iii** $y = \frac{2}{3x}$
iv $y = \frac{4}{x}$ **v** $y = \frac{-1}{x}$ **vi** $y = \frac{-1}{x}$

d **i** $y = \sqrt{2x}$ **ii** $y = \sqrt{\frac{x}{5}}$
iii $y = \frac{2\sqrt{x}}{3}$ **iv** $y = 4\sqrt{x}$
v $y = -\sqrt{x}$ **vi** $y = \sqrt{-x}, x \leq 0$



Exercise 7C

1 a $y = 3\sqrt{x-2}$ **b** $y = -\sqrt{x+3}$
c $y = -3\sqrt{x}$ **d** $y = -\sqrt{\frac{x}{2}}$
e $y = 2\sqrt{x-2} - 3$ **f** $y = \sqrt{\frac{x+2}{2}} - 3$

2 a $y = \frac{3}{x-2}$ **b** $y = \frac{-1}{x+3}$
c $y = -\frac{3}{x}$ **d** $y = -\frac{2}{x}$
e $y = \frac{2}{x-2} - 3$ **f** $y = \frac{2}{x+2} - 3$

3 a $y = 3(x-2)^{\frac{1}{3}}$ **b** $y = -(x+3)^{\frac{1}{3}}$
c $y = -3x^{\frac{1}{3}}$ **d** $y = -\left(\frac{x}{2}\right)^{\frac{1}{3}}$
e $y = 2(x-2)^{\frac{1}{3}} - 3$ **f** $y = \left(\frac{x+2}{2}\right)^{\frac{1}{3}} - 3$

Exercise 7D

- 1 a** **i** A dilation of factor 2 from the x -axis, then a translation of 1 unit in the positive direction of the x -axis and 3 units in the positive direction of the y -axis
ii A reflection in the x -axis, then a translation of 1 unit in the negative direction of the x -axis and 2 units in the positive direction of the y -axis
iii A dilation of factor $\frac{1}{2}$ from the y -axis, then a translation of $\frac{1}{2}$ unit in the negative direction of the x -axis and 2 units in the negative direction of the y -axis
- b** **i** A dilation of factor 2 from the x -axis, then a translation of 3 units in the negative direction of the x -axis
ii A translation of 3 units in the negative direction of the x -axis and 2 units in the positive direction of the y -axis
iii A translation of 3 units in the positive direction of the x -axis and 2 units in the negative direction of the y -axis
- c** **i** A translation of 3 units in the negative direction of the x -axis and 2 units in the positive direction of the y -axis
ii A dilation of factor $\frac{1}{3}$ from the y -axis, then a dilation of factor 2 from the x -axis

- iii A reflection in the x -axis, then a translation of 2 units in the positive direction of the y -axis

2 a $(x, y) \rightarrow (x - 3, \frac{y+7}{5})$
 b $(x, y) \rightarrow (3x + 2, y - 5)$
 c $(x, y) \rightarrow (3x + 1, -\frac{y-7}{3})$
 d $(x, y) \rightarrow (-(x-4), \frac{y}{2})$
 e $(x, y) \rightarrow (-(x-4), \frac{15-y}{2})$

Exercise 7E

1 $\mathbf{X} + \mathbf{Y} = \begin{bmatrix} 4 \\ -2 \end{bmatrix}$ $2\mathbf{X} = \begin{bmatrix} 2 \\ -4 \end{bmatrix}$ $4\mathbf{Y} + \mathbf{X} = \begin{bmatrix} 13 \\ -2 \end{bmatrix}$
 $\mathbf{X} - \mathbf{Y} = \begin{bmatrix} -2 \\ -2 \end{bmatrix}$ $-3\mathbf{A} = \begin{bmatrix} -3 & 3 \\ -6 & -9 \end{bmatrix}$
 $-3\mathbf{A} + \mathbf{B} = \begin{bmatrix} 1 & 3 \\ -7 & -7 \end{bmatrix}$
 2 $2\mathbf{A} = \begin{bmatrix} 2 & -2 \\ 0 & 4 \end{bmatrix}$ $-3\mathbf{A} = \begin{bmatrix} -3 & 3 \\ 0 & -6 \end{bmatrix}$
 $-6\mathbf{A} = \begin{bmatrix} -6 & 6 \\ 0 & -12 \end{bmatrix}$
 3 a $\begin{bmatrix} 6 & 4 \\ -4 & -4 \end{bmatrix}$ b $\begin{bmatrix} 0 & -9 \\ 12 & 3 \end{bmatrix}$
 c $\begin{bmatrix} 6 & -5 \\ 8 & -1 \end{bmatrix}$ d $\begin{bmatrix} -6 & -13 \\ 16 & 7 \end{bmatrix}$
 4 a $\begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix}$ b $\begin{bmatrix} -2 & 3 \\ 6 & 3 \end{bmatrix}$ c $\begin{bmatrix} 3 & 3 \\ -1 & 7 \end{bmatrix}$
 5 $\mathbf{X} = \begin{bmatrix} 2 & 4 \\ 0 & -3 \end{bmatrix}$ $\mathbf{Y} = \begin{bmatrix} -\frac{9}{2} & -\frac{23}{2} \\ -1 & 11 \end{bmatrix}$
 6 $\mathbf{AX} = \begin{bmatrix} 4 \\ -5 \end{bmatrix}$ $\mathbf{BX} = \begin{bmatrix} 6 \\ -1 \end{bmatrix}$ $\mathbf{IX} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$
 $\mathbf{AI} = \begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix}$ $\mathbf{IB} = \begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix}$
 $\mathbf{AB} = \begin{bmatrix} 3 & -2 \\ -3 & 3 \end{bmatrix}$ $\mathbf{BA} = \begin{bmatrix} 3 & -6 \\ -1 & 3 \end{bmatrix}$
 $\mathbf{A}^2 = \begin{bmatrix} 3 & -8 \\ -4 & 11 \end{bmatrix}$ $\mathbf{B}^2 = \begin{bmatrix} 9 & 0 \\ 0 & 1 \end{bmatrix}$
 7 $\mathbf{AX} = \begin{bmatrix} 4 \\ -9 \end{bmatrix}$ $\mathbf{BX} = \begin{bmatrix} -3 \\ -4 \end{bmatrix}$ $\mathbf{CX} = \begin{bmatrix} -5 \\ -4 \end{bmatrix}$
 $\mathbf{AC} = \begin{bmatrix} 4 & 5 \\ -9 & 1 \end{bmatrix}$ $\mathbf{CB} = \begin{bmatrix} -3 & 6 \\ 6 & 2 \end{bmatrix}$
 $\mathbf{AB} = \begin{bmatrix} -6 & -2 \\ 3 & 8 \end{bmatrix}$ $\mathbf{BA} = \begin{bmatrix} -6 & 3 \\ -2 & 8 \end{bmatrix}$
 $\mathbf{A}^2 = \begin{bmatrix} 5 & -6 \\ -6 & 17 \end{bmatrix}$ $\mathbf{B}^2 = \begin{bmatrix} 9 & 0 \\ 0 & 4 \end{bmatrix}$

Exercise 7F

1 a 1 b $\begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}$ c 2 d $\frac{1}{2} \begin{bmatrix} 2 & 2 \\ -3 & -2 \end{bmatrix}$
 2 a $\begin{bmatrix} -1 & 1 \\ -4 & 3 \end{bmatrix}$ b $\begin{bmatrix} \frac{2}{7} & -1 \\ 1 & \frac{3}{14} \end{bmatrix}$ c $\begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{k} \end{bmatrix}$
 3 a $\mathbf{A}^{-1} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ 0 & -1 \end{bmatrix}$ $\mathbf{B}^{-1} = \begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix}$
 b $\mathbf{AB} = \begin{bmatrix} 5 & 1 \\ -3 & -1 \end{bmatrix}$ $(\mathbf{AB})^{-1} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ -\frac{3}{2} & -\frac{5}{2} \end{bmatrix}$
 c $\mathbf{A}^{-1}\mathbf{B}^{-1} = \begin{bmatrix} -1 & \frac{1}{2} \\ 3 & -1 \end{bmatrix}$ $\mathbf{B}^{-1}\mathbf{A}^{-1} = (\mathbf{AB})^{-1}$
 4 a $\begin{bmatrix} -\frac{1}{2} & \frac{3}{2} \\ 1 & -2 \end{bmatrix}$ b $\begin{bmatrix} 0 & 7 \\ 1 & -8 \end{bmatrix}$ c $\begin{bmatrix} \frac{5}{2} & -7 \\ \frac{11}{2} & -21 \\ \frac{2}{2} & \frac{2}{2} \end{bmatrix}$
 5 a $\begin{bmatrix} -3 & 11 \\ 8 & 8 \\ 1 & 7 \\ 16 & 16 \end{bmatrix}$ b $\begin{bmatrix} -11 & 17 \\ 16 & 16 \\ -1 & 3 \\ 4 & 4 \end{bmatrix}$

Exercise 7G

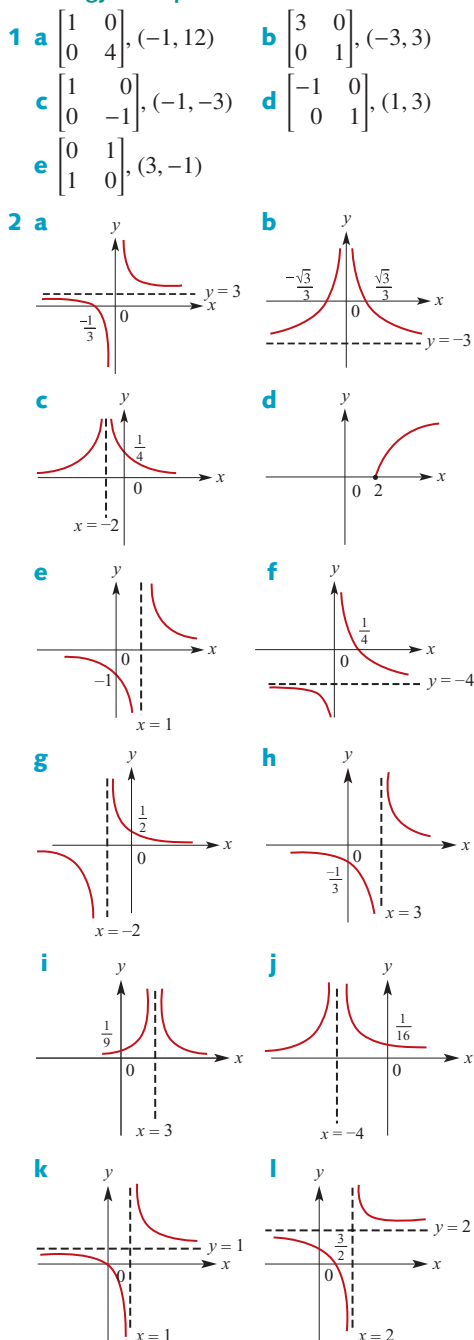
1 a $\begin{bmatrix} -8 \\ 3 \end{bmatrix}$ b $\begin{bmatrix} -3a-b \\ -a+3b \end{bmatrix}$
 2 $(1, 0) \rightarrow (2, -4)$, $(0, 1) \rightarrow (-1, 3)$,
 $(3, 2) \rightarrow (4, -6)$
 3 a $(2, 1), (-4, 1)$ b $(-2, 0), (2, 2)$
 c $(2, 3), (4, -5)$
 4 a $(6, 21)$ b $(12, 7)$ c $(6, -7)$
 d $(-6, 7)$ e $(7, 6)$
 5 a $\begin{bmatrix} 2 & 3 \\ 3 & -1 \end{bmatrix}$ c $\begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix}$
 6 $\begin{bmatrix} 1 & 0 \\ 0 & -2 \end{bmatrix}$
 7 a $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$ b $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ c $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$
 d $\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$ e $\begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{2} \end{bmatrix}$ f $\begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix}$
 8 a $\begin{bmatrix} 0 & -2 \\ -1 & 0 \end{bmatrix}$ b $(-4, -3)$ c $a = -2, b = -3$
 9 $\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \mathbf{X} + \begin{bmatrix} 3 \\ 4 \end{bmatrix} = \mathbf{X}'$
 $\Rightarrow \mathbf{X} = \frac{1}{2} \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} (\mathbf{X}' - \begin{bmatrix} 3 \\ 4 \end{bmatrix})$
 $\Rightarrow x = \frac{1}{2}(x' - 3), y = y' - 4$
 10 $\frac{1}{2} \begin{bmatrix} -1 & -1 \\ -21 & 5 \end{bmatrix}$

Exercise 7H

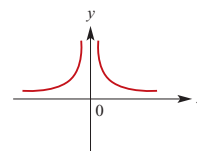
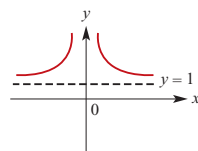
1 $y = \frac{-2x^2}{9} - \frac{2x}{3} - 4$ 2 $y = \frac{-x^3}{32} - x$
 3 $y = \frac{-3x+18}{4}$ 4 $y = \frac{x}{4} - 4$ 5 $y = \frac{x}{4} + 3$
 6 $y = \frac{x+21}{4}$ 7 $y = \frac{-3x^3}{4} - \frac{9x^2}{2} + 14$

Chapter 7 review

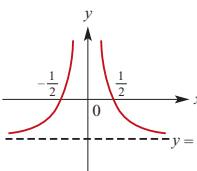
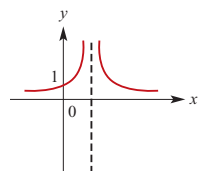
Technology-free questions



3 a Range = $(1, \infty)$ b Range = $(0, \infty)$



c Range = $(0, \infty)$ d Range = $(-4, \infty)$



4 $\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} x' \\ y' \end{bmatrix}, x = x' - 2, y = \frac{y' - 3}{2}$

5 a i $(x, y) \rightarrow (x - 1, 3y + 2)$
 ii $(x, y) \rightarrow (x - 2, -2y + 3)$

iii $(x, y) \rightarrow \left(\frac{x-1}{3}, y-1\right)$

b i $(x, y) \rightarrow (x - 2, 4y)$

ii $(x, y) \rightarrow (x - 6, y - 12)$

iii $(x, y) \rightarrow (x + 3, 4y - 5)$

c i $(x, y) \rightarrow (x + 4, y + 2)$

ii $(x, y) \rightarrow \left(\frac{x}{2}, 2y\right)$

iii $(x, y) \rightarrow (x, -2y + 3)$

6 $\begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -2 \\ 3 \end{bmatrix} = \begin{bmatrix} x' \\ y' \end{bmatrix}, x = \frac{x' + 2}{3}, y = y' - 3$

Multiple-choice questions

1 C 2 D 3 A 4 A
 5 E 6 D 7 B 8 C

Extended-response questions

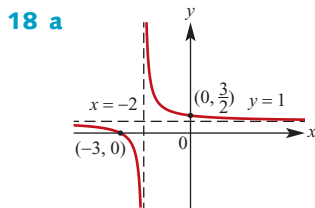
1 a $k = \frac{1}{4}$ b $h = \frac{-1}{4}$
 2 a $h = -1 \pm 2\sqrt{2}$ b $a = \pm 2\sqrt{2}$
 c $a = -8, b = 16$
 3 a $k = 10$
 b i $h > 2 + \sqrt{10}$ ii $h < 2 - \sqrt{10}$
 iii $2 - \sqrt{10} < h < 2 + \sqrt{10}$
 4 a $x = 5$ or $x = -2$ or $x = 7$
 b $x = 1$ or $x = -6$ or $x = 3$
 c $k = -60,$
 $x = 0$ or $x = 2 + \sqrt{21}$ or $x = 2 - \sqrt{21}$
 d $h = -3$ or $h = -5$ or $h = 4$
 e $-5 < h < -3$

Chapter 8

Technology-free questions

1 a $M\left(\frac{1}{2}, -\frac{3}{2}\right), N\left(\frac{3}{2}, 4\right)$ b $m_{BC} = m_{MN} = \frac{11}{2}$
 2 a -6 b 69 c -15
 3 a $12a^2 - 4$ b $3a^2 - 6a - 1$ c $12a$

- 4 a** No **b** $x = -\frac{3}{7}$ **c** $x \leq -\frac{3}{7}$ **d** $k = -\frac{3}{16}$
5 $x = 2$ and $y = 3$, or $x = 3$ and $y = 2$
6 $AB = BC = CD = DA = 5\sqrt{2}$,
 $m_{BC} = m_{AD} = 1$ and $m_{AB} = m_{CD} = -7$
7 a $y = (x+2)^2 - 13$ **b** $y = \left(x - \frac{3}{2}\right)^2 - \frac{53}{4}$
c $y = 2\left(x - \frac{3}{4}\right)^2 + \frac{79}{8}$
8 a $\left(\frac{1-\sqrt{41}}{2}, 3-2\sqrt{41}\right), \left(\frac{\sqrt{41}+1}{2}, 2\sqrt{41}+3\right)$
b $(2, 6)$ **c** $(-4, 14)$
9 a $x < \frac{-3-\sqrt{29}}{2}$ or $x > \frac{-3+\sqrt{29}}{2}$
b $x \leq \frac{5-\sqrt{65}}{4}$ or $x \geq \frac{5+\sqrt{65}}{4}$ **c** $x \geq -4$
d $\frac{1}{2} \leq x \leq 3$ or $x \leq -4$ **e** $x \leq 4$
10 a $\mathbb{R} \setminus \{\frac{5}{2}\}$ **b** $(-\infty, 5]$ **c** $\mathbb{R}$
d $\mathbb{R} \setminus \{2\}$ **e** $\mathbb{R}$ **f** $\mathbb{R} \setminus \{\frac{2}{3}\}$
11 $p = -38, (x-3)(x+4)(3x-2)$
12 $a = -5, R = -35$
13 a $f^{-1}: [1, 4] \rightarrow \mathbb{R}, f^{-1}(x) = \sqrt{x}$
b $f^{-1}: [0, 3] \rightarrow \mathbb{R}, f^{-1}(x) = 2-x$
c $f^{-1}: (-4, \infty) \rightarrow \mathbb{R}, f^{-1}(x) = -\sqrt{x+4}$
d $f^{-1}: [3, \infty) \rightarrow \mathbb{R}, f^{-1}(x) = 2 - (x-3)^2$
e $f^{-1}: \mathbb{R} \rightarrow \mathbb{R}, f^{-1}(x) = (x-8)^{\frac{1}{3}} + 2$
14 a $3b + 2f = 18.20$ **b** \$2.80
15 a $k = 1$ **b** $k = -16$
16 a $\frac{2}{5}$ **b** $2y + 5x - 17 = 0$
17 a $a\left(x + \frac{1}{a}\right)^2 + \frac{a^2-1}{a}$ **b** $\left(-\frac{1}{a}, \frac{a^2-1}{a}\right)$
c $a = \pm 1$ **d** $a \in (-1, 1)$



- b** $A\left(0, \frac{3}{2}\right), B(-3, 0)$ **c** $y = \frac{1}{2}x + \frac{3}{2}$
d $\left(-\frac{3}{2}, \frac{3}{4}\right)$ **e** $y = -2x - \frac{9}{4}$

Multiple-choice questions

- 1** B **2** D **3** C **4** A **5** D
6 D **7** C **8** C **9** C **10** A
11 E **12** B **13** A **14** E **15** B
16 D **17** D **18** E **19** B **20** D
21 E **22** D **23** B **24** D **25** D
26 A **27** B **28** D **29** D **30** B
31 C **32** A **33** C **34** A **35** B
36 C **37** D **38** E **39** E **40** C
41 C **42** C **43** A

Extended-response questions

- 1 a** $4b - 5c - d = 41, 2b - 7c - d = 53,$
 $-4b + 3c - d = 25$

b $x^2 + y^2 - 2x - 4y - 29 = 0$

- 2 a** $c = -b - 8$ **b** $x = 0$ or $x = -b$

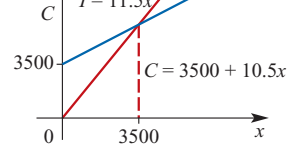
c $y = 0$ or $y = b + 8$ **d** $b = -8$

- 3 a** $x \leq a$ **b** $\left(\frac{\sqrt{4a+1}-1}{2}, \frac{\sqrt{4a+1}-1}{2}\right)$

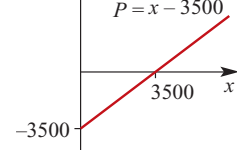
c $a = 2$ **d** $a = 6$ **e** $a = c^2 + c$

- 4 a** $C = 3500 + 10.5x$ **b** $I = 11.5x$

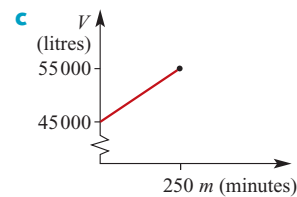
c I and C **d** 3500



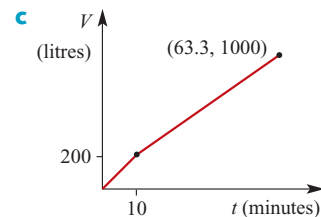
- e** Profit **f** 5500



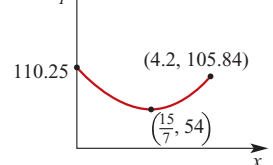
- 5 a** $V = 45\,000 + 40m$ **b** 4 hours 10 minutes



- 6 a** 200 L **b** $V = \begin{cases} 20t & 0 \leq t \leq 10 \\ 15t + 50 & 10 < t \leq \frac{190}{3} \end{cases}$



- 7 a** $A_R = 6x^2$
b $A_S = (10.5 - 2.5x)^2$
c $0 \leq x \leq 4.2$
d $A_T = 12.25x^2 - 52.5x + 110.25$
e A_T



- f** 110.25 cm² (when area of rectangle = 0)

- g** Rectangle 9×6 and square 3×3 ,
 or rectangle $\frac{27}{7} \times \frac{18}{7}$ and square $\frac{51}{7} \times \frac{51}{7}$

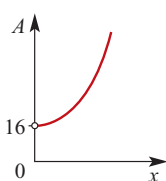
8 a 20 m b 20 m c 22.5 m

9 a $A = 10x^2 + 28x + 16$

b i 54 cm² ii 112 cm²

c 3 cm

d



e $V = 2x^3 + 8x^2 + 8x$ f $x = 3$ g $x = 6.66$

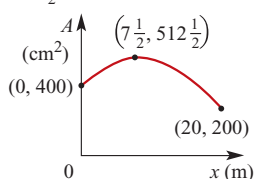
10 a i $A = (10 + x)y - x^2$ ii $P = 2(y + x + 10)$

b i $A = 400 + 30x - 2x^2$

ii 512 $\frac{1}{2}$ m²

iii $0 \leq x \leq 20$

iv

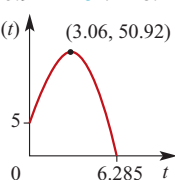


11 a $A = 6x^2 + 7xy + 2y^2$

c i $x = 0.5$ m ii $y = 0.25$ m

12 a 50.9 m b $t = 6.12$ seconds

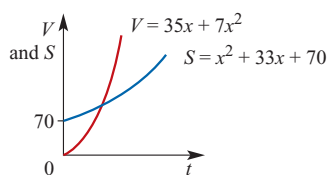
c $h(t)$ d 6.285 seconds



13 a $x + 5$ b $V = 35x + 7x^2$

c $S = x^2 + 33x + 70$

d



e 3.25 cm f 10 cm

14 a $2y + 3x = 22$

b i $B(0, 11)$ ii $D(8, -1)$

c 52 units²

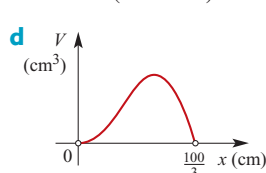
d 6.45 units

15 a 25 km/h b Tap A 60 min; Tap B 75 min

c 4 cm

16 a $h = 100 - 3x$ b $V = 2x^2(100 - 3x)$

c $0 < x < \frac{100}{3}$



e i $x = 18.142$ or $x = 25.852$

ii $x = 12.715$ or $x = 29.504$

f $V_{\max} = 32\,921.811$ cm³ when $x = 22.222$

g i $S = 600x - 14x^2$

ii $S_{\max} = \frac{45000}{7}$ cm² when $x = \frac{150}{7}$

h $x = 3.068$ or $x = 32.599$

17 a $y = (7.6 \times 10^{-5})x^3 - 0.0276x^2 + 2.33x$

b $y = (7.6 \times 10^{-5})x^3 - 0.0276x^2 + 2.33x + 5$

c 57.31 m

18 a $y = \frac{3}{4}x - 4$

b $y = -\frac{4}{3}x + \frac{38}{3}$

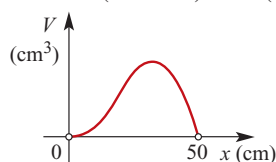
c $D(8, 2)$ d 5 units

e 50 units²

19 a i $y = 250 - 5x$

ii $V = x^2(250 - 5x) = 5x^2(50 - x)$

b



c (0, 50)

d $x = 11.378$ or $x = 47.813$

e $V_{\max} = 92\,592.59$ cm³ when $x = 33.33$ and $y = 83.33$

Chapter 9

Exercise 9A

1 $\{H, T\}$

2 $\{1, 2, 3, 4, 5, 6\}$

3 a 52 b 4

c clubs ♣, hearts ♥, spades ♠, diamonds ♦

d clubs and spades are black;

diamonds and hearts are red

e 13

f ace, king, queen, jack

g 4 h 16

4 a $\{0, 1, 2, 3, 4, 5\}$

b $\{0, 1, 2, 3, 4, 5, 6\}$

c $\{0, 1, 2, 3\}$

5 a $\{0, 1, 2, 3, \dots\}$

b $\{0, 1, 2, 3, \dots, 41\}$

c $\{1, 2, 3, \dots\}$

6 a $\{2, 4, 6\}$ b $\{FFF\}$

c $\emptyset$

7 a $\frac{1}{2}$

b $\frac{3}{10}$

c $\frac{3}{20}$

8 a $\frac{4}{15}$

b $\frac{2}{3}$

c $\frac{4}{15}$

9 a $\frac{1}{4}$

b $\frac{1}{2}$

c $\frac{4}{13}$

d $\frac{2}{13}$

10 a $\frac{9}{13}$

b $\frac{10}{13}$

c $\frac{5}{13}$

d $\frac{1}{13}$

11 a $\frac{1}{365}$

b $\frac{30}{365}$

c $\frac{30}{365}$

d $\frac{90}{365}$

12 a $\frac{1}{9}$

b $\frac{1}{3}$

c $\frac{5}{9}$

d $\frac{4}{9}$

13 $\frac{1}{3}$

14 0.4

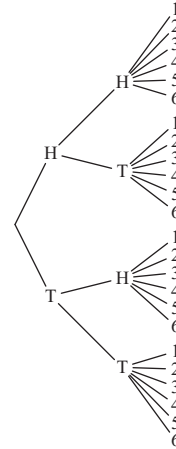
- 15 a** $\frac{1}{3}$ **b** $\frac{1}{8}$ **c** $\frac{1}{4}$
16 a $\frac{1}{7}$ **b** $\frac{5}{7}$
17 a $\frac{3}{4}$ **b** $\frac{1}{2}$ **c** $\frac{10}{13}$ **d** $\frac{23}{26}$
18 $\frac{1}{5}, \frac{1}{5}, \frac{1}{5}, \frac{2}{5}$
19 a $\frac{1}{13}, \frac{2}{13}, \frac{2}{13}, \frac{2}{13}, \frac{2}{13}, \frac{4}{13}$ **b** $\frac{9}{13}$

Exercise 9B

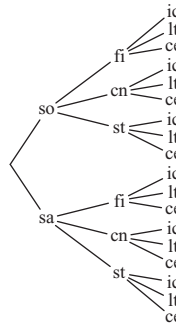
- 1 a** $\frac{17}{50}$ **b** $\frac{1}{10}$ **c** $\frac{4}{15}$ **d** $\frac{1}{200}$
2 a No **b** Answers will vary
c Answers will vary **d** Yes
e As the number of trials approaches infinity, the relative frequency approaches the value of the probability
3 $\Pr(\text{a 6 from first die}) \approx \frac{78}{500} = 0.156$
 $\Pr(\text{a 6 from second die}) \approx \frac{102}{700} \approx 0.146$
 Choose first die, as higher probability of a 6
4 a $\frac{17}{20}$ **b** $\frac{4}{5}$ **c** $\frac{9}{10}$ **d** 51
5 0.445
6 a $\frac{\pi}{4}$ **b** $\frac{\pi}{4} \approx 0.7855$
7 $\frac{3}{4}$
8 a $\frac{1}{3}$ **b** $\frac{1}{6}$ **c** $\frac{5}{6}$
9 a $\frac{4\pi}{25}$ **b** $1 - \frac{4\pi}{25} \approx 0.4973$
10 a i x^2 **ii** $\frac{1}{4}\pi x^2$ **iii** $\frac{1}{16}\pi x^2$
b i $\frac{\pi}{16}$ **ii** $\frac{3\pi}{16}$ **iii** $1 - \frac{\pi}{4}$

Exercise 9C

- 1 a** $\frac{1}{4}$ **b** $\frac{1}{4}$
2 a $\frac{1}{2}$ **b** $\frac{1}{2}$ **c** $\frac{1}{4}$
3 a $\frac{1}{2}$ **b** $\frac{1}{18}$ **c** $\frac{5}{18}$
4 a $\frac{1}{12}$ **b** $\frac{1}{2}$ **c** $\frac{7}{12}$
5 a $\frac{3}{8}$ **b** $\frac{3}{8}$ **c** $\frac{1}{8}$ **d** $\frac{1}{8}$
6 a $\frac{1}{2}$ **b** $\frac{1}{4}$ **c** $\frac{3}{4}$
7 $\frac{1}{4}$

8 a


- b i** $\frac{1}{24}$ **ii** $\frac{1}{4}$ **iii** $\frac{1}{8}$ **iv** $\frac{1}{2}$

9 a


- b i** $\frac{1}{18}$ **ii** $\frac{1}{3}$ **iii** $\frac{1}{6}$ **iv** $\frac{2}{3}$
c i $\frac{1}{36}$ **ii** $\frac{1}{2}$ **iii** $\frac{15}{36}$ **iv** $\frac{1}{12}$

10 a

		2nd ball				
		1	2	3	4	5
1st ball	1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)
	2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)
	3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)
	4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)
	5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)

- b i** $\frac{4}{25}$ **ii** $\frac{4}{5}$ **iii** $\frac{3}{25}$

Exercise 9D

- 1 a** {1, 2, 3, 4, 6} **b** {2, 4}
c {5, 6, 7, 8, 9, 10} **d** {1, 3}
e {1, 3, 5, 6, 7, 8, 9, 10} **f** {5, 7, 8, 9, 10}
2 a {1, 2, 3, 5, 6, 7, 9, 10, 11}
b {1, 3, 5, 7, 9, 11} **c** {2, 4, 6, 8, 10, 12}
d {1, 3, 5, 7, 9, 11} **e** {1, 3, 5, 7, 9, 11}
3 a {E, H, M, S} **b** {C, H, I, M}
c {A, C, E, I, S, T} **d** {H, M}
e {C, E, H, I, M, S} **f** {H, M}

- 4 a 20 b 45
 5 a $\frac{2}{3}$ b 0 c $\frac{1}{2}$ d $\frac{5}{6}$
 6 a $\frac{1}{2}$ b $\frac{1}{3}$ c $\frac{1}{6}, \frac{2}{3}$
 7 a $\frac{7}{18}$ b $\frac{4}{18} = \frac{2}{9}$ c $\frac{2}{18} = \frac{1}{9}$ d $\frac{1}{2}$
 8 a $\frac{3}{5}$ b $\frac{1}{5}$ c $\frac{1}{10}$ d $\frac{2}{5}$
 9 $\Pr(A \cup B) = 0.7$
 10 $\Pr(A \cup B) = 0.47$
 11 a $\Pr(A \cap B) = 0.28$ b $\Pr(A \cup B) = 0.45$
 12 a $\Pr(A \cap B) = 0.45$ b $\Pr(A \cup B) = 0.58$
 13 a $\Pr(A \cap B) = 0$ b $\Pr(A \cup B) = 0.7$
 14 a $\Pr(A \cap B) = 0$ b $\Pr(A \cup B) = 0.23$
 15 $\Pr(A \cap B) = 0.2$
 16 $\Pr(A \cap B) = 0.05$
 17 $\Pr(A \cup B) = 0.7$
 18 0.32
 19 a 0.43 b 0.29

Exercise 9E

- 1 a 0.2 b 0.5 c 0.3 d 0.7
 2 a 0.75 b 0.4 c 0.87 d 0.48
 3 a 0.63 b 0.23 c 0.22 d 0.77
 4 a 0.45 b 0.40 c 0.25 d 0.70
 5 a 0.9 b 0.6 c 0.1 d 0.9
 6 a 95% b 5%
 7 a $A = \{J\heartsuit, Q\heartsuit, K\heartsuit, A\heartsuit, J\clubsuit, Q\clubsuit, K\clubsuit, A\clubsuit, J\spadesuit, Q\spadesuit, K\spadesuit, A\spadesuit, J\diamondsuit, Q\diamondsuit, K\diamondsuit, A\diamondsuit, J\heartsuit, Q\heartsuit, K\heartsuit, A\heartsuit\}$
 $C = \{2\heartsuit, 3\heartsuit, 4\heartsuit, 5\heartsuit, 6\heartsuit, 7\heartsuit, 8\heartsuit, 9\heartsuit, 10\heartsuit, J\heartsuit, Q\heartsuit, K\heartsuit, A\heartsuit\}$
 b i $\Pr(\text{a picture card}) = \frac{4}{13}$
 ii $\Pr(\text{a heart}) = \frac{1}{4}$
 iii $\Pr(\text{a heart picture card}) = \frac{1}{13}$
 iv $\Pr(\text{a picture card or a heart}) = \frac{25}{52}$
 v $\Pr(\text{a picture card or a club, diamond or spade}) = \frac{43}{52}$
 8 a $\frac{8}{15}$ b $\frac{7}{10}$ c $\frac{2}{15}$ d $\frac{1}{3}$
 9 a 0.8 b 0.57 c 0.28 d 0.08
 10 a 0.81 b 0.69 c 0.74 d 0.86
 11 a 0 b 1 c $\frac{1}{5}$ d $\frac{1}{3}$
 12 a 0.88 b 0.58 c 0.30 d 0.12

Exercise 9F

- 1 $\frac{1}{4}$ 2 $\frac{1}{3}$ 3 $\frac{1}{13}$ 4 $\frac{7}{19}$ 5 $\frac{1}{6}$
 6 a $\frac{4}{17}$ b $\frac{4}{7}$
 7 $\frac{7}{12}$

- 8 a $\frac{375}{500} = \frac{3}{4}$ b $\frac{225}{300} = \frac{3}{4}$
 9 a $\frac{65}{284}$ b $\frac{137}{568}$ c $\frac{21}{65}$ d $\frac{61}{246}$
 10 a 0.06 b 0.2
 11 a $\frac{4}{7}$ b 0.3 c $\frac{15}{22}$
 12 a 0.2 b 0.5 c 0.4
 13 a 0.2 b $\frac{10}{27}$ c $\frac{1}{3}$
 14 a 0.3 b 0.75
 15 16%
 16 $\frac{1}{5}$
 17 a $\frac{1}{16}$ b $\frac{1}{169}$ c $\frac{1}{4}$ d $\frac{16}{169}$
 18 a $\frac{1}{17}$ b $\frac{1}{221}$ c $\frac{13}{51}$ d $\frac{20}{221}$
 19 $0.230\ 808 \approx 0.231$
 20 a $\frac{15}{28}$ b $\frac{1}{2}$ c $\frac{1}{2}$ d $\frac{2}{5}$
 e $\frac{3}{7}$ f $\frac{8}{13}$ g $\frac{5}{28}$ h $\frac{3}{14}$
 21 a 0.85 b 0.6 c 0.51 d 0.51
 22 0.4, 68%
 23 a i 0.444 ii 0.4 iii 0.35 iv 0.178 v 0.194
 b 0.372
 c i 0.478 ii 0.425
 24 a i 0.564 ii 0.05 iii 0.12 iv 0.0282 v 0.052
 b 0.081
 c 0.35
 25 a $\frac{1}{6}$ b $\frac{53}{90}$ c $\frac{15}{53}$
 26 a $B \subseteq A$ b $A \cap B = \emptyset$ c $A \subseteq B$

Exercise 9G

- 1 $\Pr(\text{male and support guns}) = 0.35$;
 $\Pr(\text{male}) \times \Pr(\text{support guns}) = \frac{60}{100} \times \frac{65}{100} = 0.39 \neq 0.35$;
 therefore not independent
 2 $\Pr(\text{male and prefer sport}) = 0.45$;
 $\Pr(\text{male}) \times \Pr(\text{prefer sport}) = 0.45$;
 therefore independent
 3 $\Pr(\text{speeding and serious}) = \frac{42}{376} \approx 0.112$;
 $\Pr(\text{speeding}) \times \Pr(\text{serious}) = \frac{130}{376} \times \frac{103}{376} \approx 0.095 \neq 0.112$;
 therefore not independent
 4 a Yes b Yes c No
 5 $\Pr(A) \times \Pr(B) = \frac{3}{6} \times \frac{2}{6} = \frac{1}{6} = \Pr(A \cap B)$
 6 No
 7 a 0.6 b 0.42 c 0.88
 8 0.6
 9 a 0.35 b 0.035 c 0.1225 d 0.025

$$10 \text{ a } \frac{4}{15} \quad \text{b } \frac{1}{15} \quad \text{c } \frac{133}{165} \quad \text{d } \frac{6}{11} \quad \text{e } \frac{4}{15}$$

No, as $\Pr(L|F) \neq \Pr(L)$

$$11 \Pr(A) \times \Pr(B) = \frac{20}{36} \times \frac{9}{36} = \frac{5}{36} = \Pr(A \cap B)$$

$$12 \text{ a } 0.35 \quad \text{b } 0.875$$

$$13 \text{ a } \frac{18}{65} \quad \text{b } \frac{12}{65} \quad \text{c } \frac{23}{65} \quad \text{d } \frac{21}{65} \quad \text{e } \frac{4}{65}$$

$$\text{f } \frac{8}{65} \quad \text{g } \frac{2}{15} \quad \text{h } \frac{8}{21}; \text{ No}$$

$$14 \text{ a i } 0.75 \quad \text{ii } 0.32 \quad \text{iii } 0.59 \quad \text{b No} \quad \text{c No}$$

Exercise 9H

$$1 \text{ Approx. } 0.125 \quad 2 \text{ Approx. } 0.172$$

$$3 \text{ Approx. } 0.0009 \quad 4 \text{ Approx. } 29.29$$

$$5 \text{ a Approx. } 1.75 \quad \text{b Approx. } 2.19$$

Chapter 9 review

Technology-free questions

$$1 \text{ a } \frac{1}{6} \quad \text{b } \frac{5}{6}$$

$$2 \text{ } 0.007$$

$$3 \text{ a } \frac{1}{3} \quad \text{b } \frac{1}{4} \quad \text{c } \frac{1}{2}$$

$$4 \text{ a } 0.36 \quad \text{b } \frac{87}{245}$$

$$5 \frac{4}{15}$$

$$6 \text{ a } \{156, 165, 516, 561, 615, 651\} \quad \text{b } \frac{2}{3} \quad \text{c } \frac{1}{3}$$

$$7 \text{ a } \frac{5}{12} \quad \text{b } \frac{1}{4}$$

$$8 \text{ a } 0.036 \quad \text{b } 0.027 \quad \text{c } 0.189 \quad \text{d } 0.729$$

$$9 \text{ a } \frac{1}{27} \quad \text{b } \frac{4}{27} \quad \text{c } \frac{4}{9} \quad \text{d } \frac{20}{27}$$

$$10 \text{ No}$$

$$11 \text{ a } 0.5 \quad \text{b } 0 \quad \text{c } 1$$

$$12 \text{ a } \frac{7}{18} \quad \text{b } \frac{1}{2}$$

$$13 \text{ a } \frac{2}{7} \quad \text{b } \frac{32}{63} \quad \text{c } \frac{9}{16}$$

$$14 \text{ a } 0.2 \quad \text{b } 0.4$$

$$15 \text{ a } 0.7 \quad \text{b } 0.3 \quad \text{c } \frac{1}{3} \quad \text{d } \frac{2}{3}$$

$$16 \text{ a } B \subseteq A \quad \text{b } A \cap B = \emptyset$$

c A and B are independent

Multiple-choice questions

$$1 \text{ B} \quad 2 \text{ C} \quad 3 \text{ A} \quad 4 \text{ C} \quad 5 \text{ D} \quad 6 \text{ A}$$

$$7 \text{ D} \quad 8 \text{ E} \quad 9 \text{ A} \quad 10 \text{ B} \quad 11 \text{ B} \quad 12 \text{ B}$$

$$13 \text{ A} \quad 14 \text{ E} \quad 15 \text{ C} \quad 16 \text{ A} \quad 17 \text{ B} \quad 18 \text{ D}$$

Extended-response questions

$$1 \text{ a } 0.15 \quad \text{b } 0.148$$

$$2 \text{ a } A: \frac{3}{28}; \quad B: \frac{3}{4} \quad \text{b } A: \frac{9}{64}; \quad B: \frac{49}{64}$$

$$\text{c } \frac{1}{8} \quad \text{d } \frac{9}{58}$$

$$3 \text{ a } \frac{4}{5} \quad \text{b } 0.69 \quad \text{c } 0.208$$

$$4 \text{ a } 1.6 \quad \text{b } 2.9$$

$$5 \text{ A and B: } \frac{3}{8}; \quad \text{C and D: } \frac{1}{8}$$

Chapter 10

Exercise 10A

$$1 \text{ a } 11 \quad \text{b } 12 \quad \text{c } 37 \quad \text{d } 29$$

$$2 \text{ a } 60 \quad \text{b } 500 \quad \text{c } 350 \quad \text{d } 512$$

$$3 \text{ a } 128 \quad \text{b } 160$$

$$4 \text{ } 20 \quad 5 \text{ } 63 \quad 6 \text{ } 26 \quad 7 \text{ } 240 \quad 8 \text{ } 260\,000$$

$$9 \text{ } 17\,576\,000 \quad 10 \text{ } 30$$

Exercise 10B

$$1 \text{ a } 6 \quad \text{b } 120 \quad \text{c } 5040 \quad \text{d } 2 \quad \text{e } 1 \quad \text{f } 1$$

$$2 \text{ a } 20 \quad \text{b } 72 \quad \text{c } 6 \quad \text{d } 56 \quad \text{e } 120 \quad \text{f } 720$$

$$3 \text{ } 120 \quad 4 \text{ } 5040 \quad 5 \text{ } 24$$

$$6 \text{ } 720 \quad 7 \text{ } 720 \quad 8 \text{ } 336$$

$$9 \text{ a } 5040 \quad \text{b } 210$$

$$10 \text{ a } 120 \quad \text{b } 120$$

$$11 \text{ a } 840 \quad \text{b } 2401$$

$$12 \text{ a } 480 \quad \text{b } 1512$$

$$13 \text{ a } 60 \quad \text{b } 24 \quad \text{c } 252$$

$$14 \text{ a } 150 \quad \text{b } 360 \quad \text{c } 1560$$

$$15 \text{ a } 720 \quad \text{b } 48$$

Exercise 10C

$$1 \text{ a } 3 \quad \text{b } 3 \quad \text{c } 6 \quad \text{d } 4$$

$$2 \text{ a } 10 \quad \text{b } 10 \quad \text{c } 35 \quad \text{d } 35$$

$$3 \text{ a } 190 \quad \text{b } 100 \quad \text{c } 4950 \quad \text{d } 31\,125$$

$$4 \text{ a } 20 \quad \text{b } 7 \quad \text{c } 28 \quad \text{d } 1225$$

$$5 \text{ } 1716 \quad 6 \text{ } 2300$$

$$7 \text{ } 133\,784\,560 \quad 8 \text{ } 8\,145\,060 \quad 9 \text{ } 18$$

$$10 \text{ a } 5\,852\,925 \quad \text{b } 1\,744\,200$$

$$11 \text{ } 100\,386$$

$$12 \text{ a } 792 \quad \text{b } 336$$

$$13 \text{ a } 150 \quad \text{b } 75 \quad \text{c } 6 \quad \text{d } 462 \quad \text{e } 81$$

$$14 \text{ a } 8\,436\,285 \quad \text{b } 3003 \quad \text{c } 66 \quad \text{d } 2\,378\,376$$

$$15 \text{ } 186 \quad 16 \text{ } 32 \quad 17 \text{ } 256 \quad 18 \text{ } 31 \quad 19 \text{ } 57$$

$$20 \text{ a } 20 \quad \text{b } 21$$

Exercise 10D

$$1 \text{ a } 0.5 \quad \text{b } 0.5$$

$$2 \text{ } 0.375$$

$$3 \text{ a } 0.2 \quad \text{b } 0.6 \quad \text{c } 0.3$$

$$4 \text{ } 0.2 \quad 5 \text{ } \frac{329}{858}$$

$$6 \text{ a } \frac{27}{2^8 - 1} \approx 0.502 \quad \text{b } \frac{56}{255} \quad \text{c } \frac{73}{85}$$

$$7 \text{ a } \frac{5}{204} \quad \text{b } \frac{35}{136}$$

$$8 \text{ a } \frac{1}{6} \quad \text{b } \frac{5}{6} \quad \text{c } \frac{17}{21} \quad \text{d } \frac{34}{35}$$

- 9 a $\frac{25}{49}$ b $\frac{24}{49}$ c $\frac{3}{7}$ d $\frac{1}{6}$
 10 a 0.659 b 0.341 c 0.096 d 0.282
 11 a $\frac{5}{42}$ b $\frac{20}{21}$ c $\frac{15}{37}$

Exercise 10E

- 1 a $x^4 + 8x^3 + 24x^2 + 32x + 16$
 b $16x^4 + 32x^3 + 24x^2 + 8x + 1$
 c $16x^4 - 96x^3 + 216x^2 - 216x + 81$
 d $27x^3 - 27x^2 + 9x - 1$
 e $16x^4 - 32x^3 + 24x^2 - 8x + 1$
 f $-32x^5 + 80x^4 - 80x^3 + 40x^2 - 10x + 1$
 g $-243x^5 + 405x^4 - 270x^3 + 90x^2 - 15x + 1$
 h $16x^4 - 96x^3 + 216x^2 - 216x + 81$

Chapter 10 review

Technology-free questions

- 1 a 499 500 b 1 000 000 c 1 000 000
 2 648 3 120 4 $8n$
 5 5416 6 36 750 7 50 400
 8 a 10 b 32
 9 1200
 10 a $\frac{1}{8}$ b $\frac{3}{8}$ c $\frac{3}{28}$

Multiple-choice questions

- 1 E 2 D 3 A 4 D 5 C
 6 B 7 C 8 A 9 E 10 E

Extended-response questions

- 1 a 2880 b 80 640
 2 a 720 b 48 c 336
 3 a 60 b 45
 4 a 210 b 100 c 80
 5 a 1365 b 210 c 1155
 6 a 3060 b 330 c 1155
 7 Div. 1: 1.228×10^{-7} Div. 2: 1.473×10^{-6}
 Div. 3: 2.726×10^{-5} Div. 4: 1.365×10^{-3}
 Div. 5: 3.362×10^{-3}
 8 a 1.290×10^{-4} b 6.449×10^{-4}

Chapter 11

Exercise 11A

- 1 a No b No c Yes d No e No
 2 a $\Pr(X = 2)$ b $\Pr(X > 2)$ c $\Pr(X \geq 2)$
 d $\Pr(X < 2)$ e $\Pr(X \geq 2)$ f $\Pr(X \geq 2)$
 g $\Pr(X \leq 2)$ h $\Pr(X \geq 2)$ i $\Pr(X \leq 2)$
 j $\Pr(X \geq 2)$ k $\Pr(2 < X < 5)$
 3 a {2} b {3, 4, 5} c {2, 3, 4, 5}
 d {0, 1} e {0, 1, 2} f {2, 3, 4, 5}
 g {3, 4, 5} h {2, 3, 4} i {3, 4}
 4 a 0.2 b 0.5 c 0.3 d 0.35 e 0.9

- 5 a $\frac{1}{15}$ b $\frac{3}{5}$
 6 a 0.09 b 0.69
 7 a 0.49 b 0.51 c 0.74
 8 a 0.6 b 0.47 c $\frac{2}{3}$

- 9 a {HHH, HTH, HHT, HTT, THH, TTH, THT, TTT} b $\frac{3}{8}$

c

x	0	1	2	3
p(x)	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

d $\frac{7}{8}$ e $\frac{4}{7}$

- 10 a {2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12} b $\frac{1}{6}$

c

y	2	3	4	5	6	7	8	9	10	11	12
p(y)	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

- 11 a {1, 2, 3, 4, 5, 6} b $\frac{7}{36}$

c

x	1	2	3	4	5	6
p(x)	$\frac{1}{36}$	$\frac{3}{36}$	$\frac{5}{36}$	$\frac{7}{36}$	$\frac{9}{36}$	$\frac{11}{36}$

- 12 a
- | | | | | |
|------|---------------|---------------|---------------|---------------|
| y | -3 | -2 | 1 | 3 |
| p(y) | $\frac{1}{8}$ | $\frac{3}{8}$ | $\frac{3}{8}$ | $\frac{1}{8}$ |
- b $\frac{7}{8}$

Exercise 11B

- 1 0.378 2 $\frac{28}{57} \approx 0.491$ 3 $\frac{12}{13} \approx 0.923$
 4 $\frac{60}{253} \approx 0.237$ 5 0.930 6 0.109

Exercise 11C

- 1 a 0.185 b 0.060
 2 a 0.194 b 0.930
 3 a 0.137 b 0.446 c 0.554
 4 a 0.008 b 0.268 c 0.468
 5 a 0.056 b 0.391
 6 0.018

- 7 a $\Pr(X = x) = \binom{5}{x} (0.1)^x (0.9)^{5-x}$
 for $x = 0, 1, 2, 3, 4, 5$

x	0	1	2	3	4	5
p(x)	0.591	0.328	0.073	0.008	0.000	0.000

- b Most probable number is 0

- 8 0.749 9 0.021 10 0.5398 11 $\frac{175}{256}$

- 12 a 0.988 b 0.9999 c 8.1×10^{-11}

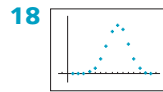
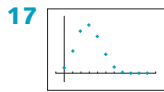
- 13 a 0.151 b 0.302

- 14 5.8%

- 15 a i 0.474 ii 0.224 iii 0.078

- b Answers will vary – about 5 or more

16 0.014



19 a 5 b 8

20 a 13 b 22

21 a 16 b 29

22 a 45 b 59

23 a 0.3087 b $\frac{0.3087}{1 - (0.3)^5} \approx 0.3095$

24 a 0.3020 b 0.6242 c 0.3225

Chapter 11 review

Technology-free questions

1 a 0.92 b 0.63 c 0.8

2

x	1	2	3	4
$p(x)$	0.25	0.28	0.30	0.17

3

x	2	3	4
$p(x)$	$\frac{2}{5}$	$\frac{8}{15}$	$\frac{1}{15}$

4 a

		2nd choice							
		1	2	3	6	7	9		
1st choice	1	2	3	4	7	8	10		
	2	3	4	5	8	9	11		
	3	4	5	6	9	10	12		
	6	7	8	9	12	13	15		
	7	8	9	10	13	14	16		
	9	10	11	12	15	16	18		

b {2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 18}

c

x	2	3	4	5	6	7	8	9
$p(x)$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{4}{36}$	$\frac{4}{36}$

x	10	11	12	13	14	15	16	18
$p(x)$	$\frac{4}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

5 a 0.051 b 0.996 c $\frac{243}{256} \approx 0.949$

6 a $\frac{9}{64}$ b $\frac{37}{64}$

7 a $\frac{16}{81}$ b $\frac{32}{81}$ c $\frac{11}{27}$

8 a $\binom{7}{3} \left(\frac{1}{4}\right)^3 \left(\frac{3}{4}\right)^4$

b $\left(\frac{3}{4}\right)^7 + 7 \left(\frac{1}{4}\right) \left(\frac{3}{4}\right)^6 + 21 \left(\frac{1}{4}\right)^2 \left(\frac{3}{4}\right)^5$

9 a $\left(\frac{p}{100}\right)^{15}$ b $15 \left(\frac{p}{100}\right)^{14} \left(1 - \frac{p}{100}\right)$

c $\left(\frac{p}{100}\right)^{15} + 15 \left(\frac{p}{100}\right)^{14} \left(1 - \frac{p}{100}\right) + 105 \left(\frac{p}{100}\right)^{13} \left(1 - \frac{p}{100}\right)^2$

10 a $\frac{117}{125}$ b $m = 5$

Multiple-choice questions

1 B 2 A 3 C 4 A 5 E
6 C 7 A 8 D 9 B 10 E

Extended-response questions

1 a

x	1	2	3	4
$p(x)$	0.54	0.16	0.06	0.24

b 0.46

2 a i 0.1 ii 0.6 iii $\frac{2}{3}$ b i 0.0012 ii 0.2508

3 a $\frac{3}{5}$ b i $\frac{7}{40}$ ii $\frac{3}{10}$ c i $\frac{11}{20}$ ii $\frac{3}{8}$

4 a 0.003 b 5.320×10^{-6}

5 0.8

6 a 0.401 b $n \geq 45$

7 a $1 - q^2$ b $1 - 4q^3 + 3q^4$ c $\frac{1}{3} < q < 1$

8 a 0.734 (exact answer)

b About 7 (by simulation)

Chapter 12

Technology-free questions

1 a $\frac{1}{9}$ b $\frac{8}{9}$

2 a {348, 384, 438, 483, 834, 843}, $n(\epsilon) = 6$

b $\frac{2}{3}$ c $\frac{2}{3}$

3 a $\frac{1}{2}$ b $\frac{12}{13}$

4 a $\frac{1}{4}$ b $\frac{3}{8}$

5 a $\Pr(1) = \Pr(2) = \Pr(3) = \Pr(5) = \frac{2}{17}$,

$\Pr(4) = \frac{8}{17}$, $\Pr(6) = \frac{1}{17}$

b $\frac{9}{17}$

6 $\frac{1}{4}$

7 a 0.6 b $\frac{1}{3}$

8 a $\frac{1}{2}$ b $\frac{3}{4}$ c $\frac{1}{2}$ d $\frac{1}{4}$

9 a 0.48 b 0.56

10 a $\Pr(A \cap B) = 0.05$ b $\Pr(A | B) = 0.25$

11 a 0.4 b 0.2 c 0.7

12 720

13 $\frac{52!}{7!45!}$

14 a $\frac{7}{22}$ b $\frac{21}{44}$

15 a 0.7 b 0.7 c 0.7

16 a $\Pr(X = 4) = 0.0016$ b $\Pr(X \geq 3) = 0.0272$

- 17 a $\Pr(X = 3) = {}^7C_3 0.2^3 0.8^4$
 b $\Pr(X < 3) = {}^7C_0 0.2^0 0.8^7 + {}^7C_1 0.2^1 \cdot 0.8^6 + {}^7C_2 0.2^2 0.8^5$
 c $\Pr(X \geq 4) = {}^7C_4 0.2^4 0.8^3 + {}^7C_5 0.2^5 \cdot 0.8^2 + {}^7C_6 0.2^6 0.8^1 + {}^7C_7 0.2^7 0.8^0$
- 18 a $\left(\frac{x}{100}\right)^{10}$ b $10\left(\frac{x}{100}\right)^9 \left(1 - \frac{x}{100}\right)$
 c $\left(\frac{x}{100}\right)^{10} + 10\left(\frac{x}{100}\right)^9 \left(1 - \frac{x}{100}\right) + 45\left(\frac{x}{100}\right)^8 \left(1 - \frac{x}{100}\right)^2$

Multiple-choice questions

- 1 E 2 C 3 E 4 B 5 E
 6 E 7 C 8 C 9 B 10 D
 11 D 12 D 13 E 14 A 15 E
 16 E 17 B 18 C 19 C 20 A
 21 E 22 E 23 C 24 D 25 D

Extended-response questions

- 1 a i $\frac{15}{28}$ ii $\frac{37}{56}$ iii $\frac{43}{49}$
 b i $\frac{9}{14}$ ii $\frac{135}{392}$
- 2 a $\frac{1}{2}$ b $\frac{13}{36}$
 3 a $\frac{3}{8}$ b $\frac{1}{56}$ c $\frac{3}{28}$ d $\frac{6}{7}$
 4 a 0.0027 b 0.12 c 0.17 d 0.72
 5 a $\frac{59}{120}$ b $\frac{45}{59}$
 6 a $\frac{167}{360}$
 b i $\frac{108}{193}$ ii $\frac{45}{193}$
 7 a i $\frac{1}{9}$ ii $\frac{5}{18}$
 b i $\frac{1}{81}$ ii $\frac{13}{324}$
 8 a i $m = 30, q = 35, s = 25$
 ii $m + q = 65$
 b $\frac{3}{10}$
 c $\frac{7}{12}$
 9 a 0.084 b 0.52 c 0.68
 10 a 60 b 8 c 0.1
 11 a $\frac{1}{60}$ b $\frac{1}{5}$ c $\frac{3}{5}$ d $\frac{6}{13}$
 12 a i 10 000 cm² ii 400 cm² iii 6400 cm²
 b i 0.04 ii 0.12 iii 0.64
 c i 0.0016 ii 0.000 64
 13 a $\frac{7}{18}$ b $\frac{13}{36}$ c $\frac{23}{108}$
 14 a i 0.328 ii 0.205 iii 0.672
 b i 11 ii 18
 15 a i 0.121 ii 0.851 iii 0.383
 b i 9 ii 14

Chapter 13

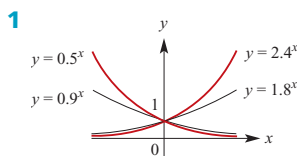
Exercise 13A

- 1 a x^5 b $8x^7$ c x^2 d $2x^3$ e a^6
 f 2^6 g x^2y^2 h x^4y^6 i $\frac{x^3}{y^3}$ j $\frac{x^6}{y^4}$
- 2 a 3^{17} b x^7y^5 c 3^{4x+3} d $30a^5b^6$
- 3 a x^2y b b^{4x+1} c $4a^5b$
- 4 a $\frac{1}{49}$ b 64 c $\frac{8}{125}$
- 5 a b^{10} b 729 c b^4
- 6 a $\frac{27a^8b}{16}$ b $\frac{125b^6}{c^9}$
- 7 a 64 b $-27a^3$ c $-96a^3$
- 8 a 2^{-2n} b 2^4 c $\frac{5^{2n}}{2^{2n}}$
- 9 a x^9 b 2^{16} c 3^{17} d q^8p^9 e $a^{11}b^3$
 f 2^8x^{18} g $m^{11}n^{12}p^{-2}$ h $2a^5b^{-2}$
- 10 a x^2y^3 b $8a^8b^3$ c x^5y^2 d $\frac{9}{2}x^2y^3$
- 11 a $\frac{1}{n^4p^5}$ b $\frac{2x^8z}{y^4}$ c $\frac{b^5}{a^5}$ d $\frac{a^3b}{c}$
 e $a^{n+2}b^{n+1}c^{n-1}$
- 12 a 3^{17n} b 2^{3-n} c $\frac{3^{4n-11}}{2^2}$
 d $2^{n+1}3^{3n-1}$ e 5^{3n-2} f $2^{3x-3} \times 3^{-4}$
 g $3^{6-n} \times 2^{-5n}$ h $3^3 = 27$ i 6
- 13 a $2^{12} = 4096$ b $5^5 = 3125$ c $3^3 = 27$

Exercise 13B

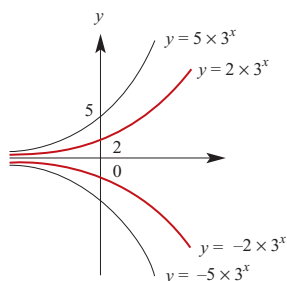
- 1 a 25 b 27 c $\frac{1}{9}$ d 16
 e $\frac{1}{2}$ f $\frac{1}{4}$ g $\frac{1}{25}$ h 16
 i $\frac{1}{10\,000}$ j 1000 k 27 l $\frac{3}{5}$
 m -2 n $\frac{1}{625}$ o 16 p 343
- 2 a $a^{\frac{1}{6}}b^{-\frac{7}{6}}$ b $a^{-6}b^{\frac{9}{2}}$ c $3^{-\frac{7}{3}} \times 5^{-\frac{7}{6}}$
 d $\frac{1}{4}$ e x^6y^{-8} f $a^{\frac{14}{15}}$
- 3 a $(2x-1)^{\frac{3}{2}}$ b $(x-1)^{\frac{5}{2}}$ c $(x^2+1)^{\frac{3}{2}}$
 d $(x-1)^{\frac{7}{2}}$ e $x(x-1)^{-\frac{1}{2}}$ f $(5x^2+1)^{\frac{4}{3}}$

Exercise 13C

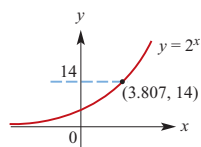
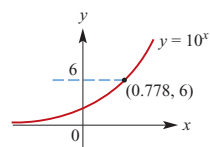


- All pass through (0, 1)
- Horizontal asymptote $y = 0$
- Increasing for base > 1
- Decreasing for base < 1

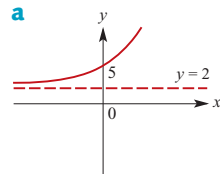
2



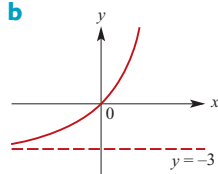
- $y = a \times b^x$ has y-axis intercept at $(0, a)$
- Horizontal asymptote $y = 0$
- Graphs c and d are reflections in the x-axis of graphs a and b

 3 $x = 3.807$

 4 $x = 0.778$


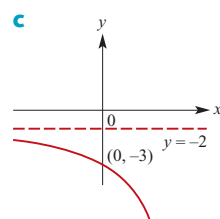
5 a



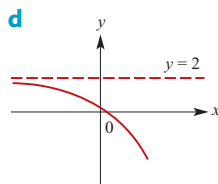
b



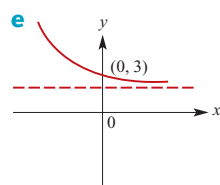
c



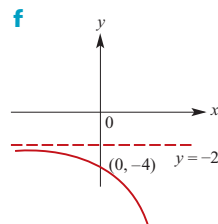
d



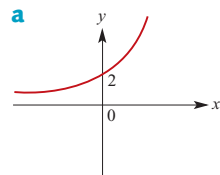
e



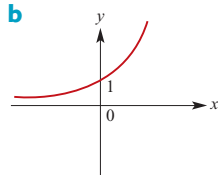
f



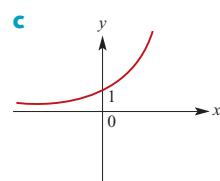
6 a



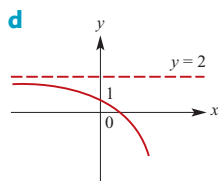
b



c



d



Exercise 13D

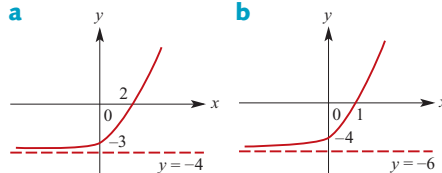
- 1 a 3 b 3 c $\frac{1}{2}$ d $\frac{3}{4}$ e $\frac{1}{3}$
 f 4 g 2 h 3 i 3
 2 a 1 b 2 c $-\frac{3}{2}$ d $\frac{4}{3}$ e -1
 f 8 g 3 h -4 i 8 j 4
 k $3\frac{1}{2}$ l 6 m $7\frac{1}{2}$
 3 a $\frac{4}{5}$ b $\frac{3}{2}$ c $5\frac{1}{2}$
 4 a 0 b 0, -2 c 1, 2 d 0, 1
 5 a 2.32 b 1.29 c 1.26 d 1.75
 6 a $x > 2$ b $x > \frac{1}{3}$ c $x \leq \frac{1}{2}$ d $x < 3$
 e $x < \frac{3}{4}$ f $x > 1$ g $x \leq 3$

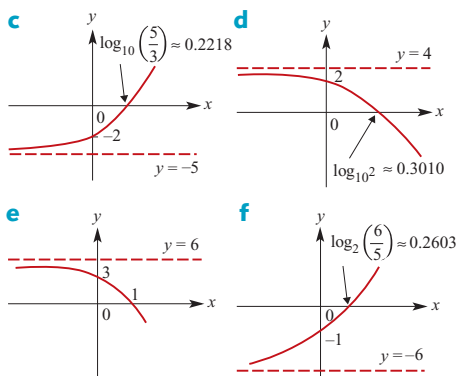
Exercise 13E

- 1 a 7 b 4 c 3 d -1
 2 a $\log_2(10a)$ b 1 c $\log_2\left(\frac{9}{4}\right)$ d 1
 e $3\log_2 a$ f 9 g $-\log_5 6$ h -2
 3 a 3 b 4 c -7 d -3
 e 4 f -3 g 4 h -6
 i -9 j -1 k 4 l -2
 4 a 2 b 7 c 9 d 1
 e $\frac{5}{2}$ f $\log_x(a^5)$ g 3 h 1
 5 a 2 b 27 c $\frac{1}{125}$ d 8
 e 30 f $\frac{2}{3}$ g 8 h 64
 i 4 j 10
 6 a 5 b 32.5 c 22 d 20
 e $\frac{3 + \sqrt{17}}{2}$ f 3 or 0
 7 $2 + 3a - \frac{5c}{2}$ 9 10
 10 a 4 b $\frac{6}{5}$ c 3 d 10
 e 9 f 2

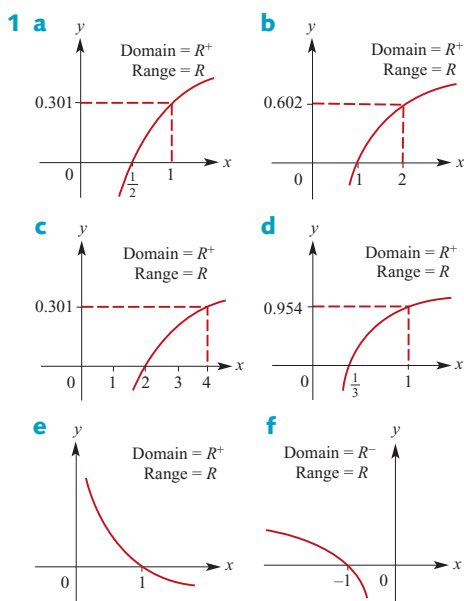
Exercise 13F

- 1 a 2.81 b -1.32 c 2.40 d 0.79
 e -2.58 f -0.58
 2 a 1.90 b 3.10 c -0.68
 3 a $x > 3$ b $x < 1.46$ c $x < -1.15$
 d $x \leq 2.77$ e $x \geq 1.31$
 4 a b

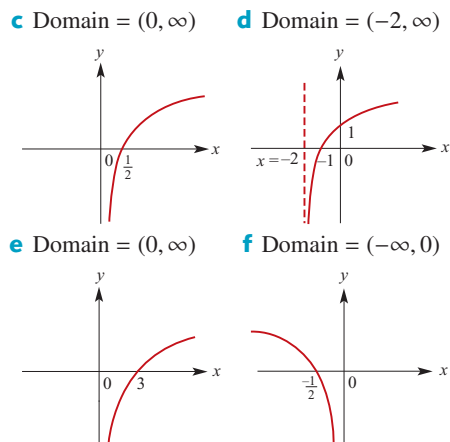
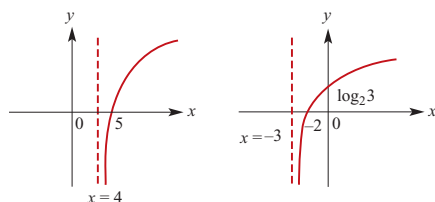




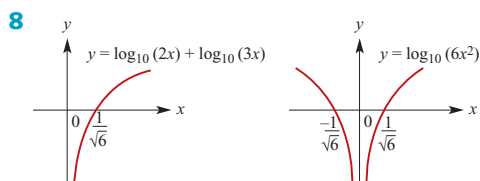
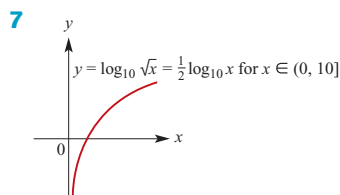
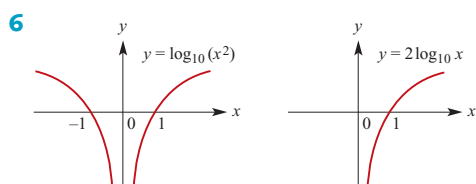
Exercise 13G



- 2 a** $y = 2 \log_{10} x$ **b** $y = 10^{\frac{1}{3}x}$
- c** $y = \frac{1}{3} \log_{10} x$ **d** $y = \frac{1}{3} 10^{\frac{1}{2}x}$
- 3 a** $f^{-1}(x) = \log_3(x-2)$ **b** $f^{-1}(x) = 2^x + 3$
- c** $f^{-1}(x) = \log_3\left(\frac{x-2}{4}\right)$ **d** $f^{-1}(x) = \log_5(x+2)$
- e** $f^{-1}(x) = \frac{1}{3} \times 2^x$ **f** $f^{-1}(x) = 3 \times 2^x$
- g** $f^{-1}(x) = 2^x - 3$
- h** $f^{-1}(x) = \log_3\left(\frac{x+2}{5}\right)$
- 4 a** Domain = $(4, \infty)$ **b** Domain = $(-3, \infty)$



- 5 a** 0.64 **b** 0.40



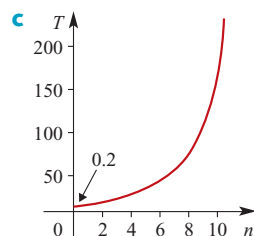
Exercise 13H

- 1 a** $N = 1000 \times 2^{\frac{t}{15}}$ **b** 50 minutes
- 2** 79 726 years
- 3** 7575 years
- 4 a** 535 millibars **b** 7331 metres
- 5** 22 hours
- 6** 6.4°C
- 7** $t > 18.668 \dots$
- 8 a**
- b i** $t = 12.56 \dots$ (mid 1962)
- ii** $t = 37.56 \dots$ (mid 1987)
- 9 a** $y = 3 \times 5^x$ **b** $y = 4 \times (\frac{1}{2})^x$ **c** $y = 5 \times (\frac{3}{2})^x$

- 10 a** $k = \log_{10}\left(\frac{5}{4}\right)$ **b** 7.212 hours
11 a $N = 1000 \times 10^{\frac{t}{5}}$ **b** 210 minutes
c 15 hours **d** 251 189 bacteria
12 $a = 6 \times \left(\frac{10}{3}\right)^{-\frac{2}{3}}$ and $k = \frac{1}{3} \log_{10}\left(\frac{10}{3}\right)$
13 $y = 1.5 \times 0.575^x$ **14** $p = 2.5 \times 1.35^t$

Cuts, n	Sheets	Thickness, T (mm)
0	1	0.2
1	2	0.4
2	4	0.8
3	8	1.6
4	16	3.2
5	32	6.4
6	64	12.8
7	128	25.6
8	256	51.2
9	512	102.4
10	1024	204.8

b $T = 0.2 \times 2^n$

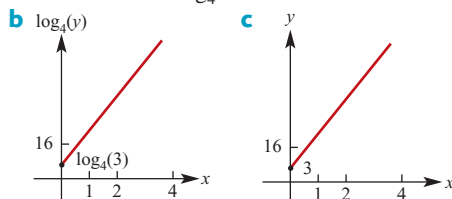


d 214 748.4 m

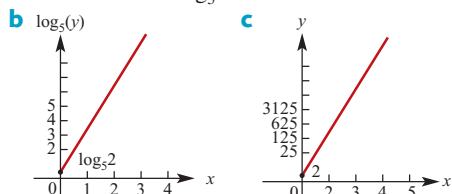
16 $d_0 = 41.92$, $m = 0.094$

Exercise 131

1 a $m = 2$ and $c = \log_4 3$



2 a $m = 3$ and $c = \log_5 2$



3 Street 10^{-9} watt/cm²; Quiet car 10^{-11} watt/cm²

4 a Increases by $10 \log_{10} 2 \approx 3$ dB

b Increases by $10 \log_{10} 10 = 10$ dB

c $P_{\text{new}} = (P_{\text{old}})^3 \times 10^{32}$

d $P = 10^{-16}$ **e** $P = 10^{-6}$

5 $P_1 = 10^{\frac{\lambda}{10}} \times P_2$

7 $7.3 - \log_{10} 4$

6 $5 + \log_{10} 5$

8 $[10^{-4}, 10^{-2}]$

Chapter 13 review

Technology-free questions

1 a a^4 **b** $\frac{1}{b^2}$ **c** $\frac{1}{m^2 n^2}$ **d** $\frac{1}{ab^6}$

e $\frac{3a^6}{2}$ **f** $\frac{5}{3a^2}$ **g** a^3 **h** $\frac{n^8}{m^4}$

i $\frac{1}{p^2 q^4}$ **j** $\frac{8}{5a^{11}}$ **k** $2a$ **l** $a^2 + a^6$

2 a $\log_2 7$ **b** $\frac{1}{2} \log_2 7$ **c** $\log_{10} 2$

d $\log_{10}\left(\frac{18}{5}\right)$ **e** $1 + \log_{10} 11$ **f** $1 + \log_{10} 101$

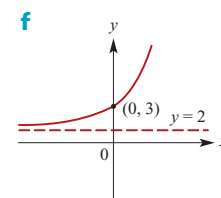
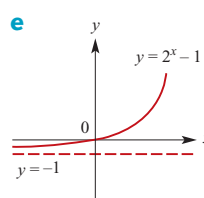
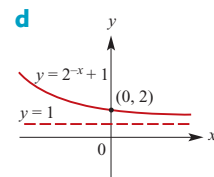
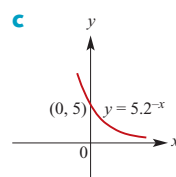
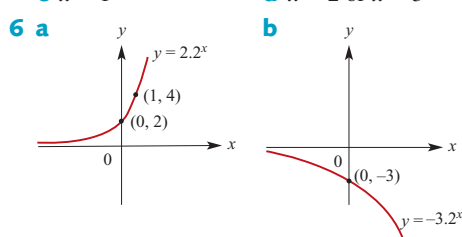
g $\frac{1}{5} \log_2 100$ **h** $-\log_2 10$

3 a 6 **b** 7 **c** 2 **d** 0
e 3 **f** -2 **g** -3 **h** 4

4 a $\log_{10} 6$ **b** $\log_{10} 6$ **c** $\log_{10}\left(\frac{a^2}{b}\right)$

d $\log_{10}\left(\frac{a^2}{25000}\right)$ **e** $\log_{10} y$ **f** $\log_{10}\left(\frac{a^2 b^3}{c}\right)$

5 a $x = 3$ **b** $x = 3$ or $x = 0$
c $x = 1$ **d** $x = 2$ or $x = 3$



7 $x = 1$ **9** 3

10 a $k = \frac{1}{7}$ **b** $q = \frac{3}{2}$

11 a $a = \frac{1}{2}$ **b** $y = -4$ or $y = 20$

Multiple-choice questions

1 C **2** A **3** C **4** C **5** A

6 B **7** A **8** A **9** A **10** A

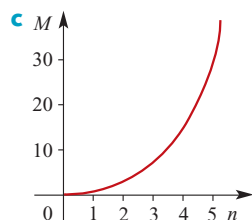
Extended-response questions

1 a

n	1	2	3	4
M	1	3	7	15

b $M = 2^n - 1$

n	5	6	7
M	31	63	127



d

Three discs	1	2	3
Times moved	4	2	1

Four discs	1	2	3	4
Times moved	8	4	2	1

2 $n = 2$

3 a $(\frac{1}{2})^{3n}$ **b** $(\frac{1}{2})^{5n-2}$ **c** $n = 3$

4 a $729(\frac{1}{4})^n$ **b** $128(\frac{1}{2})^n$ **c** 4 times

5 a Batch 1: $15(0.95)^n$; Batch 2: $20(0.94)^n$

b 32 years

6 a X \$1.82 Y \$1.51 Z \$2.62

b X \$4.37 Y \$4.27 Z \$3.47

c Intersect at $t = 21.784 \dots$ and $t = 2.090 \dots$; therefore Feb 2014 until Sept 2015

d Feb 2015 until Sept 2015; approx. 8 months

7 a 13.81 years

b 7.38 years

8 a Temperature = 87.065×0.94^t

b i 87.1°C **ii** 18.56°C

c Temperature = 85.724×0.94^t

d i 85.72°C **ii** 40.82°C

e 28.19 minutes

9 a $a = 0.2$ and $b = 5$

b i $z = x \log_{10} b$ **ii** $a = 0.2$ and $k = \log_{10} 5$

10 a $y = 2 \times 1.585^x$ **b** $y = 2 \times 10^{0.2x}$

c $x = 5 \log_{10} \left(\frac{y}{2} \right)$

Chapter 14

Exercise 14A

1 a $\frac{\pi}{3}$ **b** $\frac{4\pi}{5}$ **c** $\frac{4\pi}{3}$ **d** $\frac{11\pi}{6}$ **e** $\frac{7\pi}{3}$ **f** $\frac{8\pi}{3}$

2 a 120° **b** 150° **c** 210° **d** 162°
e 100° **f** 324° **g** 220° **h** 324°

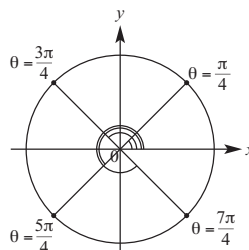
3 a 34.38° **b** 108.29° **c** 166.16° **d** 246.94°
e 213.14° **f** 296.79° **g** 271.01° **h** 343.78°

4 a 0.66 **b** 1.27 **c** 1.87 **d** 2.81
e 1.47 **f** 3.98 **g** 2.38 **h** 5.74

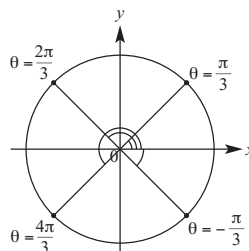
5 a -60° **b** -720° **c** -540° **d** -180°
e 300° **f** -330° **g** 690° **h** -690°

6 a -2π **b** -3π **c** $-\frac{4\pi}{3}$ **d** -4π
e $-\frac{11\pi}{6}$ **f** $-\frac{7\pi}{6}$

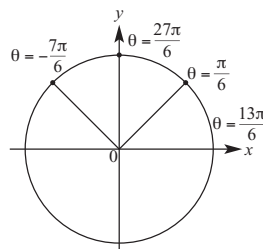
7 a



b



c



Exercise 14B

1 a 0, 1 **b** -1, 0 **c** 1, 0 **d** 1, 0
e 0, -1 **f** 1, 0 **g** -1, 0 **h** 0, 1
2 a 0.95 **b** 0.75 **c** -0.82 **d** 0.96
e -0.5 **f** -0.03 **g** -0.86 **h** 0.61
3 a 0, -1 **b** -1, 0 **c** -1, 0 **d** -1, 0
e -1, 0 **f** 0, -1 **g** 0, -1 **h** 0, -1

Exercise 14C

1 a 0 **b** 0 **c** Undefined **d** 0
e Undefined **f** Undefined
2 a -34.23 **b** -2.57 **c** -0.97 **d** -1.38
e 0.95 **f** 0.75 **g** 1.66
3 a 0 **b** 0 **c** 0 **d** 0 **e** 0 **f** 0

Exercise 14D

1 a $67^\circ 59'$ **b** 4.5315 **c** 2.5357
d 6.4279 **e** $50^\circ 12'$ **f** 3.4202
g 2.3315 **h** 6.5778 **i** 6.5270

Exercise 14E

1 a -0.42 **b** -0.7 **c** -0.42 **d** -0.38
e 0.42 **f** -0.38 **g** -0.7 **h** 0.7
2 a $\frac{5\pi}{6}$ **b** $\frac{7\pi}{6}$ **c** $\frac{11\pi}{6}$

- 3 a** $-\frac{1}{2}$ **b** $\frac{\sqrt{3}}{2}$ **c** $\frac{1}{2}$ **d** $-\frac{\sqrt{3}}{2}$
e $-\sqrt{3}$ **f** $-\sqrt{3}$
4 a $-\frac{\sqrt{3}}{2}$ **b** $\frac{1}{2}$ **c** $-\sqrt{3}$ **d** $-\frac{\sqrt{3}}{2}$ **e** $-\frac{1}{2}$
5 a $a = 0.7660, b = 0.6428$
b $c = -0.7660, d = 0.6428$
c i $\cos 140^\circ = -0.7660, \sin 140^\circ = 0.6428$
ii $\cos 140^\circ = -\cos 40^\circ$
6 a -0.7 **b** -0.6 **c** -0.4 **d** -0.6
e -0.7 **f** -0.7 **g** 0.4 **h** 0.6
7 a 120° **b** 240° **c** -60° **d** 120°
e 240° **f** 300°

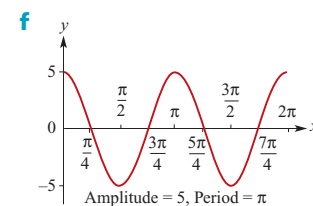
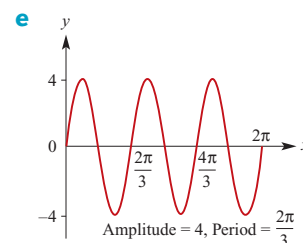
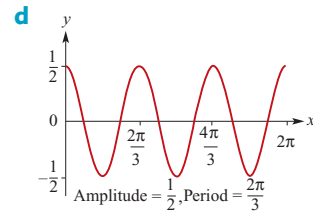
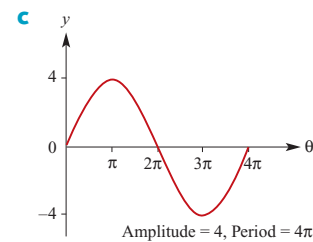
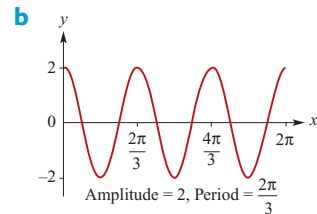
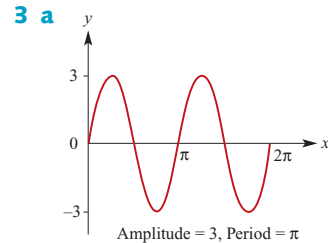
Exercise 14F

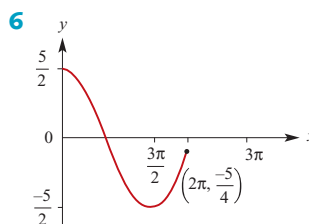
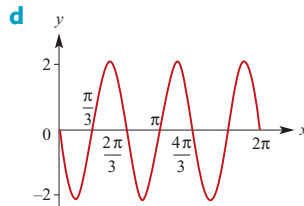
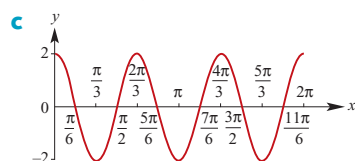
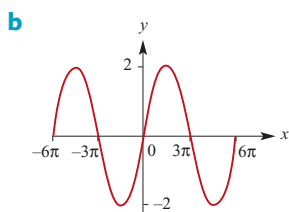
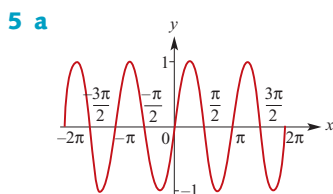
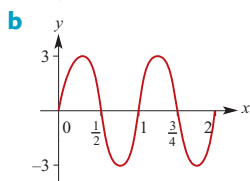
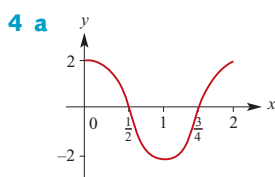
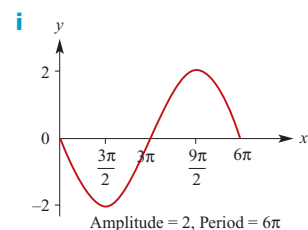
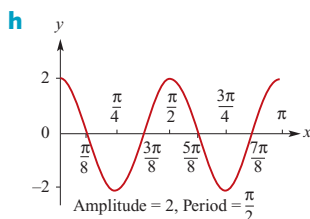
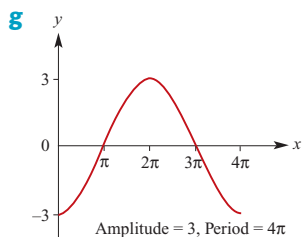
- 1 a** $\sin = \frac{\sqrt{3}}{2}, \cos = -\frac{1}{2}, \tan = -\sqrt{3}$
b $\sin = \frac{1}{\sqrt{2}}, \cos = -\frac{1}{\sqrt{2}}, \tan = -1$
c $\sin = -\frac{1}{2}, \cos = -\frac{\sqrt{3}}{2}, \tan = \frac{1}{\sqrt{3}}$
d $\sin = -\frac{\sqrt{3}}{2}, \cos = -\frac{1}{2}, \tan = \sqrt{3}$
e $\sin = -\frac{1}{\sqrt{2}}, \cos = \frac{1}{\sqrt{2}}, \tan = -1$
f $\sin = \frac{1}{2}, \cos = \frac{\sqrt{3}}{2}, \tan = \frac{1}{\sqrt{3}}$
g $\sin = \frac{\sqrt{3}}{2}, \cos = \frac{1}{2}, \tan = \sqrt{3}$
h $\sin = -\frac{1}{\sqrt{2}}, \cos = -\frac{1}{\sqrt{2}}, \tan = 1$
i $\sin = \frac{\sqrt{3}}{2}, \cos = \frac{1}{2}, \tan = \sqrt{3}$
j $\sin = -\frac{\sqrt{3}}{2}, \cos = \frac{1}{2}, \tan = -\sqrt{3}$
2 a $\frac{\sqrt{3}}{2}$ **b** $-\frac{1}{\sqrt{2}}$ **c** $-\frac{1}{\sqrt{3}}$ **d** $-\frac{1}{2}$ **e** $-\frac{1}{\sqrt{2}}$
f $\sqrt{3}$ **g** $-\frac{\sqrt{3}}{2}$ **h** $\frac{1}{\sqrt{2}}$ **i** $-\frac{1}{\sqrt{3}}$
3 a $-\frac{\sqrt{3}}{2}$ **b** $-\frac{1}{\sqrt{2}}$ **c** $\frac{1}{\sqrt{3}}$ **d** Undefined
e 0 **f** $-\frac{1}{\sqrt{2}}$ **g** $\frac{1}{\sqrt{2}}$ **h** -1

Exercise 14G

- 1 a i** 2π **ii** 2 **b i** π **ii** 3 **c i** $\frac{2\pi}{3}$ **ii** $\frac{1}{2}$
d i 4π **ii** 3 **e i** $\frac{2\pi}{3}$ **ii** 4 **f i** $\frac{\pi}{2}$ **ii** $\frac{1}{2}$
g i 4π **ii** 2 **h i** 2 **ii** 2 **i i** 4 **ii** 3
2 a Dilation of factor 3 from the x -axis;
 Amplitude = 3; Period = 2π

- b** Dilation of factor $\frac{1}{5}$ from the y -axis;
 Amplitude = 1; Period $\frac{2\pi}{5}$
c Dilation of factor 3 from the y -axis;
 Amplitude = 1; Period = 6π
d Dilation of factor 2 from the x -axis and
 dilation of factor $\frac{1}{5}$ from the y -axis;
 Amplitude = 2; Period = $\frac{2\pi}{5}$





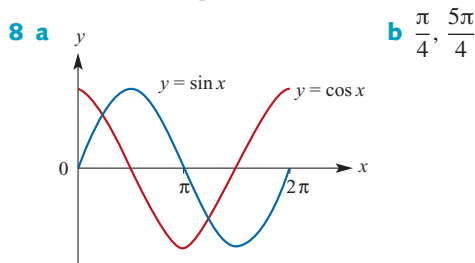
7 a Dilation of factor $\frac{1}{5}$ from the y -axis and reflection in the x -axis;
Amplitude = 1; Period = $\frac{2\pi}{5}$

b Reflection in the y -axis;
Amplitude = 1; Period = 2π

c Dilation of factor 3 from the y -axis and dilation of factor 2 from the x -axis;
Amplitude = 2; Period = 6π

d Dilation of factor 2 from the y -axis, dilation of factor 4 from the x -axis and reflection in the x -axis; Amplitude = 4; Period = 4π

e Dilation of factor 3 from the y -axis, dilation of factor 2 from the x -axis and reflection in the y -axis; Amplitude = 2; Period = 6π



Exercise 14H

1 a $\frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \frac{11\pi}{3}$ **b** $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{9\pi}{4}, \frac{11\pi}{4}$

c $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{7\pi}{3}, \frac{8\pi}{3}$

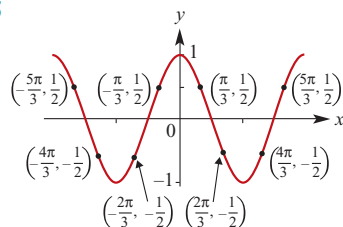
2 a 0.93, 2.21 **b** 4.30, 1.98 **c** 3.50, 5.93
d 0.41, 2.73 **e** 2.35, 3.94 **f** 1.77, 4.51

3 a 150, 210 **b** 30, 150 **c** 120, 240
d 120, 240 **e** 60, 120 **f** 45, 135

4 a $\frac{\pi}{6}, \frac{11\pi}{6}$ **b** $\frac{5\pi}{4}, \frac{7\pi}{4}$ **c** $\frac{\pi}{4}, \frac{7\pi}{4}$

5 a $\frac{3\pi}{4}, -\frac{3\pi}{4}$ **b** $\frac{\pi}{3}, \frac{2\pi}{3}$ **c** $\frac{2\pi}{3}, -\frac{2\pi}{3}$

6



7 a $\frac{7\pi}{12}, \frac{11\pi}{12}, \frac{19\pi}{12}, \frac{23\pi}{12}$

b $\frac{\pi}{12}, \frac{11\pi}{12}, \frac{13\pi}{12}, \frac{23\pi}{12}$

c $\frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}$

d $\frac{5\pi}{12}, \frac{7\pi}{12}, \frac{13\pi}{12}, \frac{15\pi}{12}, \frac{21\pi}{12}, \frac{23\pi}{12}$

e $\frac{5\pi}{12}, \frac{7\pi}{12}, \frac{17\pi}{12}, \frac{19\pi}{12}$

f $\frac{5\pi}{8}, \frac{7\pi}{8}, \frac{13\pi}{8}, \frac{15\pi}{8}$

8 a 2.034, 2.678, 5.176, 5.820

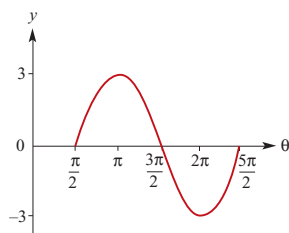
b 1.892, 2.820, 5.034, 5.961

c 0.580, 2.562, 3.721, 5.704

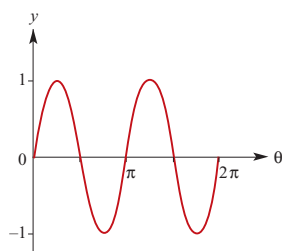
d 0.309, 1.785, 2.403, 3.880, 4.498, 5.974

Exercise 141

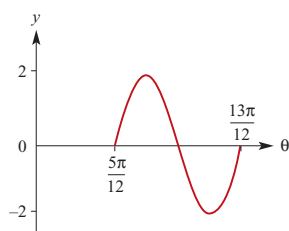
1 a Period = 2π , Amplitude = 3, $y = \pm 3$



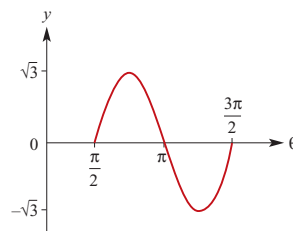
b Period = π , Amplitude = 1, $y = \pm 1$



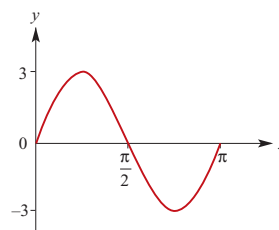
c Period = $\frac{2\pi}{3}$, Amplitude = 2, $y = \pm 2$



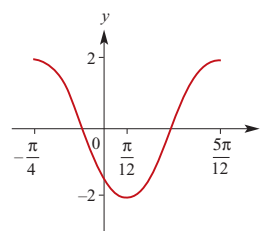
d Period = π , Amplitude = $\sqrt{3}$, $y = \pm\sqrt{3}$



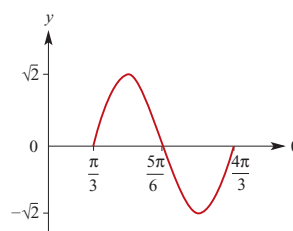
e Period = π , Amplitude = 3, $y = \pm 3$



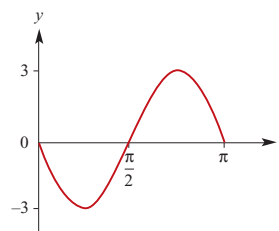
f Period = $\frac{2\pi}{3}$, Amplitude = 2, $y = \pm 2$



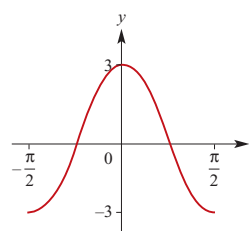
g Period = π , Amplitude = $\sqrt{2}$, $y = \pm\sqrt{2}$



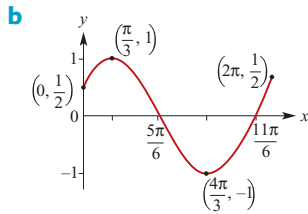
h Period = π , Amplitude = 3, $y = \pm 3$



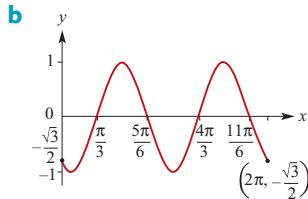
i Period = π , Amplitude = 3, $y = \pm 3$



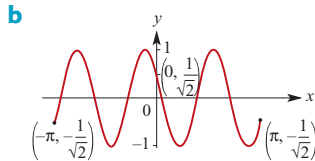
2 a $f(0) = \frac{1}{2}, f(2\pi) = \frac{1}{2}$



3 a $f(0) = -\frac{\sqrt{3}}{2}, f(2\pi) = -\frac{\sqrt{3}}{2}$



4 a $f(-\pi) = -\frac{1}{\sqrt{2}}, f(\pi) = -\frac{1}{\sqrt{2}}$



5 a $y = 3 \sin \frac{x}{2}$

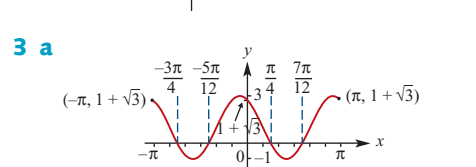
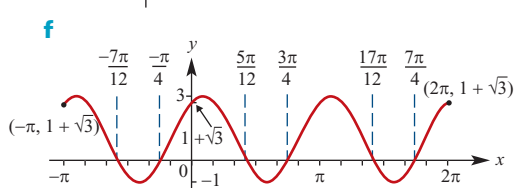
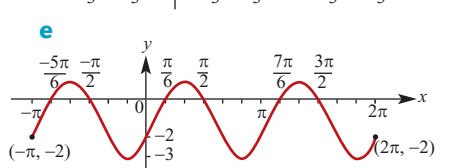
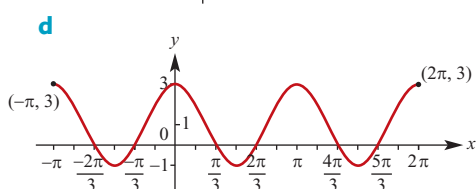
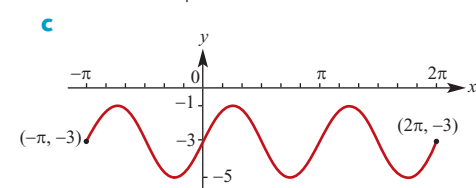
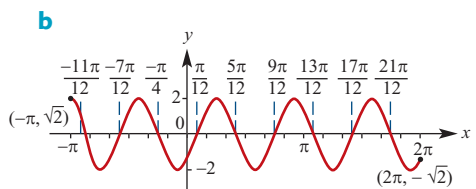
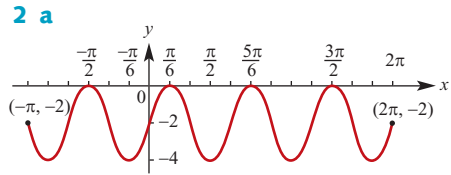
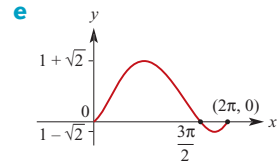
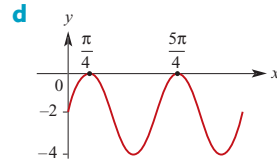
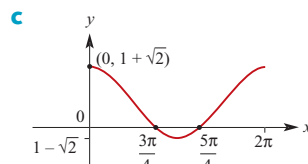
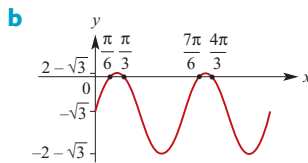
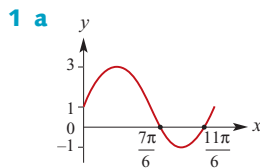
b $y = 3 \sin(2x)$

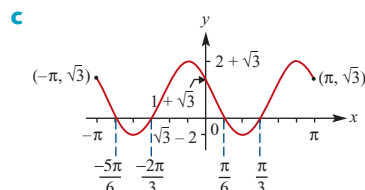
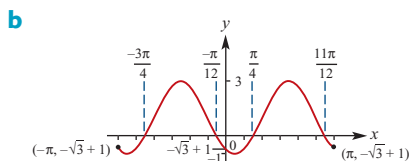
c $y = 2 \sin \frac{x}{3}$

d $y = \sin 2\left(x - \frac{\pi}{3}\right)$

e $y = \sin \frac{1}{2}\left(x + \frac{\pi}{3}\right)$

Exercise 14J



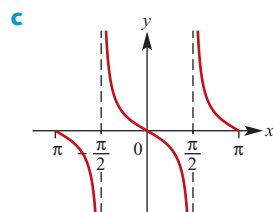
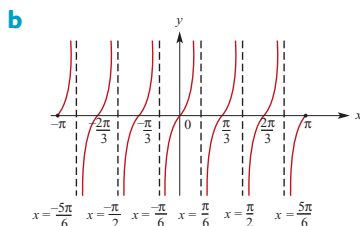
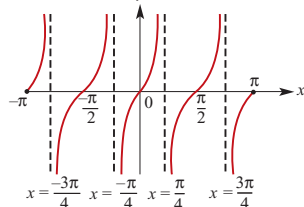


Exercise 14K

- 1 a** 0.6 **b** 0.6 **c** -0.7 **d** 0.3
e -0.3 **f** $\frac{10}{7}$ (1.49) **g** -0.3
h 0.6 **i** -0.6 **j** -0.3
2 a $\frac{\pi}{3}$ **b** $\frac{\pi}{3}$ **c** $\frac{5\pi}{12}$ **d** $\frac{\pi}{14}$
3 $\sin x = \frac{-4}{5}$, $\tan x = \frac{-4}{3}$
4 $\cos x = \frac{-12}{13}$, $\tan x = \frac{-5}{12}$
5 $\sin x = \frac{-2\sqrt{6}}{5}$, $\tan x = -2\sqrt{6}$

Exercise 14L

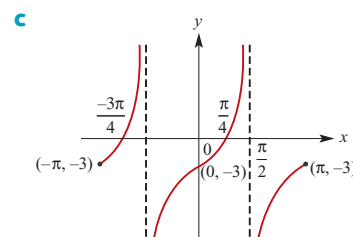
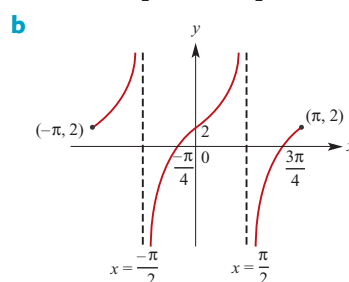
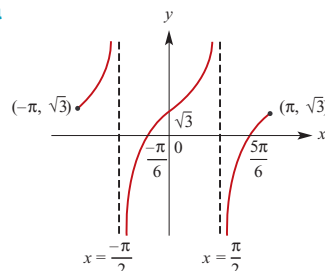
- 1 a** $\frac{\pi}{4}$ **b** $\frac{3\pi}{2}$ **c** $\frac{\pi}{2}$
2 a



- 3 a** $\frac{-7\pi}{8}$, $\frac{-3\pi}{8}$, $\frac{\pi}{8}$, $\frac{5\pi}{8}$

- b** $\frac{-17\pi}{18}$, $\frac{-11\pi}{18}$, $\frac{-5\pi}{18}$, $\frac{\pi}{18}$, $\frac{7\pi}{18}$, $\frac{13\pi}{18}$
c $\frac{-5\pi}{6}$, $\frac{-\pi}{3}$, $\frac{\pi}{6}$, $\frac{2\pi}{3}$
d $\frac{-13\pi}{18}$, $\frac{-7\pi}{18}$, $\frac{-\pi}{18}$, $\frac{5\pi}{18}$, $\frac{11\pi}{18}$, $\frac{17\pi}{18}$

4 a



Exercise 14M

- 1 a** 0.74 **b** 0.51 **c** 0.82, -0.82 **d** 0, 0.88
2 $y = a \sin(b\theta + c) + d$
a $a = 1.993$, $b = 2.998$, $c = 0.003$, $d = 0.993$
b $a = 3.136$, $b = 3.051$, $c = 0.044$, $d = -0.140$
c $a = 4.971$, $b = 3.010$, $c = 3.136$, $d = 4.971$

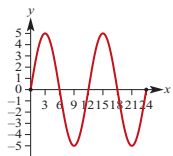
Exercise 14N

- 1 a** $x = \frac{(12n+1)\pi}{6}$ or $x = \frac{(12n+5)\pi}{6}$, $n \in \mathbb{Z}$
b $x = \frac{(12n \pm 1)\pi}{18}$, $n \in \mathbb{Z}$
c $x = \frac{(3n+2)\pi}{3}$, $n \in \mathbb{Z}$
2 a $\frac{\pi}{6}$, $\frac{5\pi}{6}$ **b** $\frac{\pi}{18}$, $\frac{11\pi}{18}$ **c** $\frac{2\pi}{3}$, $\frac{5\pi}{3}$
3 $x = n\pi$ or $x = \frac{(4n-1)\pi}{4}$, $n \in \mathbb{Z}$;
 $x = -\frac{5\pi}{4}$, $-\pi$, $-\frac{\pi}{4}$, 0 , $\frac{3\pi}{4}$, π or $\frac{7\pi}{4}$
4 $x = \frac{n\pi}{3}$, $n \in \mathbb{Z}$; $x = -\pi$, $-\frac{2\pi}{3}$, $-\frac{\pi}{3}$ or 0

$$5 \quad x = \frac{6n-1}{12} \text{ or } x = \frac{3n+2}{6}, n \in \mathbb{Z};$$

$$x = -\frac{2}{3}, -\frac{7}{12}, -\frac{1}{6}, -\frac{1}{12}, \frac{1}{3}, \frac{5}{12}, \frac{5}{6} \text{ or } \frac{11}{12}$$

Exercise 140

- 1 a  b $t = 3$ and $t = 15$

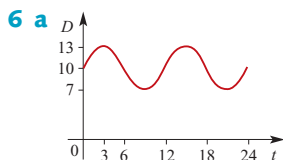
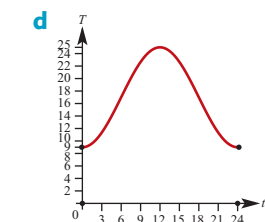
- c 5 m above mean sea level
d $\frac{5\sqrt{3}}{2}$ m above mean sea level
e $\frac{5\sqrt{3}}{2}$ m above mean sea level
f $t \in [1, 5] \cup [13, 17]$

- 2 a 5 metres b 1 metres
c $t = 0.524, 2.618$ or 4.712 seconds
d $t = 0, 1.047$ or 2.094 seconds
e Particle oscillates between $x = 1$ and $x = 5$

- 3 a 7 m b $\frac{3}{4}$ m
c $t = \frac{1}{4}, \frac{5}{4}, \frac{9}{4}, \frac{13}{4}$ or $\frac{17}{4}$
d $t = \frac{1}{12}, \frac{5}{12}, \frac{13}{12}, \frac{17}{12}, \frac{25}{12}$ or $\frac{29}{12}$
e Particle oscillates between $x = 3$ and $x = 7$

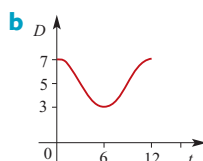
- 4 a i 10 ii $10 + 5\sqrt{3}$ iii $10 + 5\sqrt{3}$
iv $10 - 5\sqrt{3}$ v $10 - 5\sqrt{3}$
b 6 seconds c 20 metres
d $\frac{1}{2}, \frac{5}{2}, \frac{13}{2}, \frac{17}{2}$ s e $\frac{7}{2}, \frac{11}{2}, \frac{19}{2}, \frac{23}{2}$ s

- 5 a 9°C b Maximum = 25°C ; Minimum = 9°C
c Between 7:28 and 16:32



- b $\{t : D(t) \geq 8.5\} = [0, 7] \cup [11, 19] \cup [23, 24]$
c 12.9 m

- 7 a $p = 5, q = 2$

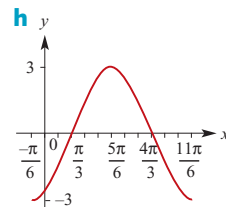
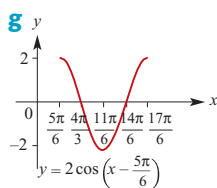
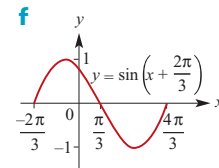
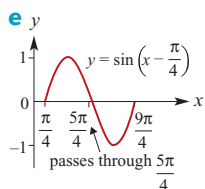
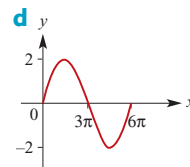
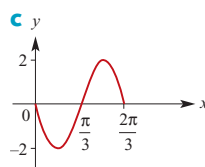
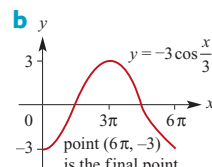
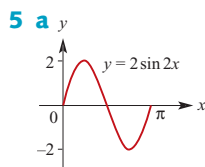


- c A ship can enter 2 hours after low tide

Chapter 14 review

Technology-free questions

- 1 a $\frac{11\pi}{6}$ b $\frac{9\pi}{2}$ c 6π d $\frac{23\pi}{4}$ e $\frac{3\pi}{4}$
f $\frac{9\pi}{4}$ g $\frac{13\pi}{6}$ h $\frac{7\pi}{3}$ i $\frac{4\pi}{9}$
2 a 150° b 315° c 495° d 45°
e 1350° f -135° g -45° h -495°
i -1035°
3 a $\frac{1}{\sqrt{2}}$ b $\frac{1}{\sqrt{2}}$ c $-\frac{1}{2}$ d $-\frac{\sqrt{3}}{2}$
e $\frac{\sqrt{3}}{2}$ f $-\frac{1}{2}$ g $\frac{1}{2}$ h $-\frac{1}{\sqrt{2}}$
4 a 2, 4π b 3, $\frac{\pi}{2}$ c $\frac{1}{2}, \frac{2\pi}{3}$ d 3, π
e 4, 6π f $\frac{2}{3}, 3\pi$



- 6 a $-\frac{2\pi}{3}, -\frac{\pi}{3}$ b $-\frac{\pi}{3}, -\frac{\pi}{6}, \frac{2\pi}{3}, \frac{5\pi}{6}$
c $\frac{\pi}{6}, \frac{3\pi}{2}$ d $\frac{7\pi}{6}$ e $\frac{\pi}{2}, \frac{7\pi}{6}$

Multiple-choice questions

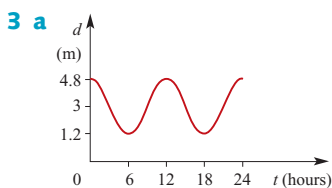
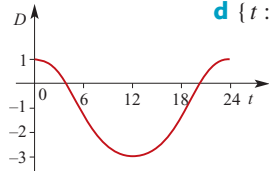
- 1 C 2 D 3 E 4 C 5 E
6 D 7 E 8 E 9 C 10 B

Extended-response questions

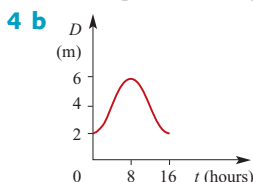
- 1 a i 1.83×10^{-3} hours ii 11.79 hours
b 26 April ($t = 3.86$), 14 August ($t = 7.48$)

- 2 a 19.5°C b $D = -1 + 2 \cos\left(\frac{\pi t}{12}\right)$

- c $\{t : 4 < t < 20\}$



- b 3 a.m., 3 p.m., 3 a.m.
c 9 a.m., 9 p.m. d 10:03 a.m.
e i 6:12 p.m. ii 5 trips



- c $t = 16$ (8 p.m.)
d $t = 4$ and $t = 12$ (8 a.m. and 4 p.m.)
e i 1.5 m ii 2.086 m
f 9 hours 17 minutes

Chapter 15

Technology-free questions

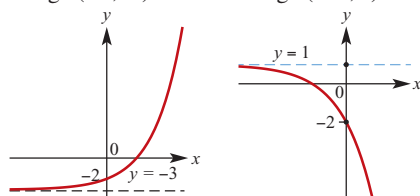
- 1 a $-24a^{10}$ b $\frac{a^3}{2b^2}$ c $\frac{3}{4x^5}$ d 8

- e $\frac{y^{\frac{2}{3}}}{x^{\frac{1}{6}}}$ f $\frac{1}{(2x-1)^{\frac{1}{2}}}$

- 2 a $\frac{25}{9}$ b 16 c 81 d $-\frac{1}{3}$

- 3 a $2^{6n} \times 3^{3n}$ b 12 c $\log_{10} 36$ d -3

- 4 a Range $(-3, \infty)$ b Range $(-\infty, 1)$

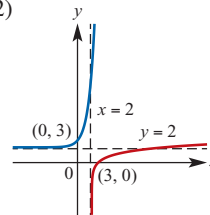


- 5 a $x = 3$ b $x = 0, 2$ c $x > 4$ d $x = 7$ e $x = 1$

- 6 a $x = \log_2 5$ b $x = \frac{1}{3}(\log_5(10) - 1)$

- c $x > \frac{\log_{10} 0.2}{\log_{10} 0.6}$

- 7 $f^{-1}(x) = \log_3(x - 2)$



- 8 a $\frac{\pi}{3}$ b $\frac{3\pi}{2}$ c $\frac{7\pi}{9}$

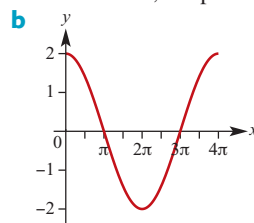
- 9 a -1 b 0 c 0 d Undefined

- 10 a 21.80° b 3.06 c 9.97

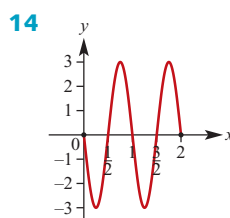
- 11 a -0.3 b -0.5 c 1.6 d -0.6 e 0.1 f $\frac{4}{5}$

- 12 a $\frac{\sqrt{3}}{2}$ b $-\frac{\sqrt{3}}{2}$ c -1 d $\frac{1}{2}$ e $\frac{1}{\sqrt{2}}$ f $-\sqrt{3}$

- 13 a Period = 4π ; Amplitude = 2



- c Dilation of factor 2 from the x-axis and dilation of factor 2 from the y-axis

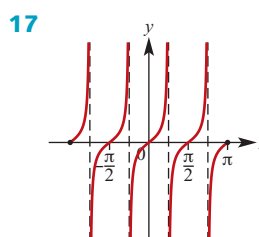
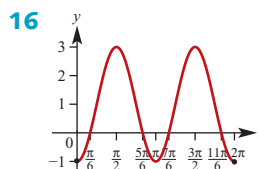


- 15 a $\frac{-7\pi}{6}, \frac{-5\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}$

- b $\frac{-7\pi}{4}, \frac{-5\pi}{4}, \frac{\pi}{4}, \frac{3\pi}{4}$

- c $\frac{-17\pi}{12}, \frac{-13\pi}{12}, \frac{-5\pi}{12}, \frac{-\pi}{12}, \frac{7\pi}{12}, \frac{11\pi}{12}, \frac{19\pi}{12}, \frac{23\pi}{12}$

- d $\frac{-4\pi}{3}, \frac{-\pi}{3}, \frac{2\pi}{3}, \frac{5\pi}{3}$



18 a $x = \frac{\pi}{6} + 2n\pi$ or $x = \frac{5\pi}{6} + 2n\pi, n \in \mathbb{Z}$

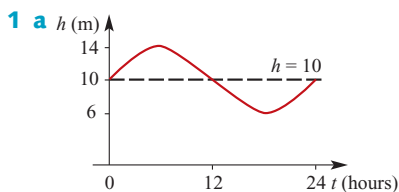
b $x = \pm \frac{\pi}{6} + 2n\pi, n \in \mathbb{Z}$

c $x = -\frac{\pi}{8} + \frac{n\pi}{2}, n \in \mathbb{Z}$

Multiple-choice questions

- 1** B **2** B **3** B **4** E **5** D **6** A
7 D **8** C **9** B **10** A **11** A **12** D
13 A **14** D **15** D **16** D **17** A **18** E
19 D **20** D **21** E **22** A **23** E **24** B
25 D **26** B

Extended-response questions

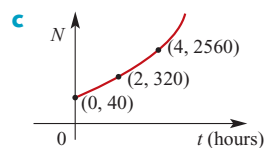


b $t = 3.2393$ and $t = 8.7606$

c $t \in [0.9652, 11.0348]$

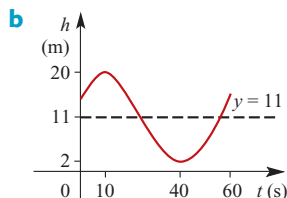
2 a 40 bacteria

b i 320 **ii** 2560 **iii** 10 485 760



d 40 minutes ($= \frac{2}{3}$ hours)

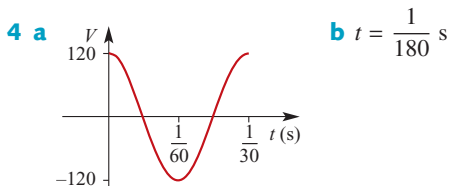
3 a 60 seconds



c $[2, 20]$

d First at height 2 metres after 40 seconds; then every 60 seconds after this first time

e At $t = 0, t = 20$ and $t = 60$, for $t \in [0, 60]$



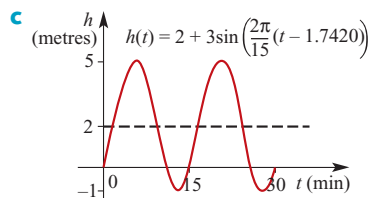
b $t = \frac{1}{180}$ s

c $t = \frac{k}{30}$ s, for $k = 0, 1, 2, \dots$

5 a i Period = 15 seconds

ii Amplitude = 3 **iii** $c = \frac{2\pi}{15}$

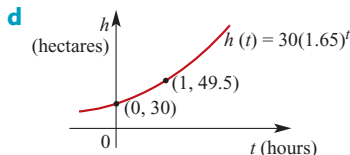
b $h = 1.74202$



6 a i 30 **ii** 49.5 **iii** 81.675

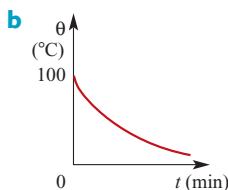
b $k = 1.65$

c 6.792 hours



7 a

t	0	1	2	3	4	5
θ	100	60	40	30	25	22.5



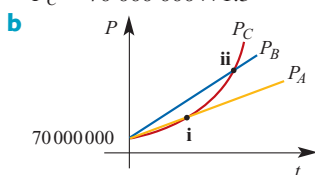
c 1 minute

d 27.071

8 a $P_A = 70\,000\,000 + 3\,000\,000t$

$P_B = 70\,000\,000 + 5\,000\,000t$

$P_C = 70\,000\,000 \times 1.3^{\frac{t}{10}}$



c i 35 years **ii** 67 years

9 a i 4 billion **ii** 5.944 billion **iii** 7.25 billion

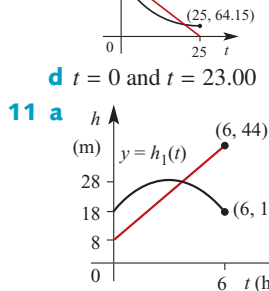
b 2032

10 a $V_1(0) = V_2(0) = 1000$

b

c 64.15 litres

d $t = 0$ and $t = 23.00$

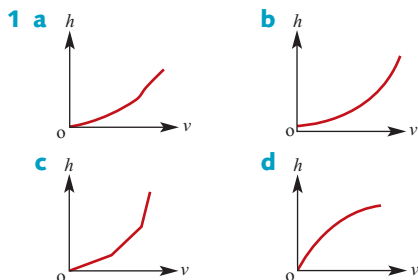


b 3:19 a.m. to nearest minute ($t = 3.31$)

c i 9 a.m. **ii** $8 + 6t$ metres

Chapter 16

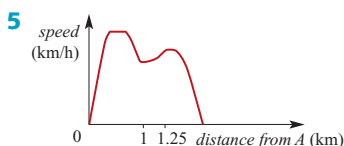
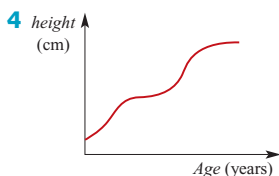
Exercise 16A



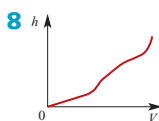
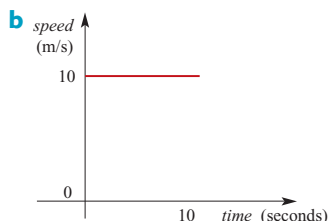
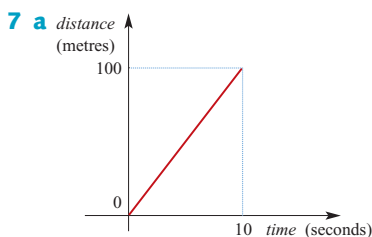
- 2 For the first 2 minutes, the particle travels a distance of 4 m with its speed increasing. For the next 4 minutes, it travels 4 m at constant speed. Then it turns back and returns to its starting point O , travelling at a constant speed and taking 8 minutes to reach O .

Note: For questions 3–6, there may be more than one correct answer.

- 3 C is the most likely



- 6 C and B are the most likely



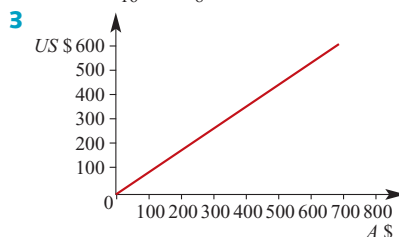
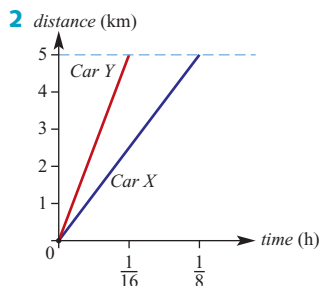
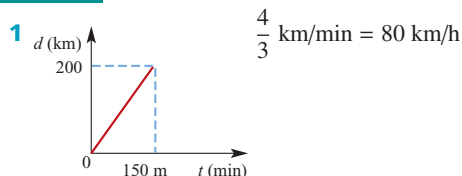
- 9 D

- 10 C

- 11 a $(-4, 0)$ b $[-7, -4) \cup (0, 3]$

- 12 a $(-3, 0)$ b $[-5, -3) \cup (0, 2]$

Exercise 16B

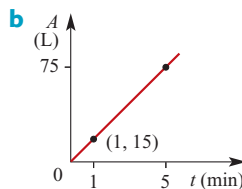


- 4 a 60 km/h b 3 m/s
c 400 m/min = 24 km/h = $6\frac{2}{3}$ m/s
d 35.29 km/h (correct to 2 d.p.)
e 20.44 m/s (correct to 2 d.p.)

- 5 a 8 litres/min b 50 litres/min
c $\frac{200}{17}$ litres/min d $\frac{135}{13}$ litres/min

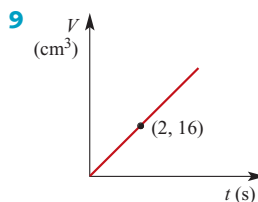
6 a

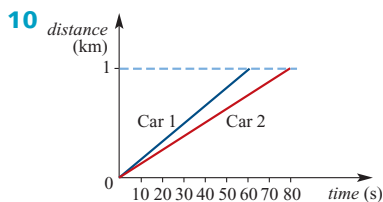
t	0	0.5	1	1.5	2	3	4	5
A	0	7.5	15	22.5	30	45	60	75



- 7 $\frac{\$200}{13}$ per hour = \$15.38 per hour

- 8 $208\frac{1}{3}$ m/s





Exercise 16C

1 3.2 m/s

2 a 2 **b** 7 **c** $-\frac{1}{2}$ **d** $\frac{1-\sqrt{5}}{4}$

3 a $-\frac{25}{7}$ **b** $-\frac{18}{7}$ **c** 4 **d** $\frac{4b}{3a}$

4 a 4 m/s **b** 32 m/s

5 a \$2450.09 **b** \$150.03 per year

6 3.125 cm/min

7 C

Exercise 16D

1 7.19

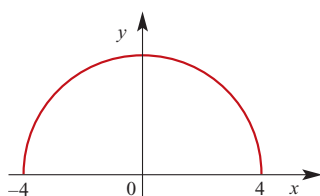
2 a 0.015 **b** $\frac{1}{60} \approx 0.0167$

3 a i 9 **ii** 4.3246 **iii** 2.5893
b 2.30

4 a 25°C at 16:00 **b** $\approx 3^\circ\text{C/h}$ **c** -2.5°C/h

5 -0.5952

6



a 0 **b** -0.6 **c** -1.1

7 4

8 a 16 m³/min **b** 10 m³/min

9 a 18 million/min **b** 8.3 million/min

10 a 620 m³/min flowing out
b 4440 m³/min flowing out
c 284 000 m³/min flowing out

11 a 7 **b** 9 **c** 2 **d** 35

12 a 28 **b** 12

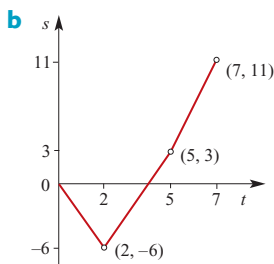
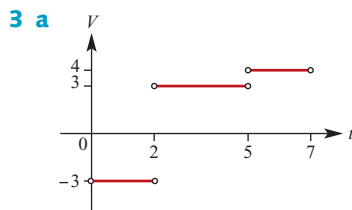
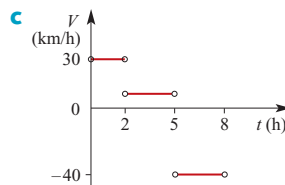
13 a 10 **b** 4

14 a i $\frac{2}{\pi} \approx 0.637$ **ii** $\frac{2\sqrt{2}}{\pi} \approx 0.9003$
iii 0.959 **iv** 0.998
b 1

Exercise 16E

1 a 4 m/s **b** 1.12 m/s

2 a i 30 km/h **ii** $\frac{20}{3}$ km/h **iii** -40 km/h



4 a $t = 2.5$ **b** $0 \leq t < 2.5$ **c** 6 m
d 5 s **e** 3 m/s

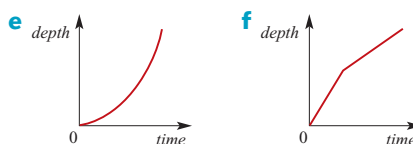
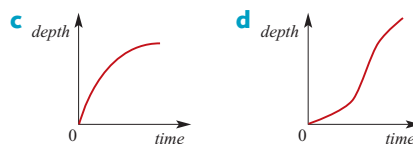
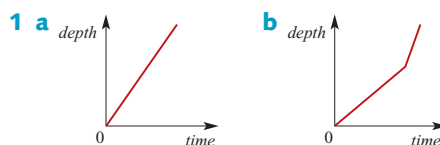
5 a $t = 6$ **b** 15 m/s **c** 17.5 m/s
d 20 m/s **e** -10 m/s **f** -20 m/s

6 a 11 m/s **b** 15 m **c** 1 s **d** 2.8 s **e** 15 m/s

7 a $t = 2, t = 3$ and $t = 8$
b $0 < t < 2.5$ and $t > 6$
c $t = 2.5$ and $t = 6$

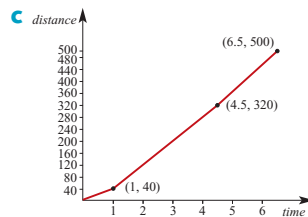
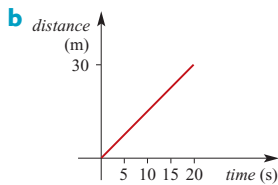
Chapter 16 review

Technology-free questions



2 a

Speed = $\frac{200}{3}$ km/h
= $\frac{10}{9}$ km/min



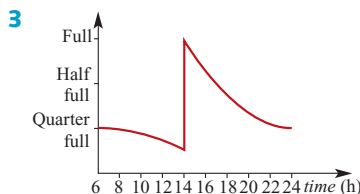
- 3** $36 \text{ cm}^2/\text{cm}$
4 a 1 **b** 13
5 a -2 m/s **b** -12.26 m/s **c** -14 m/s

Multiple-choice questions

- 1** C **2** B **3** D **4** E
5 D **6** B **7** C **8** E

Extended-response questions

- 1 a** Yes, the relation is linear
b $0.05 \text{ ohm}/^\circ\text{C}$
2 a i 9.8 m/s **ii** 29.4 m/s
b i $4.9(8h - h^2)$
ii $4.9(8 - h)$
iii 38.22 m/s , 38.71 m/s , 38.995 m/s ,
 39.151 m/s , 39.1951 m/s



- 4 a** $b + a$ ($a \neq b$) **b** 3 **c** 4.01
5 a $2\frac{2}{3}$, $1\frac{3}{5}$; Gradient = $-1\frac{1}{15}$
b 2.1053, 1.9048; Gradient = -1.003
c -1.000025 **d** -1.0000003
6 a $\approx 3\frac{1}{3} \text{ kg/year}$ **b** $\approx 4.4 \text{ kg/year}$
c $(0, 5) \cup (10, 12)$ **d** $(5, 7) \cup (11, 17\frac{1}{2})$
7 a i 2.5×10^8 **ii** 5×10^8
b 0.007 billion/year
c i 0.004 billion/year **ii** 0.015 billion/year
d 25 years after 2020
8 a i 1049.1 **ii** 1164.3 **iii** 1297.7 **iv** 1372.4
b At 2.8 the gradient is 1452.8
9 a $a^2 + ab + b^2$ **b** 7 **c** 12.06
d $3b^2$
10 a B **b** A **c** 25 m **d** 45 s
e 0.98 m/s, 1.724 m/s, 1.136 m/s
11 a m **b** cm **c** -m

Chapter 17

Exercise 17A

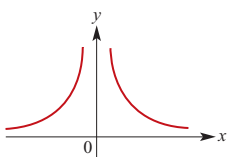
- 1 a** $-2 - h$ **b** -2
2 a $5 + h$ **b** 5
3 $2x - 2$ **4** 32 **5** 2000 m/s
6 7 per day
7 a 1 **b** $3x^2 + 1$ **c** 20
d $30x^2 + 1$ **e** 5 **f** $30x^3 + 4$
8 a $2x + 2$ **b** 13 **c** $3x^2 + 4x$
9 a $5 + 3h$ **b** 5.3 **c** 5
10 a $\frac{-1}{2+h}$ **b** -0.48 **c** $\frac{-1}{2}$
11 a $6 + h$ **b** 6.1 **c** 6
12 a $6x$ **b** 4 **c** 0 **d** $6x + 4$
e $6x^2$ **f** $8x - 5$ **g** $-2 + 2x$
13 $4x^3$

Exercise 17B

- 1 a** $2x + 4$ **b** 2 **c** $3x^2 - 1$ **d** $x - 3$
e $15x^2 + 6x$ **f** $-3x^2 + 4x$
2 a $12x^{11}$ **b** $21x^6$ **c** 5 **d** 5
e 0 **f** $10x - 3$ **g** $50x^4 + 12x^3$
h $8x^3 - x^2 - \frac{1}{2}x$
3 a 6 **b** 20 **c** 5 **d** 10 **e** 0
f 7 **g** 31 **h** 7 **i** -34
4 a 60 **b** -16 **c** 57 **d** 168
5 a 7 **b** 2 **c** -16 **d** 11
6 a -1 **b** 0 **c** $12x^2 - 3$
d $x^2 - 1$ **e** $2x + 3$ **f** $18x^2 - 8$
g $15x^2 + 3x$
7 a $2(x + 4)$ **b** $48t^2 + 16t - 7$ **c** $2x$
8 a i 3 **ii** $3a^2$ **b** $3x^2$
9 a $\frac{dy}{dx} = 3(x - 1)^2 \geq 0$ for all x ;
 therefore gradient of graph ≥ 0 for all x
b $\frac{dy}{dx} = 1$ for $x \neq 0$
c $18x + 6$
10 a 1, Gradient = 2 **b** 1, Gradient = 1
c 3, Gradient = -4 **d** -5 , Gradient = 4
e 28, Gradient = -36 **f** 9, Gradient = -24
11 a i $4x - 1$, 3, $(\frac{1}{2}, 0)$
ii $\frac{1}{2} + \frac{2}{3}x$, $\frac{7}{6}$, $(\frac{3}{4}, \frac{25}{16})$
iii $3x^2 + 1$, 4, $(0, 0)$
iv $4x^3 - 31$, -27 , $(2, -46)$
b Coordinates of the point where gradient is 1
12 a $6t - 4$ **b** $-2x + 3x^2$ **c** $-4z - 4z^3$
d $6y - 3y^2$ **e** $6x^2 - 8x$ **f** $19.6t - 2$
13 a $(4, 16)$ **b** $(2, 8)$, $(-2, -8)$ **c** $(0, 0)$
d $(\frac{3}{2}, -\frac{5}{4})$ **e** $(2, -12)$ **f** $(-\frac{1}{3}, \frac{4}{27})$, $(1, 0)$

Exercise 17C

- 1 a $-\frac{1}{(x-3)^2}$ b $-\frac{1}{(x+2)^2}$
 2 a $-\frac{2}{x^3}$ b $-\frac{4}{x^5}$
 3 a $-6x^{-3} - 5x^{-2}$ b $-6x^{-3} + 10x$
 c $-15x^{-4} - 8x^{-3}$ d $6x - \frac{20}{3}x^{-5}$
 e $-12x^{-3} + 3$ f $3 - 2x^{-2}$
 4 a $-2z^{-2} - 8z^{-3}, z \neq 0$ b $-9z^{-4} - 2z^{-3}, z \neq 0$
 c $\frac{1}{2}, z \neq 0$ d $18z + 4 - 18z^{-4}, z \neq 0$
 e $2z^{-3}, z \neq 0$ f $-\frac{3}{5}, z \neq 0$
 5 a $f'(x) = 12x^3 + 18x^{-4} - x^{-2}$
 b $f'(x) = 20x^3 - 8x^{-3} - x^{-2}$
 6



a Gradient of $PQ = \frac{-2-h}{(1+h)^2}$ b -2

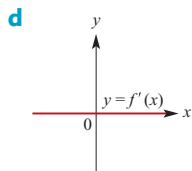
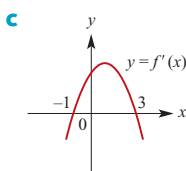
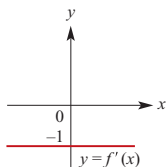
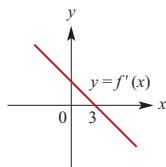
- 7 a $11\frac{3}{4}$ b $\frac{1}{8}$ c -1 d 5

- 8 a $-\frac{1}{2}$ b $\frac{1}{2}$

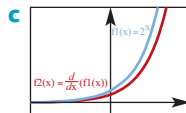
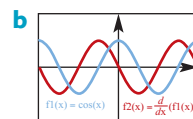
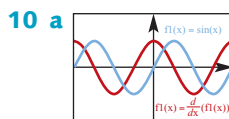
- 9 $f'(x) = -\frac{1}{x^2} < 0$ for all $x \neq 0$

Exercise 17D

- 1 Graphs b and d
 2 Graphs a, b and e
 3 a $x = 1$ b $x = 1$ c $x > 1$ d $x < 1$ e $x = \frac{1}{2}$
 4 a $(-\infty, -3) \cup (\frac{1}{2}, 4)$ b $(-3, \frac{1}{2}) \cup (4, \infty)$
 c $\{-3, \frac{1}{2}, 4\}$
 5 a B b C c D d A e F f E
 6 a



- 7 a $(-1, 1.5)$ b $(-\infty, -1) \cup (1.5, \infty)$ c $\{-1, 1.5\}$
 8 a $(3, 0)$ b $(4, 2)$
 9 a $(\frac{1}{2}, -6\frac{1}{4})$ b $(0, -6)$



- 11 a i 66.80° ii 42.51°
 b $(0.5352, 0.2420)$
 c No
 12 a $(0.6)t^2$ b $0.6 \text{ m/s}, 5.4 \text{ m/s}, 15 \text{ m/s}$
 13 a $a = 2, b = -5$ b $(\frac{5}{4}, -\frac{25}{8})$
 14 a Height = 450 000 m; Speed = 6000 m/s
 b $t = 25 \text{ s}$

Exercise 17E

- 1 a $\frac{x^4}{8} + c$ b $x^3 - 2x + c$ c $\frac{5x^4}{4} - x^2 + c$
 d $\frac{x^4}{5} - \frac{2x^3}{3} + c$ e $\frac{x^3}{3} - x^2 + x + c$
 f $\frac{x^3}{3} + x + c$ g $\frac{z^4}{2} - \frac{2z^3}{3} + c$
 h $\frac{4t^3}{3} - 6t^2 + 9t + c$ i $\frac{t^4}{4} - t^3 + \frac{3t^2}{2} - t + c$
 2 $f(x) = x^4 + 2x^3 + 2x$
 3 $y = 2x^3 + 12$
 4 a $y = x^2 - x$ b $y = 3x - \frac{x^2}{2} + 1$
 c $y = \frac{x^3}{3} + x^2 + 2$ d $y = 3x - \frac{x^3}{3} + 2$
 e $y = \frac{2x^5}{5} + \frac{x^2}{2}$
 5 a $V = \frac{t^3}{3} - \frac{t^2}{2} + \frac{9}{2}$ b $\frac{1727}{6} \approx 287.83$
 6 $f(x) = x^3 - x + 2$
 7 a B b $w = 2000t - 10t^2 + 100\,000$
 8 $f(x) = 5x - \frac{x^2}{2} + 4$
 9 $f(x) = \frac{x^4}{4} - x^3 - 2$
 10 a $k = 8$ b $(0, 7)$
 11 $8\frac{2}{3}$
 12 a $k = -4$ b $y = x^2 - 4x + 9$
 13 a $k = -32$ b $f(7) = 201$
 14 $y = \frac{1}{3}(x^3 - 5)$

Exercise 17F

- 1 a 15 b 1 c $-3\frac{1}{2}$ d $-2\frac{1}{2}$
 e 0 f 4 g 2 h $2\sqrt{3}$
 i -2 j 12 k $\frac{11}{9}$ l $\frac{1}{4}$

2 a 3, 4 b 7

3 a Discontinuity at 0, as $f(0) = 0$,
 $\lim_{x \rightarrow 0^+} f(x) = 0$ but $\lim_{x \rightarrow 0^-} f(x) = 2$

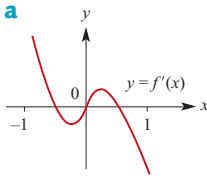
b Discontinuity at 1, as $f(1) = 3$,
 $\lim_{x \rightarrow 1^+} f(x) = 3$ but $\lim_{x \rightarrow 1^-} f(x) = -1$

c Discontinuity at 0, as $f(0) = 1$,
 $\lim_{x \rightarrow 0^+} f(x) = 1$ but $\lim_{x \rightarrow 0^-} f(x) = 0$

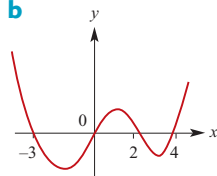
4 $x = 1$

Exercise 17G

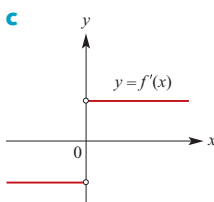
1 a



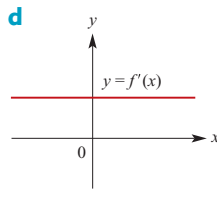
b



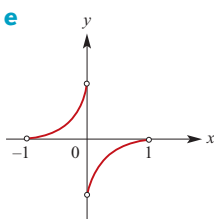
c



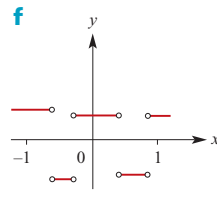
d



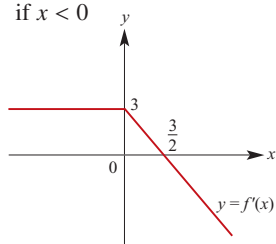
e



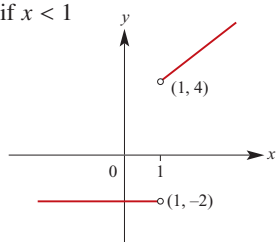
f



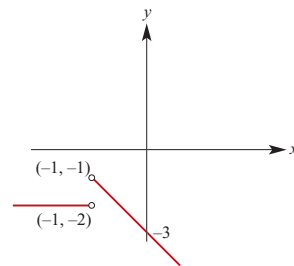
2
$$f'(x) = \begin{cases} -2x + 3 & \text{if } x \geq 0 \\ 3 & \text{if } x < 0 \end{cases}$$



3
$$f'(x) = \begin{cases} 2x + 2 & \text{if } x > 1 \\ -2 & \text{if } x < 1 \end{cases}$$



4
$$f'(x) = \begin{cases} -2x - 3 & \text{if } x > -1 \\ -2 & \text{if } x < -1 \end{cases}$$



Chapter 17 review

Technology-free questions

1 a 3 b $-2x$ c $2x + 5$
 d $3x^2 + 1$ e $2x + 2$ f $6x - 1$

2 a $6x - 2$ b 0 c $4 - 4x$
 d $4(20x - 1)$ e $6x + 1$ f $-6x - 1$

3 a -1 b 0 c $\frac{4x+7}{4}$ d $\frac{4x-1}{3}$ e x

4 a 1; 2 b 3; -4 c -5; 4 d 28; -36

5 a $(\frac{3}{2}, -\frac{5}{4})$ b $(2, -12)$ c $(-\frac{1}{3}, \frac{4}{27})$, $(1, 0)$

d $(-1, 8)$, $(1, 6)$ e $(0, 1)$, $(\frac{3}{2}, -\frac{11}{16})$

f $(3, 0)$, $(1, 4)$

6 a $x = \frac{1}{2}$ b $x = \frac{1}{2}$ c $x > \frac{1}{2}$ d $x < \frac{1}{2}$

e $x \in \mathbb{R} \setminus \{\frac{1}{2}\}$ f $x = \frac{5}{8}$

7 a $-4x^{-5}$ b $-6x^{-4}$ c $\frac{2}{3x^3}$ d $\frac{4}{x^5}$

e $-\frac{15}{x^6}$ f $-\frac{2}{x^3} - \frac{1}{x^2} = -\frac{2+x}{x^3}$

g $-\frac{2}{x^2}$ h $10x + \frac{2}{x^2}$

8 a $a = 2$, $b = -1$ b $(\frac{1}{4}, -\frac{1}{8})$

9 a $\frac{x}{2} + c$ b $\frac{x^3}{6} + c$ c $\frac{x^3}{3} + \frac{3x^2}{2} + c$

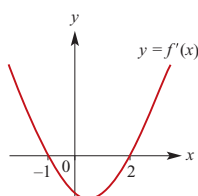
d $\frac{4x^3}{3} + 6x^2 + 9x + c$ e $\frac{at^2}{2} + c$ f $\frac{t^4}{12} + c$

g $\frac{t^3}{3} - \frac{t^2}{2} - 2t + c$ h $\frac{-t^3}{3} + \frac{t^2}{2} + 2t + c$

10 $f(x) = x^2 + 5x - 25$

11 a $f(x) = x^3 - 4x^2 + 3x$ b 0, 1, 3

12



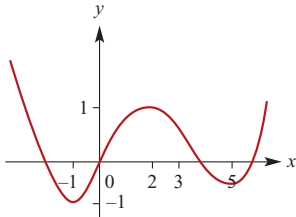
13 a $(-1, 4)$ b $(-\infty, -1) \cup (4, \infty)$ c $\{-1, 4\}$

Multiple-choice questions

- 1 D 2 B 3 E 4 B 5 C
6 C 7 A 8 E 9 A 10 D

Extended-response questions

1



- 2 $y = \frac{7}{36}x^3 + \frac{1}{36}x^2 - \frac{20}{9}x$
3 a i 71.57° ii 89.58° b 2 km
4 a 0.12, -0.15
b $x = 2$, $y = 2.16$; Height 2.16 km
5 a $t = \sqrt[3]{250}$, 11.9 cm/s b 3.97 cm/s
6 a At $x = 0$, gradient is -2; at $x = 2$, gradient is 2. Angles of inclination to the positive direction of the x -axis are supplementary.

Chapter 18

Exercise 18A

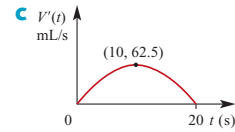
- 1 a $y = 4x - 4$, $4y + x = 18$
b $y = 12x - 15$, $12y + x = 110$
c $y = -x + 4$, $y = x$
d $y = 6x + 2$, $6y + x = 49$
2 $y = 2x - 10$
3 $y = 2x - 1$, $y = 2x - \frac{8}{3}$;
Both have gradient 2; Distance apart = $\frac{\sqrt{5}}{3}$
4 $y = 3x + 2$, $y = 3x + 6$
5 a Tangents both have gradient 2 b (0, -3)
6 (3, 12), (1, 4)
7 a $y = 10x - 16$ b (-4, -56)
8 a $y = 5x - 1$ b (2, 4), (4, -8)

Exercise 18B

- 1 a 36 ; $\frac{36}{1} = 36$ b $48 - 12h$ c 48
2 a $1200t - 200t^2$ b \$1800 per month
c At $t = 0$ and $t = 6$
3 a $30 - 4P$
b 10, -10
c For $P < 7.5$ revenue increases as P increases
4 a 50 people per year b 0 people per year
c Decreasing by 50 people per year

- 5 a i 0 mL ii $833\frac{1}{3}$ mL

b $V'(t) = \frac{5}{8}(20t - t^2)$

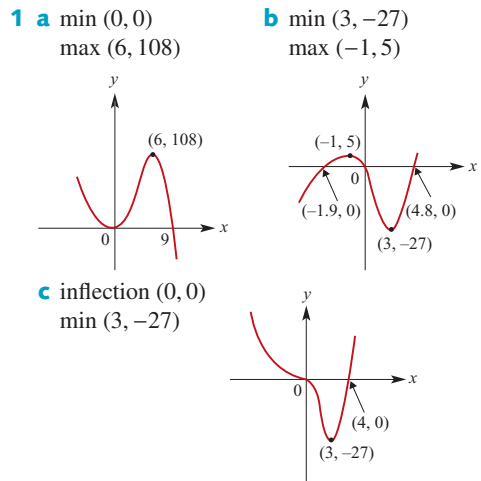


- 6 a 0.6 km^2 b $0.7 \text{ km}^2/\text{h}$

Exercise 18C

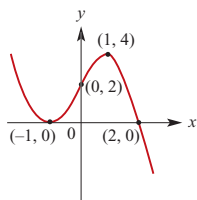
- 1 a (3, -6) b (3, 2) c (2, 2) d (4, 48)
e (0, 0), (2, -8) f (0, -10), (2, 6)
2 $a = 2$, $b = -8$, $c = -1$
3 $a = -\frac{1}{2}$, $b = 1$, $c = 1\frac{1}{2}$
4 a $a = 2$, $b = -5$ b $(\frac{5}{4}, -\frac{25}{8})$
5 $a = -8$
6 $a = 6$
7 a (2.5, -12.25) b $(\frac{7}{48}, -\frac{625}{96})$
c (0, 27), (3, 0) d (-2, 48), (4, -60)
e (-3, 4), (-1, 0) f (-1.5, 0.5)
8 $a = -1$, $b = 2$
9 $a = -\frac{2}{9}$, $b = \frac{3}{2}$, $c = -3$, $d = 7\frac{1}{2}$

Exercise 18D

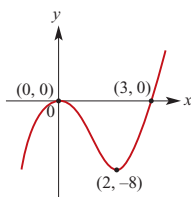


- 2 a (0, 0) max; $(\frac{8}{3}, -\frac{256}{27})$ min
b (0, 0) min; (2, 4) max c (0, 0) min
d $(\frac{10}{3}, -\frac{200\,000}{729})$ min; (0, 0) inflection
e (3, -7) min; $(\frac{1}{3}, \frac{67}{27})$ max
f (6, -36) min; $(\frac{4}{3}, \frac{400}{27})$ max

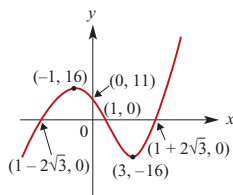
- 3 a** max at (1, 4)
min at (-1, 0)
intercepts (2, 0), (-1, 0)



- b** min at (2, -8)
max at (0, 0)
intercepts (3, 0), (0, 0)

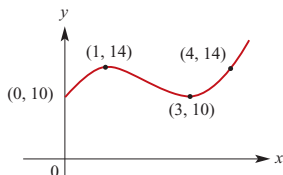


- c** min at (3, -16)
max at (-1, 16)
intercepts (0, 11),
(1 ± 2√3, 0), (1, 0)

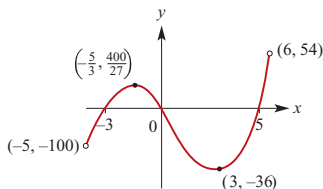


- 4 a** Local maximum
b Stationary point of inflection

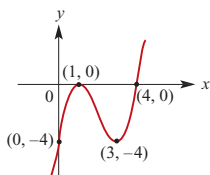
- 5 a** $(-\infty, 1) \cup (3, \infty)$
b (1, 14) max; (3, 10) min
c



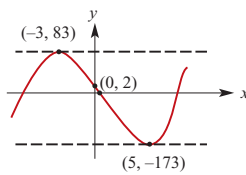
- 6** $\{x : -2 < x < 2\}$
7 a $x \in (-1, 1)$ **b** $x \in (-\infty, -1) \cup (1, \infty)$
8 a $x = -\frac{5}{3}$, $x = 3$
b max at $(-\frac{5}{3}, \frac{400}{27})$, min at (3, -36)
intercepts (5, 0), (0, 0), (-3, 0)



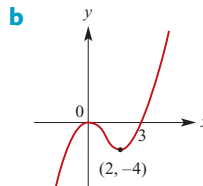
9



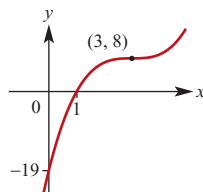
10



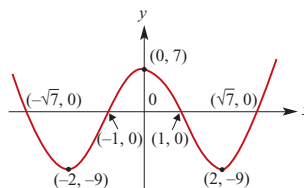
- 11 a i** (0, 2) **ii** $(-\infty, 0) \cup (2, \infty)$ **iii** {0, 2}



- 12** Stationary point of inflection at (3, 8)



- 13** min at (-2, -9) and (2, -9); max at (0, 7)
intercepts $(\pm\sqrt{7}, 0)$, $(\pm 1, 0)$, (0, 7)



Exercise 18E

- 1** 2500 cm²
2 25
3 2
4 a $V(x) = (6 - 2x)^2 x$
b $V_{\max} = 16 \text{ m}^3$ when $x = 1$
5 a i 0.9375 m **ii** 2.5 m **iii** 2.8125 m
b $x = \frac{40}{3}$, $y = \frac{80}{27}$
c i $x = 11.937$, $x = 1.396$ **ii** $x = 14.484$
6 b $V = \frac{75x - x^3}{2}$ **c** 125 cm³ **d** 118 cm³
7 256π
8 $x = \frac{5}{3}(9 - \sqrt{21})$
9 Absolute max = 2; Absolute min = -30
10 Absolute max = 6; Absolute min = -9
11 Absolute max = 32; Absolute min = -8
12 Absolute max = 1050; Absolute min = -8
13 b $\frac{dV}{dx} = 30x - 36x^2$ **c** $\frac{125}{36}$ **d** $\frac{432}{125}$
e $\frac{125}{36}$ when $x = \frac{5}{6}$
14 a $15 \leq y \leq 18$ **b** Max 75, min 36
15 a $\frac{125\,000}{27}$ **b** 3000 **c** $\frac{125\,000}{27}$
16 b $\frac{dA}{dx} = \frac{1}{8}(2x - 10)$ **c** $x = 5$ **d** $\frac{25}{8} \text{ m}^2$

Exercise 18F

- 1 **a** $x = 11$ **b** $x = -16$
 2 **a** -12 cm/s **b** $t = 6, x = -25$
c -9 cm/s **d** 9 cm/s
 3 **a** -3 cm/s **b** $2\sqrt{3} \text{ s}$
 4 **a** $x = 5 \text{ cm}, v = 0 \text{ cm/s}, a = -12 \text{ cm/s}^2$
b $t = 0, x = 5, a = -12; t = 1, x = 3, a = 12$
 5 **a** 2 m/s^2 **b** 50 m/s^2
 6 **a** $(15 - 9.8t) \text{ m/s}$ **b** -9.8 m/s^2
 7 **a** 3.5 s **b** 2 m/s^2 **c** 14.5 m
d When $t = 2.5 \text{ s}$; the particle is 1.25 m to the left of O
 8 **a** $0 \text{ s}, 1 \text{ s}, 2 \text{ s}$
b $2 \text{ m/s}, -1 \text{ m/s}, 2 \text{ m/s}; -6 \text{ m/s}^2, 0 \text{ m/s}^2, 6 \text{ m/s}^2$
c 0 m/s
 9 **a** 12 cm to the right of O
b 2 cm to the right of O
c Moving to the left at 7 cm/s
d $t = 3.5 \text{ s}$; particle is 0.25 cm to the left of O
e -2 cm/s **f** 2.9 cm/s
 10 **a** $18 \text{ m/s}^2, 54 \text{ m/s}^2, 114 \text{ m/s}^2$, **b** 58 m/s^2
 11 **a** 3 cm to the left of O , moving to the right at 24 cm/s
b $v = 3t^2 - 22t + 24$
c After $\frac{4}{3} \text{ s}$ and 6 s
d $11\frac{22}{27} \text{ cm}$ to the right of O and 39 cm to the left of O
e $4\frac{2}{3} \text{ s}$
f $a = 6t - 22$
g When $t = \frac{11}{3} \text{ s}$ and the particle is $13\frac{16}{27} \text{ cm}$ left of O moving to the left at $16\frac{1}{3} \text{ cm/s}$
 12 **a** When $t = \frac{2}{3} \text{ s}$ and $a = -2 \text{ cm/s}^2$, and when $t = 1$ and $a = 2 \text{ cm/s}^2$
b When $t = \frac{5}{6}$ and the particle is moving to the left at $\frac{1}{6} \text{ cm/s}$
 13 When $t = 2 \text{ s}, v = 6 \text{ cm/s}, a = -14 \text{ cm/s}^2$
 When $t = 3 \text{ s}, v = -5 \text{ cm/s}, a = -8 \text{ cm/s}^2$
 When $t = 8 \text{ s}, v = 30 \text{ cm/s}, a = 22 \text{ cm/s}^2$
 14 **a** $t = 4 \text{ s}$ and $t = -1 \text{ s}$ **b** $t = \frac{3}{2} \text{ s}$

Exercise 18G

- 1 **a** $x = 2t^2 - 6t$ **b** At the origin O
c 9 cm **d** 0 cm/s **e** 3 cm/s
 2 **a** $x = t^3 - 4t^2 + 5t + 4, a = 6t - 8$
b When $t = 1, x = 6$; when $t = \frac{5}{3}, x = 5\frac{23}{27}$
c When $t = 1, a = -2 \text{ cm/s}^2$; when $t = \frac{5}{3}, a = 2 \text{ cm/s}^2$
 3 $x = 215\frac{1}{3}, v = 73$
 4 **a** $v = -10t + 25$ **b** $x = -5t^2 + 25t$
c 2.5 s **d** $31\frac{1}{4} \text{ m}$ **e** 5 s
 5 The 29th floor

Exercise 18H

- 1 **a** $(x-2)(3x-2(b+1))$
b $(2, 0), \left(\frac{2(b+1)}{3}, \frac{-4(b-2)^3}{27}\right)$ **d** $b = 5$
 2 **a** $(0, 0)$ and $(9, -2187)$
b (a, b) and $(9+a, -2187+b)$
 3 **a** **i** $\left(-\infty, \frac{1}{2a}\right)$ **ii** $\left(\frac{1}{2a}, \infty\right)$
b $y = -x + \frac{1}{a}$ **c** $y = x - \frac{1}{a}$ **d** $\left(-\infty, \frac{1}{4a}\right]$
 4 **a** $(a, 0)$ and $\left(\frac{a+2}{3}, \frac{4(a-1)^3}{27}\right)$
b Local minimum at $(a, 0)$
 Local maximum at $\left(\frac{a+2}{3}, \frac{4(a-1)^3}{27}\right)$
c **i** $y = (a-1)^2(x-1)$ **ii** $y = 0$
iii $y = \frac{-(a-1)^2}{4}(x-a)$
 5 **a** **i** $2(a-2)$ **ii** $m = 2(a-2)$
b $(a, (a-2)^2)$
c $y = 2(a-2)x - a^2 + 4$ **d** $\frac{a+2}{2}$
 6 **a** $h = 2$ **b** $a = 3$ **c** $a = -16, b = -24$
 7 **a** $(0, 0)$ **b** (a, b)
 8 **a** $2(x-1)(x-b)(2x-b-1)$
b $(1, 0), (b, 0), \left(\frac{b+1}{2}, \frac{(b-1)^4}{16}\right)$ **c** $b = 3$

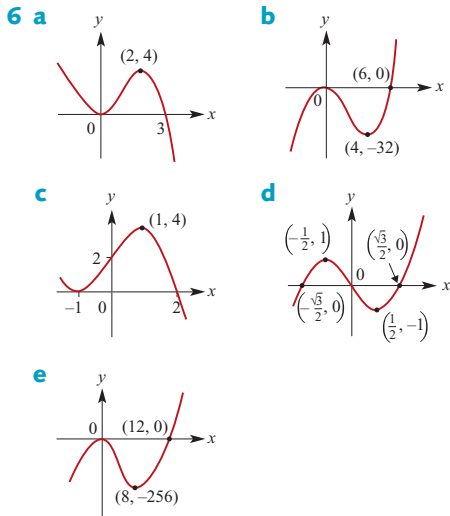
Exercise 18I

- 1 **a** 1.32 **b** 1.164 **c** 1.124 or 1.451
d 2.151 **e** -1.75
 2 1.44225 **3** 1.618

Chapter 18 review

Technology-free questions

- 1 **a** $\frac{dy}{dx} = 4 - 2x$ **b** 2 **c** $y = 2x + 1$
 2 **a** $3x^2 - 8x$ **b** -4 **c** $y = -4x$ **d** $(0, 0)$
 3 **a** $3x^2 - 12; x = \pm 2$
b Local minimum when $x = 2$
 Local maximum when $x = -2$
c $x = 2, y = -14; x = -2, y = 18$
 4 **a** Stationary point of inflection at $x = 0$
b Maximum at $x = 0$
c Min at $x = 3$, max at $x = 2$
d Min at $x = 2$, max at $x = -2$
e Max at $x = 2$, min at $x = -2$
f Max at $x = 3$, min at $x = 1$
g Max at $x = 4$, min at $x = -3$
h Max at $x = 3$, min at $x = -5$
 5 **a** $\left(-\frac{2}{3}, -\frac{16}{9}\right)$ minimum, $\left(\frac{2}{3}, \frac{16}{9}\right)$ minimum
b $(-1, 0)$ maximum, $(2, -27)$ minimum
c $\left(\frac{2}{3}, \frac{100}{27}\right)$ maximum, $(3, -9)$ minimum



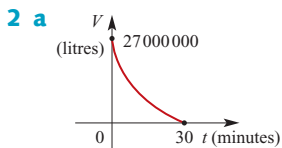
- 7 a** C **b** A **c** B
8 a 20 m **b** 6 s **c** 40 m/s
9 72
10 a $\frac{15}{2}$ m/s **b** $\frac{128}{3}$ m **c** 0 m **d** $\frac{64}{9}$ m/s

Multiple-choice questions

- 1** D **2** E **3** E **4** A **5** C **6** D
7 D **8** A **9** A **10** C **11** A **12** A

Extended-response questions

- 1 a** -14 m/s **b** -8 m/s^2

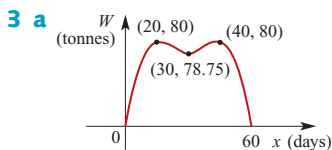
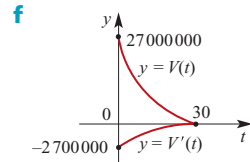


- b i** 17.4 minutes **ii** 2.9 minutes

c $\frac{dV}{dt} = -3000(30 - t)^2$

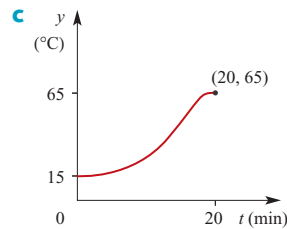
- d** 30 minutes

- e** 28.36 minutes

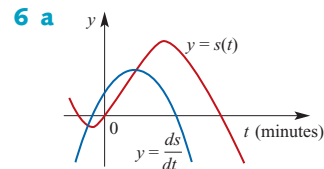
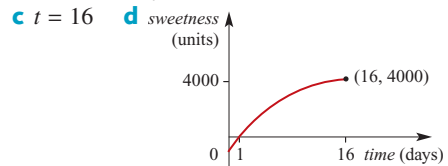


- b** From 5.71 days until 54.29 days
c When $x = 20$ and when $x = 40$, $\frac{dW}{dx} = 0$;
 When $x = 60$, $\frac{dW}{dx} = -12$ tonnes per day
d When $x = 30$, $W = 78.75$

- 4 a** 15°C
b 0°C/min , $\frac{45}{16}^\circ\text{C/min}$, $\frac{15}{4}^\circ\text{C/min}$,
 $\frac{45}{16}^\circ\text{C/min}$, 0°C/min



- 5 a** 768 units/day **b** 432, 192, 48, 0



- b** 11:59 a.m., 12:03 p.m.

- c** $\frac{5}{27}$ km, 1 km

- d** $\frac{8}{27}$ km/min = $17\frac{7}{9}$ km/h

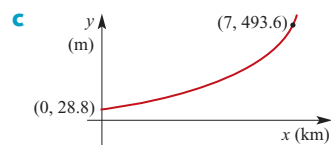
- e** $\frac{1}{3}$ km/min = 20 km/h

- 7 a** $0 \leq t \leq 12$

- b i** 27 L/h **ii** 192 L/h

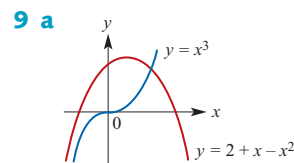
- 8 a** 28.8 m

- b** 374.4



- d** Path gets too steep after 7 km

- e i** 0.0384 **ii** 0.0504 **iii** 0.1336



- b** For $x \leq 0$, the minimum vertical distance occurs when $x = -1$; Min distance = 1 unit

- 10** 8 mm for maximum and $\frac{4}{3}$ mm for minimum

- 11 a** $y = 5 - x$ **b** $P = x(5 - x)$
c Max value is 6.25 when $x = 2.5$ and $y = 2.5$

12 a $y = 10 - 2x$ **b** $A = x^2(10 - 2x)$

c $A = \frac{1000}{27}$, $x = \frac{10}{3}$, $y = \frac{10}{3}$

13 $20\sqrt{10}$

14 a $y = 8 - x$ **b** $s = x^2 + (8 - x)^2$ **c** 32

15 $\frac{4}{3}, \frac{8}{3}$

16 Maximum area is 625 m^2 for $25 \text{ m} \times 25 \text{ m}$

17 $x = 12$ **18** 32

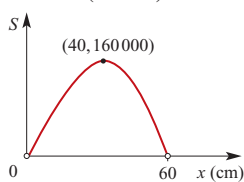
19 Maximum value of P is 2500

20 Maximum area is 2 km^2 for $2 \text{ km} \times 1 \text{ km}$

21 $p = \frac{3}{2}$, $q = \frac{8}{3}$

22 a $y = 60 - x$ **b** $S = 5x^2(60 - x)$

c $0 < x < 60$ **d**

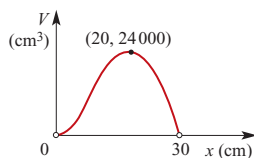


e $x = 40$, $y = 20$

f 74 005

23 12°C

24 b $0 < x < 30$ **c**



d 20 cm, 40 cm, 30 cm

e $x = 14.82$ or $x = 24.4$

25 b Maximum when $x = 3$ and $y = 18$

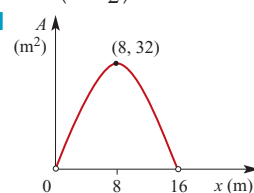
26 a Use 44 cm for circle and 56 cm for square

b Use all the wire for the circle

27 Length 7.2 metres, width 4.5 metres

28 a $A = xy$ **b** $A = \left(8 - \frac{x}{2}\right)x$

c $0 < x < 16$ **d**

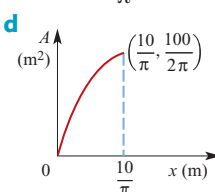


e 32 m^2

29 $h = 1188$, $a = 937$

30 a $y = 10 - \pi x$ **b** $0 \leq x \leq \frac{10}{\pi}$

c $A = \frac{x}{2}(20 - \pi x)$ **d**



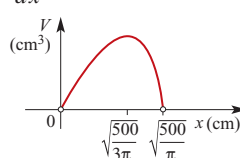
e Maximum at $x = \frac{10}{\pi}$

f A semicircle

31 a $h = \frac{500}{\pi x} - x$ **b** $V = 500x - \pi x^3$

c $\frac{dV}{dx} = 500 - 3\pi x^2$ **d** $x = 10\sqrt{\frac{5}{3\pi}} \approx 7.28$

e V **f** 2427.89 cm^3



g $x = 2.05$, $h = 75.41$ or $x = 11.46$, $h = 2.42$

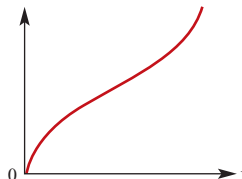
32 a $r = 4.3 \text{ cm}$, $h = 8.6 \text{ cm}$

b $r = 4.3 \text{ cm}$, $h = 8.6 \text{ cm}$

Chapter 19

Technology-free questions

1 h



2 a 1 m/s **b** 41 m/s

3 a i -4 **ii** -3 **b** $-2 - h$ **c** -2

4 $x - 1$

5 a $6x^2 - 1$ **b** $2x + 1$ **c** 1

6 a 13 **b** 10

7 a $x = 0$ or $x = \frac{1}{2}$ **b** $x = \frac{1}{4}$ **c** $x < \frac{1}{4}$

d $x > \frac{1}{4}$ **e** $x = \frac{11}{4}$

8 a $-6x^{-4} + x^{-2}$ **b** $\frac{2z - 9}{z^4}$

9 Tangent $y = -3x - 1$; Normal $y = \frac{1}{3}x - \frac{13}{3}$

10 a $t = 0$ and $t = 2$

b $t = 0$, $a = -1 \text{ cm/s}^2$; $t = 2$, $a = 1 \text{ cm/s}^2$

c $-\frac{1}{2} \text{ cm/s}$

11 Local minimum $\left(\frac{2}{\sqrt{3}}, -\frac{32}{3\sqrt{3}}\right)$

Local maximum $\left(-\frac{2}{\sqrt{3}}, \frac{32}{3\sqrt{3}}\right)$

Multiple-choice questions

1 B **2** A **3** B **4** A **5** B **6** A

7 D **8** B **9** C **10** A **11** C **12** B

13 A **14** C **15** C **16** A **17** D **18** E

19 C **20** B **21** E **22** D **23** A **24** A

25 C **26** D **27** B **28** B **29** B **30** C

31 D **32** E **33** A **34** A **35** C **36** C

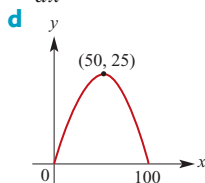
37 D **38** D **39** D

Extended-response questions

1 a 100

c $x = 50, y = 25$

b $\frac{dy}{dx} = 1 - 0.02x$



e i (25, 18.75) ii (75, 18.75)

2 a $(66\frac{2}{3}, 14\frac{22}{27})$

b i 0.28 ii -0.32 iii -1

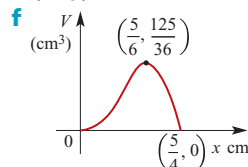
c A gradual rise to the turning point and a descent which becomes increasingly steep (in fact, alarmingly steep)

d Smooth out the end of the trip

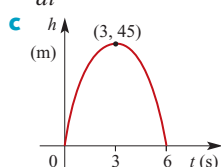
3 a $h = 5 - 4x$ c $0 < x < \frac{5}{4}$

d $\frac{dV}{dx} = 30x - 36x^2$

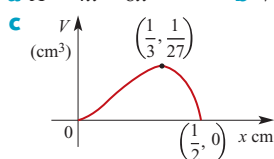
e $\{0, \frac{5}{6}\}$; Maximum volume = $3\frac{17}{36} \text{ cm}^3$



4 a $\frac{dh}{dt} = 30 - 10t$ b 45 m



5 a $A = 4x - 6x^2$ b $V = x^2 - 2x^3$

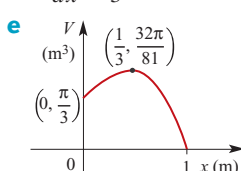


d $\frac{1}{3} \text{ cm} \times \frac{1}{3} \text{ cm} \times \frac{1}{3} \text{ cm}$; Volume = $\frac{1}{27} \text{ cm}^3$

6 a i $r = \sqrt{1 - x^2}$ ii $h = 1 + x$

c $0 < x < 1$

d i $\frac{dV}{dx} = \frac{\pi}{3}(1 - 2x - 3x^2)$ ii $\{\frac{1}{3}\}$ iii $\frac{32\pi}{81} \text{ m}^3$



7 a 1000 insects

b 1366 insects

c i $t = 40$ ii $t = 51.70$

d 63.64

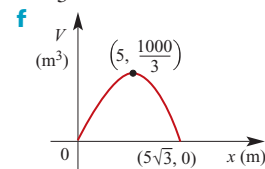
e i $\frac{1000 \times 2^{\frac{3}{4}} (2^{\frac{h}{20}} - 1)}{h}$

ii Consider h decreasing and approaching zero; instantaneous rate of change = 58.286 insects/day

8 a $h = \frac{150 - 2x^2}{3x}$ b $V = \frac{2}{3}(150x - 2x^3)$

c $\frac{dV}{dx} = 2(50 - 2x^2)$ d $0 < x < 5\sqrt{3}$

e $\frac{1000}{3} \text{ m}^3$ when $x = 5$



9 a 10

c i $h = 2.5x$

d $V = 40(420x - 135x^2)$

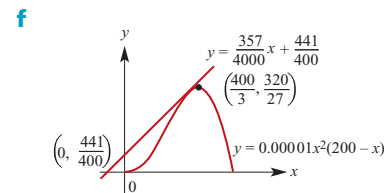
e i $x = \frac{14}{9}, y = \frac{140}{9}$ ii $13\,066\frac{2}{3} \text{ m}^3$

10 a $a = 200, k = 0.000\,01$

b i $\frac{400}{3}$ ii $\frac{320}{27}$ c i $\frac{8379}{800}$ ii $\frac{357}{4000}$

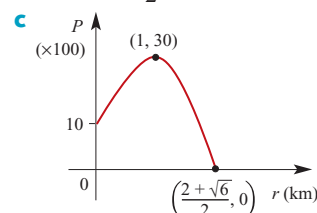
d i $y = \frac{357}{4000}x + \frac{441}{400}$ ii $\frac{441}{400}$

e 0.09975

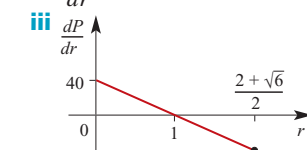


11 a 10 000 people/km²

b $0 \leq r \leq \frac{2 + \sqrt{6}}{2}$

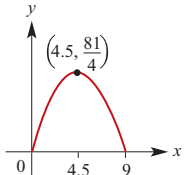


d i $\frac{dP}{dr} = 40 - 40r$ ii 20, 0, -40



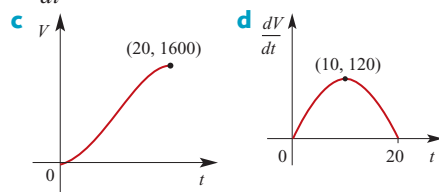
e At $r = 1$

- 12 a** $y = ax - x^2$ **b** $0 < x < a$ **c** $\frac{a^2}{4}, \frac{a}{2}$
d Negative coefficient of x^2 for quadratic function

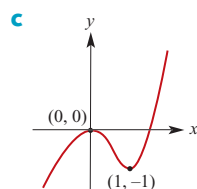
e i  **ii** $(0, \frac{81}{4})$

- 13 a i** 0 **ii** 1600

b $\frac{dV}{dt} = 0.6(40t - 2t^2)$



- 14 a** $-1 = a + b$ **b** $0 = 3a + 2b, a = 2, b = -3$



- 15 a i** $80 - 2x$ **ii** $h = \frac{\sqrt{3}}{2}x$

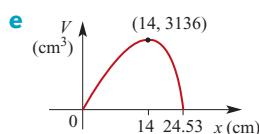
c $A = \frac{\sqrt{3}}{4}x(160 - 3x)$ **d** $x = \frac{80}{3}$

16 a $y = \frac{1400 - 2x^2 - 8x}{4x}$

b $V = -\frac{x^3}{2} - 2x^2 + 350x$

c $\frac{dV}{dx} = -\frac{3}{2}x^2 - 4x + 350$

d $x = 14$



f Maximum volume is 3136 cm³

g $x = 22.83$ and $y = 1.92$, or
 $x = 2.94$ and $y = 115.45$

Chapter 20

Exercise 20A

- 1 a** $30(x-1)^{29}$ **b** $100(x^4 - 2x^9)(x^5 - x^{10})^{19}$
c $4(1 - 3x^2 - 5x^4)(x - x^3 - x^5)^3$
d $8(x+1)^7$ **e** $-4(x+1)(x^2 + 2x)^{-3}$
f $-6(x+x^{-2})(x^2 - 2x^{-1})^{-4}$

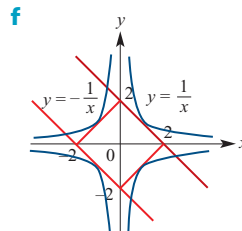
2 a $24x^2(2x^3 + 1)^3$ **b** 648

3 a $-\frac{1}{16}$ **b** $-\frac{3}{256}$

4 a $-\frac{2}{9}$ **b** $(-3, -\frac{1}{3}), (0, \frac{1}{3})$

5 a $-\frac{1}{4}$ **b** $\frac{1}{4}$ **c** $y = -x + 2$ **d** $y = x - 2$

e At $P, y = x + 2$; at $Q, y = -x - 2$; $(-2, 0)$



Exercise 20B

- 1 a** $\frac{1}{3}x^{-\frac{2}{3}}$ **b** $\frac{3}{2}x^{\frac{1}{2}}, x > 0$
c $\frac{5}{2}x^{\frac{3}{2}} - \frac{3}{2}x^{\frac{1}{2}}, x > 0$ **d** $x^{-\frac{1}{2}} - 5x^{\frac{2}{3}}, x > 0$
e $-\frac{5}{6}x^{-\frac{11}{6}}, x > 0$ **f** $-\frac{1}{2}x^{-\frac{3}{2}}, x > 0$
2 a $x(1+x^2)^{-\frac{1}{2}}$ **b** $\frac{1}{3}(1+2x)(x+x^2)^{-\frac{2}{3}}$
c $-x(1+x^2)^{-\frac{3}{2}}$ **d** $\frac{1}{3}(1+x)^{-\frac{2}{3}}$
3 a i $\frac{4}{3}$ **ii** $\frac{4}{3}$ **iii** $\frac{1}{3}$ **iv** $\frac{1}{3}$
4 a $\{x : 0 < x < 1\}$ **b** $\{x : x > (\frac{2}{3})^6\}$
5 a $-5x^{-\frac{1}{2}}(2-5\sqrt{x})$ **b** $3x^{-\frac{1}{2}}(3\sqrt{x}+2)$
c $-4x^{-3} - \frac{3}{2}x^{-\frac{5}{2}}$ **d** $\frac{3}{2}x^{\frac{1}{2}} - x^{-\frac{3}{2}}$
e $\frac{15}{2}x^{\frac{3}{2}} + 3x^{-\frac{1}{2}}$

Exercise 20C

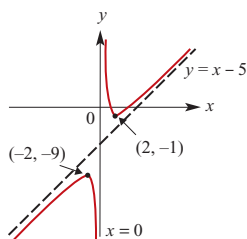
- 1 a** $-\frac{3}{x} + c$ **b** $3x^2 - \frac{2}{3x^3} + c$
c $\frac{4}{3}x^{\frac{3}{2}} + \frac{2}{5}x^{\frac{5}{2}} + c$ **d** $\frac{9}{4}x^{\frac{4}{3}} - \frac{20}{9}x^{\frac{9}{4}} + c$
e $\frac{3}{2}x^2 - \frac{2}{x} + c$ **f** $\frac{12}{7}x^{\frac{7}{4}} - \frac{14}{3}x^{\frac{3}{2}} + c$
2 a $y = \frac{2}{3}x^{\frac{3}{2}} + \frac{1}{2}x^2 - \frac{22}{3}$
b $y = \frac{3}{2} - \frac{1}{2x^2}$ **c** $y = \frac{3}{2}x^2 - \frac{1}{x} + \frac{9}{2}$
3 $f(x) = x^3 + \frac{1}{x} - \frac{17}{2}$
4 $s = \frac{3t^2}{2} + \frac{8}{t} - 8$ **5** $y = 5$
6 a 2 **b** $y = x^2 + 1$
7 $y = \frac{x^3}{3} + \frac{7}{3}$

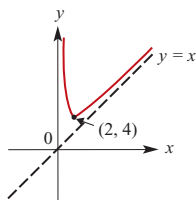
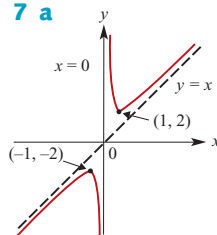
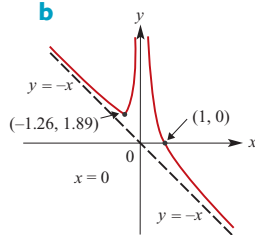
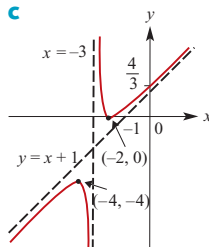
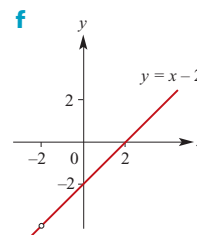
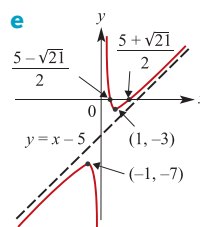
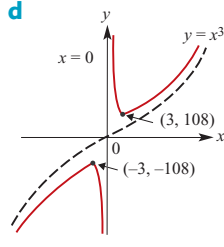
Exercise 20D

- 1** a $6x$ **b** 0 **c** $108(3x+1)^2$
d $-\frac{1}{4}x^{-\frac{3}{2}} + 18x$
e $306x^{16} + 396x^{10} + 90x^4$
f $10 + 12x^{-3} + \frac{9}{4}x^{-\frac{1}{2}}$
2 a $18x$ **b** 0 **c** 12 **d** $432(6x+1)^2$
e $300(5x+2)^2$ **f** $6x+4+6x^{-3}$
3 -9.8 m/s^2
4 a **i** -16 **ii** 4 m/s **iii** $\frac{7}{4} \text{ m/s}$ **iv** -32 m/s
b $t = 0$
c -8 m/s

Exercise 20E

- 1** a $(\frac{1}{2}, 4), (-\frac{1}{2}, -4)$ **b** $y = \frac{15}{4}x + 1$
2 $\pm \frac{1}{2}$
3 $\frac{1}{2}$
4 a $(4, 0), (1, 0)$ **b** $y = x - 5, x = 0$
c $(2, -1)$ min
 $(-2, -9)$ max


5 3

6 4

7 a

b

c

d

Chapter 20 review
Technology-free questions

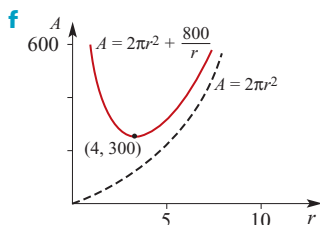
- 1** a $\frac{1}{2x^2}$ **b** $\frac{1}{3x^3}$ **c** $\frac{2}{3x^3}$ **d** $\frac{4}{3}x^{\frac{1}{3}}$
e $-\frac{1}{3x^4}$ **f** $-\frac{1}{3x^3} + \frac{6}{5x^5}$
2 a $8x+12$ **b** $24(3x+4)^3$
c $\frac{1}{(3-2x)^{\frac{3}{2}}}$ **d** $\frac{-2}{(3+2x)^2}$
e $\frac{-4}{3(2x-1)^{\frac{5}{3}}}$ **f** $\frac{-3x}{(2+x^2)^{\frac{3}{2}}}$
g $\frac{1}{3}\left(4x + \frac{6}{x^3}\right)\left(2x^2 - \frac{3}{x^2}\right)^{-\frac{2}{3}}$
3 a $-\frac{1}{x^2} + c$ **b** $\frac{2x^{\frac{5}{2}}}{5} - \frac{4x^{\frac{3}{2}}}{3} + c$
c $\frac{3x^2}{2} + 2x + c$ **d** $\frac{-6x-1}{2x^2} + c$
e $\frac{5x^2}{2} - \frac{4x^{\frac{3}{2}}}{3} + c$ **f** $\frac{20x^{\frac{7}{2}}}{7} - \frac{3x^{\frac{4}{3}}}{2} + c$
g $2x - \frac{2x^{\frac{3}{2}}}{3} + c$ **h** $-\frac{(3x+1)}{x^2} + c$
4 $s = \frac{1}{2}t^2 + 3t + \frac{1}{t} + \frac{3}{2}$
5 a $\frac{1}{6}$ **b** -2 **c** $-\frac{1}{16}$ **d** -2 **e** $\frac{1}{6}$ **f** 0
6 $(\frac{1}{2}, 2)$ and $(-\frac{1}{2}, -2)$
7 $(\frac{1}{16}, \frac{1}{4})$

Multiple-choice questions

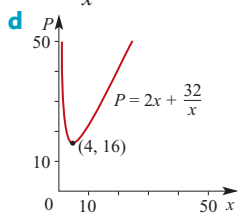
- 1** B **2** D **3** A **4** A **5** A
6 E **7** A **8** B **9** A **10** D

Extended-response questions

- 1** a $h = \frac{400}{\pi r^2}$
c $\frac{dA}{dr} = 4\pi r - \frac{800}{r^2}$
d $r = \left(\frac{200}{\pi}\right)^{\frac{1}{3}} \approx 3.99$
e $A = 301 \text{ cm}^2$



2 a $y = \frac{16}{x}$ **c** $x = 4, P = 16$



3 a $OA = \frac{120}{x}$ **b** $OX = \frac{120}{x} + 7$
c $OZ = x + 5$ **d** $y = 7x + \frac{600}{x} + 155$

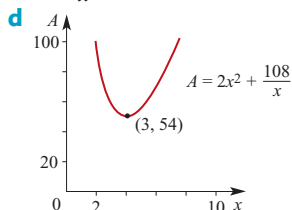
e $x = \frac{10\sqrt{42}}{7} \approx 9.26$ cm

4 a $A(-2, 0), B(0, \sqrt{2})$ **b** $\frac{1}{2\sqrt{x+2}}$

c i $\frac{1}{2}$ **ii** $2y - x = 3$ **iii** $\frac{3\sqrt{5}}{2}$

d $x > -\frac{7}{4}$

5 a $h = \frac{18}{x^2}$ **c** $x = 3, h = 2$



6 a $y = \frac{250}{x^2}$ **c** $\frac{dS}{dx} = 24x - \frac{3000}{x^2}$

d $S_{\min} = 900$ cm²

Chapter 21

Exercise 21A

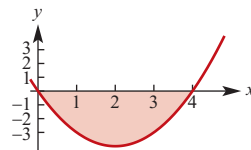
- 1** 62 square units **2** 80 square units
3 60.90623 square units **4** 68 square units
5 a 13.2 **b** 10.2 **c** 11.7
6 a 4.375 **b** 4.536
7 a 36.8 **b** 36.75
8 $\pi \approx 3.13$
9 a 4.371 **b** 1.128
10 109.5 m²

Exercise 21B

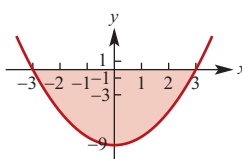
- 1 a** $\frac{7}{3}$ **b** 20 **c** $-\frac{1}{4}$ **d** 9 **e** $\frac{15}{4}$
f $\frac{297}{6} = 49.5$ **g** $15\frac{1}{3}$ **h** 30
2 4 **3** 9 **4** $\frac{4}{3}$ **5** 4

Exercise 21C

1 $-\frac{32}{3}$



2 -36

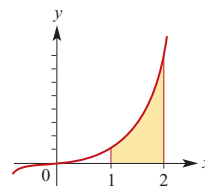


- 3** 36
4 $\frac{37}{12}$
5 a 8 **b** 16 **c** -4
6 a -12 **b** 36 **c** 20
7 a 24, 21, 45 **b** 4, -1, 3
8 4.5 square units **9** $166\frac{2}{3}$ square units
10 $\frac{37}{12}$ square units
11 a 1 **b** 1 **c** 14 **d** 31 **e** $2\frac{1}{4}$ **f** 0
12 a $\frac{4}{3}$ **b** $\frac{1}{6}$ **c** $121\frac{1}{2}$ **d** $\frac{1}{6}$
e $4\sqrt{3} \approx 6.93$ **f** 108

Chapter 21 review

Technology-free questions

- 1 a** 3 **b** 6 **c** 114 **d** $\frac{196}{3}$ **e** 5
2 a $\frac{14}{3}$ **b** $48\frac{3}{4}$ **c** $\frac{1}{2}$ **d** $\frac{15}{16}$ **e** $\frac{16}{15}$
3 $\frac{15}{4}$ square units



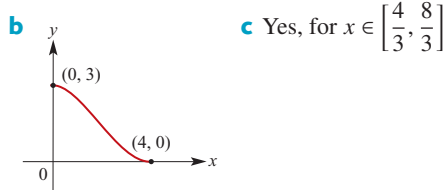
- 4** $4\frac{1}{2}$ square units **5** $21\frac{1}{12}$ square units
6 a $B(1, 3), C(3, 3)$ **b** 6 **c** $\frac{4}{3}$

Multiple-choice questions

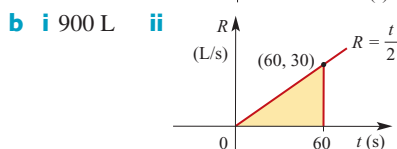
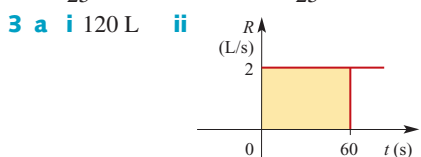
- 1** C **2** D **3** A **4** D **5** B
6 B **7** D **8** B **9** C **10** A

Extended-response questions

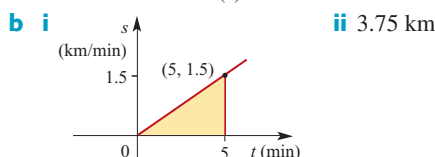
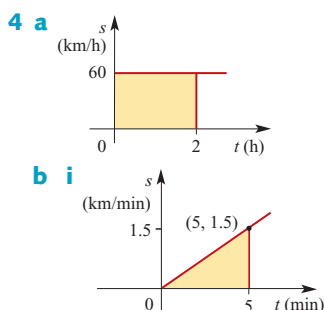
1 a $y = \frac{9}{32} \left(\frac{x^3}{3} - 2x^2 \right) + 3$

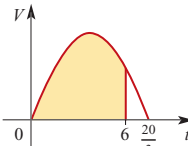


2 a 27 square units b $y = \frac{3}{25}(x-4)^2$
 c $\frac{189}{25}$ square units d $\frac{486}{25}$ square units



- iii $900a^2$ L
 c i 7200 square units
 ii Volume of water which has flowed in
 iii 66.94 s



- c i $20 - 6t$ m/s² ii  iii 144 metres

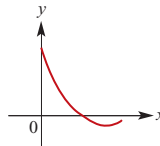
- 5 a i 4 m ii 16 m
 b i 0.7 ii -0.8
 c i $\frac{100}{3}$ ii $\frac{500}{27}$
 d $\frac{3125}{6}$ m²
 e i $(15 + 5\sqrt{33}, 12)$ ii $R = 60\sqrt{33} - 60, q = 20, p = 15 + 5\sqrt{33}$
 6 a i 9 ii $y = 9x - 3$ iii $y = 3x^2 + 3x$
 b i $12 + k$ ii $k = -7$ iii $y = 3x^2 - 7x + 12$

7 a 6 m^2 b i $y = x - \frac{1}{2}$ ii $\left(x^2 - \frac{1}{4}\right) \text{ m}^2$

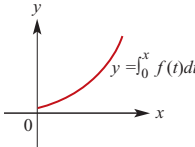
c i $y = \frac{1}{2}x^2, P(-2, 2), S(2, 2)$ ii $\frac{16}{3} \text{ m}^2$

8 a $y = 7 \times 10^{-7}x^3 - 0.00116x^2 + 0.405x + 60$

b 100 m

c i  ii (0, 60)

d 51 307 m²

9 a  b $x = 2.988$

Chapter 22

Technology-free questions

1 a $\frac{25}{12}$ b $\frac{77}{60}$ c $\frac{101}{60}$

2 a $\frac{9}{2}x^{\frac{1}{2}}$ b $\frac{1}{5}x^{-\frac{4}{5}}$ c $\frac{10}{3}x^{-\frac{8}{3}}$ d $10x^{\frac{2}{3}}$

e $-\frac{1}{5}x^{-\frac{6}{5}}$ f $-\frac{2}{3}x^{-\frac{5}{3}} - 3x^{\frac{1}{2}}$

3 a $6(3x + 5)$ b $-8(2x + 7)^3$

c $\frac{2}{3}(5 - 2x)^{-\frac{4}{3}}$ d $\frac{-12}{(5 + 3x)^2}$

e $-\frac{2}{3}(x - 1)^{-\frac{5}{3}}$ f $-9x(2 + 3x^2)^{-\frac{3}{2}}$

g $\frac{1}{3} \left(2x^3 - \frac{5}{x} \right)^{\frac{2}{3}} \left(6x^2 + \frac{5}{x^2} \right)$

4 $x = \frac{t^2}{2} + 4t + \frac{3}{t} - \frac{3}{2}$

5 a $\frac{1}{27}$ b -3 c $-\frac{3}{8}$

6 $\left(-\left(\frac{1}{2}\right)^{\frac{1}{3}}, 4^{\frac{1}{3}} \right)$

7 $\left(\left(\frac{1}{6}\right)^{\frac{3}{2}}, \left(\frac{1}{6}\right)^{\frac{1}{2}} \right)$ and $\left(-\left(\frac{1}{6}\right)^{\frac{3}{2}}, -\left(\frac{1}{6}\right)^{\frac{1}{2}} \right)$

8 a $x^3 + x + c$ b $-t^3 - \frac{1}{2}t^2 + 2t + c$

c $\frac{2}{3}x^{\frac{3}{2}} + c$ d $\frac{4}{5}x^{\frac{5}{2}} + \frac{3}{4}x^{\frac{4}{3}} + c$

9 a $\frac{2}{3}$ b $\frac{5}{6}$

10 a $\frac{1}{6}$ square units b $\frac{1}{2}$ square units

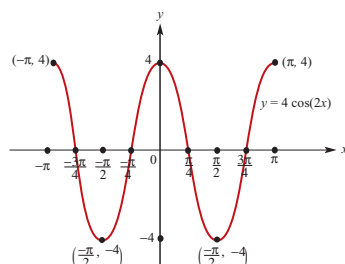
Multiple-choice questions

- 1 E 2 C 3 E 4 D 5 A
 6 A 7 E 8 C 9 E 10 C
 11 B 12 C 13 C

Chapter 23

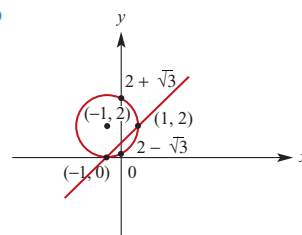
Technology-free questions

- 1 $x = 4$
- 2 $t = \frac{2d-b}{a-2c}$
- 3 $x \geq -\frac{3}{2}$
- 4 **a** -12 **b** 3 **c** 100
- 5 15
- 6 $x \leq \frac{37}{5}$
- 7 $a = 7.9$
- 8 **a** $\left(\frac{a+8}{2}, \frac{b+14}{2}\right)$ **b** $a = 2, b = 6$
- 9 **a** $4y - 3x = 30$ **b** $\frac{25}{2}$
- 10 **a** $(2, \frac{1}{2})$ **b** $\sqrt{445}$ **c** $11x + 18y = 31$
- d** $22y - 36x + 61 = 0$
- 11
- 12 $y = \frac{9}{8}(x-2)^2 - 6$
- 13 $a = -2$
- 14 **a** $w = 1500 - 9x$ **b** $V = 20x^2(1500 - 9x)$
- c** $0 \leq x \leq \frac{500}{3}$ **d** $120\,000\,000\text{ cm}^3$
- 15 **a** $\frac{16}{81}$ **b** $\frac{28}{153}$
- 16 $\frac{1}{3}$
- 17 **a** $\frac{1}{2}$ **b** $\frac{1}{3}$
- 18 0.42
- 19 $-\frac{\pi}{9}, \frac{\pi}{9}$
- 20 **a** $c = 6$ **b** $0 = -8a - 2b + 6, 0 = 3a + b$
- c** $a = 3, b = -9$
- 21 $a = -48$
- 22 **a** Amplitude = 4; Period = π
- b**

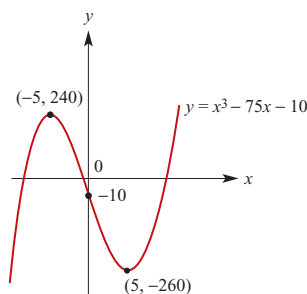


23 **a** $\frac{1}{4}$ **b** $\frac{1}{3}$ **c** $\frac{1}{4}$

24 **a** (0, 1) **b**



- 25 **a** $x = 3$ **b** $x = -\frac{5}{2}$ or $x = 1$
- 26 $k = 4$ and $y = -2x^2 + 4x + 3$
- 27 $a = \frac{1}{3}$ 28 $y = \frac{-2x^2}{9}$ 29 $b = -\frac{1}{3}$
- 30 Intersect at $(-3, -27)$; both curves have gradient 27 at this point
- 31 **a** (5, -260) and $(-5, 240)$



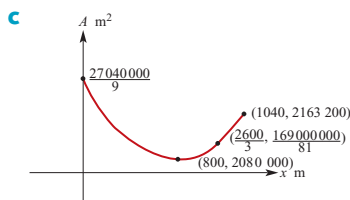
- b** $-260 \leq p \leq 240$
- 32 **a** $\mathbb{R} \setminus \{3\}$ **b** $\mathbb{R} \setminus \{2\}$ **c** $(-\infty, 2]$
- d** $[4, \infty)$ **e** $(-\infty, 5)$

Multiple-choice questions

- 1 B 2 A 3 D 4 D 5 D 6 C
- 7 C 8 A 9 B 10 A 11 B 12 A
- 13 A 14 E 15 B 16 C 17 B 18 A
- 19 C 20 A 21 B 22 B 23 D 24 D
- 25 A 26 C 27 B 28 A 29 E 30 C

Extended-response questions

- 1 **a** i $-b$ ii $x = \frac{b}{2}$
- b** i $S(b) = \frac{b}{2}(32 - b^2)$ ii $b = 2$
- 2 **a** $a = 4$ **b** $\left(\frac{1}{4}, 4\right)$ **c** $\frac{34}{15}$
- 3 **a** $2t^2 - 6t$ **b** O **c** 0 cm/s **d** 9 cm **e** 3 cm/s
- 4 **a** i $(-\infty, 0) \cup \left(\frac{2a}{3}, \infty\right)$ ii $\left(0, \frac{2a}{3}\right)$
- b** $y = -a^2(x - a)$ **c** $y = \frac{x}{a^2} - \frac{1}{a}$ **d** $\frac{a^4}{12}$
- 5 **a** $y = 13 - 9x$ **b** $A = 156x - 60x^2$
- c** $x = \frac{13}{10}$ and $y = \frac{13}{10}$
- 6 **b** 2 080 000 m² when $x = 800$ and $y = 1200$



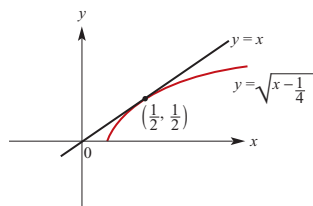
- 7 a** $\{(0, 1), (0, 3), (0, 5), (0, 7), (0, 9), (0, 11), (2, 1), (2, 3), (2, 5), (2, 7), (2, 9), (2, 11), (4, 1), (4, 3), (4, 5), (4, 7), (4, 9), (4, 11), (6, 1), (6, 3), (6, 5), (6, 7), (6, 9), (6, 11), (8, 1), (8, 3), (8, 5), (8, 7), (8, 9), (8, 11), (10, 1), (10, 3), (10, 5), (10, 7), (10, 9), (10, 11)\}$

b i $\frac{1}{36}$ **ii** $\frac{5}{36}$ **iii** $\frac{5}{36}$ **c** $\frac{2}{13}$

8 a $AB = \begin{bmatrix} -2 & 5 & 5 \\ 0 & 6 & 2 \\ 0 & 0 & -4 \end{bmatrix}$ **d** $x = 1, y = 2, z = 3$

9 a $x \geq 2a$
b $x = \frac{1 + \sqrt{1 - 8a}}{2}$ or $x = \frac{1 - \sqrt{1 - 8a}}{2}$

c $a = \frac{1}{8}$ **d**



10 a 0.343 **b** 0.399

11 a 0.4219 **b** 0.2156 **c** 0.6125

12 a $[-mb + 3, -ma + 3]$

b $f^{-1}(x) = -\frac{1}{m}x + \frac{3}{m}$

c $\left(\frac{a+b}{2}, \frac{-m(a+b)+6}{2}\right)$

d $2ym - 2x = -m^2(a+b) + 6m - (a+b)$

e $y = -mx - 3m + 8$

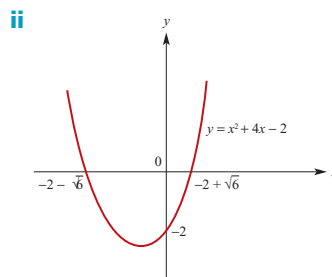
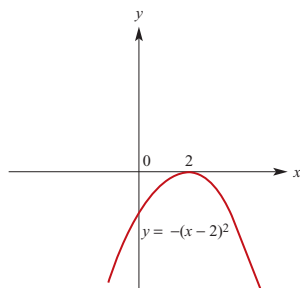
$A'(a-3, -ma+8), B'(b-3, -mb+8)$

f $y = mx + 3$

$A'(-a, -ma+3), B'(-b, -mb+3)$

g $b = 12, m = \frac{7}{6}$

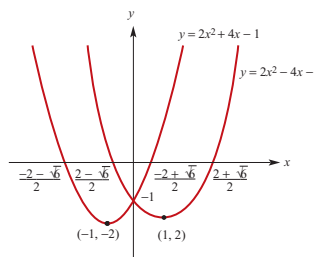
13 a i



b $\left(\frac{2}{1-p}, \frac{p^2-5p}{p-1}\right)$ **c** $p = 0$ or $p = 5$

d $0 < p < 5$ and $p \neq 1$

e $y = 2x^2 - 4x - 1$



14 a $\frac{\pi}{6}$ **b** $115\sqrt{2}$ cm **c** $115\sqrt{3}$ cm

Appendix A

Exercise A1

1 a Quotient $3x^2 + 6x + 16$; Remainder 26

b Quotient $3x^2 - 5x + 9$; Remainder -3

c Quotient $x^2 - 2x - 6$; Remainder 54

d Quotient $4x^2 - 2x + 6$; Remainder -69

e Quotient $2x^2 - x + 3$; Remainder -69

Exercise A2

1 a $\frac{3}{2}, \frac{1 \pm \sqrt{21}}{2}$

b $\frac{1}{3}, \frac{-1 \pm \sqrt{5}}{2}$

c $\frac{1}{3}, \frac{-1 \pm \sqrt{41}}{4}$

d $\frac{1}{2}, \frac{-1 \pm \sqrt{13}}{2}$

e $\frac{1}{2}, \frac{1}{3}, -\frac{2}{5}$

f $\frac{1}{2}, \frac{1}{3}, -\frac{2}{3}$

Exercise A3

1 a $x = 2, y = 3, z = 1$ **b** $x = -3, y = 5, z = 2$

c $x = 5, y = 0, z = 7$

2 $x = 6, y = 5, z = 1$

3 $a = 1, b = -2, c = 3$

4 $b = 1, c = 2, d = 5$

5 $x = 5, y = -2, z = 1$

6 $b = -2, c = 0, d = 3$

7 a $-y + 5z = 15$ and $-y + 5z = 15$

b The solution is a straight line

c $y = 5\lambda - 15$

d $x = 43 - 13\lambda$

8 a $y = -2 + 4z$

b $x = 8 - 5\lambda, y = -2 + 4\lambda, z = \lambda, \text{ for } \lambda \in \mathbb{R}$