

TOPIC 2

Algebra and equations

2.1 Overview

Why learn this?

Do you speak mathematics? Algebra is the language of mathematics; it holds the key to understanding the rules, formulas and relationships that summarise much of our understanding of the universe. Every student of mathematics needs a mathematical tool chest, a set of algebraic skills to manipulate and process mathematical information.

What do you know?

assessment

- 1 THINK** List what you know about linear equations. Use a thinking tool such as a concept map to show your list.
- 2 PAIR** Share what you know with a partner and then with a small group.
- 3 SHARE** As a class, create a thinking tool such as a large concept map that shows your class's knowledge of linear equations.

Learning sequence

- 2.1** Overview
- 2.2** Substitution
- 2.3** Adding and subtracting algebraic fractions
- 2.4** Multiplying and dividing algebraic fractions
- 2.5** Solving simple equations
- 2.6** Solving multi-step equations
- 2.7** Literal equations
- 2.8** Review

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2.2 Substitution

When the numerical values of pronumerals are known, they can be substituted into an algebraic expression and the expression can then be evaluated. It can be useful to place any substituted values in brackets when evaluating an expression.

WORKED EXAMPLE 1

If $a = 4$, $b = 2$ and $c = -7$, evaluate the following expressions.

a $a - b$

THINK

- 1** Write the expression.
- 2** Substitute $a = 4$ and $b = 2$ into the expression.
- 3** Simplify.

b $a^3 + 9b - c$

WRITE

$$\begin{aligned} \text{a } a - b &= 4 - 2 \\ &= 2 \\ \text{b } a^3 + 9b - c &= (4)^3 + 9(2) - (-7) \\ &= 64 + 18 + 7 \\ &= 89 \end{aligned}$$

WORKED EXAMPLE 2

TI

CASIO

If $c = \sqrt{a^2 + b^2}$, calculate c if $a = 12$ and $b = -5$.

THINK

- 1** Write the expression.
- 2** Substitute $a = 12$ and $b = -5$ into the expression.
- 3** Simplify.

WRITE

$$\begin{aligned} c &= \sqrt{a^2 + b^2} \\ &= \sqrt{(12)^2 + (-5)^2} \\ &= \sqrt{144 + 25} \\ &= \sqrt{169} \\ &= 13 \end{aligned}$$

Number laws

- Recall from previous studies that when dealing with numbers and pronumerals, particular rules must be obeyed. Before progressing further, let us briefly review the Commutative, Associative, Identity and Inverse Laws.
- Consider any three pronumerals x , y and z , where x , y and z are elements of the set of Real numbers.

Commutative Law

1. $x + y = y + x$ (example: $3 + 2 = 5$ and $2 + 3 = 5$)
2. $x - y \neq y - x$ (example: $3 - 2 = 1$ but $2 - 3 = -1$)
3. $x \times y = y \times x$ (example: $3 \times 2 = 6$ and $2 \times 3 = 6$)
4. $x \div y \neq y \div x$ (example: $3 \div 2 = \frac{3}{2}$, but $2 \div 3 = \frac{2}{3}$)

Therefore, the **Commutative Law** holds true for addition and multiplication, since the order in which two numbers or pronumerals are added or multiplied does not affect the result. However, the Commutative Law does not hold true for subtraction or division.

Associative Law

1. $x + (y + z) = (x + y) + z$ [example: $2 + (3 + 4) = 2 + 7 = 9$ and $(2 + 3) + 4 = 5 + 4 = 9$]
2. $x - (y - z) \neq (x - y) - z$ [example: $2 - (3 - 4) = 2 - -1 = 3$ and $(2 - 3) - 4 = -1 - 4 = -5$]
3. $x \times (y \times z) = (x \times y) \times z$ [example: $2 \times (3 \times 4) = 2 \times 12 = 24$ and $(2 \times 3) \times 4 = 6 \times 4 = 24$]
4. $x \div (y \div z) \neq (x \div y) \div z$ [example: $2 \div (3 \div 4) = 2 \div \frac{3}{4} = 2 \times \frac{4}{3} = \frac{8}{3}$ but $(2 \div 3) \div 4 = \frac{2}{3} \div 4 = \frac{2}{3} \times \frac{1}{4} = \frac{2}{12} = \frac{1}{6}$]

The **Associative Law** holds true for addition and multiplication since grouping two or more numbers or pronumerals and calculating them in a different order does not affect the result. However, the Associative Law does not hold true for subtraction or division.

Identity Law

The **Identity Law** states that in general:

$$x + 0 = 0 + x = x$$

$$x \times 1 = 1 \times x = x$$

In both of the examples above, x has not been changed (that is, it has kept its identity) when zero is added to it or it is multiplied by 1.

Inverse Law

The **Inverse Law** states that in general:

$$x + -x = -x + x = 0$$

$$x \times \frac{1}{x} = \frac{1}{x} \times x = 1$$

That is, when the additive inverse of a number or pronumeral is added to itself, it equals 0. When the multiplicative inverse of a number or pronumeral is multiplied by itself, it equals 1.

Closure Law

A law that you may not yet have encountered is the Closure Law. The **Closure Law** states that, when an operation is performed on an element (or elements) of a set, the result produced must also be an element of that set. For example, addition is closed on natural numbers (that is, positive integers: 1, 2, 3, ...) since adding a pair of natural numbers produces a natural number. Subtraction is not closed on natural numbers. For example, 5 and 7 are natural numbers and the result of adding them is 12, a natural number. However, the result of subtracting 7 from 5 is -2 , which is not a natural number.

WORKED EXAMPLE 3

Find the value of the following expressions, given the integer values $x = 4$ and $y = -12$. Comment on whether the Closure Law for integers holds for each of the expressions when these values are substituted.

a $x + y$

b $x - y$

c $x \times y$

d $x \div y$

THINK

- a** 1 Substitute each pronumeral into the expression.
- 2 Evaluate and write the answer.
- 3 Determine whether the Closure Law holds; that is, is the result an integer?

b Repeat steps 1–3 of part **a**.

c Repeat steps 1–3 of part **a**.

d Repeat steps 1–3 of part **a**.

WRITE

a $x + y = 4 + -12$

$$= -8$$

The Closure Law holds for these substituted values.

b $x - y = 4 - -12$
 $= 16$

The Closure Law holds for these substituted values.

c $x \times y = 4 \times -12$
 $= -48$

The Closure Law holds for these substituted values.

d $x \div y = 4 \div -12$
 $= \frac{4}{-12}$
 $= -\frac{1}{3}$

The Closure Law does not hold for these substituted values since the answer obtained is a fraction, not an integer.

- It is important to note that, although a particular set of numbers may be closed under a given operation, for example multiplication, another set of numbers may not be closed under that same operation. For example, in part **c** of Worked example 3, integers were closed under multiplication. However, in some cases, the set of *irrational numbers* is not closed under multiplication, since $\sqrt{3} \times \sqrt{3} = \sqrt{9} = 3$. In this example, two irrational numbers produced a rational number under multiplication.

assess on

Exercise 2.2 Substitution

INDIVIDUAL PATHWAYS

PRACTISE

Questions:
1a–f, 2a–f, 3c–d, 4a–c, 5a–c,
6a–e, 7, 8, 10, 14

CONSOLIDATE

Questions:
1c–i, 2a–f, 3c–d, 4, 5a–c, 6d–j,
9, 10, 14

MASTER

Questions:
1e–l, 2c–i, 3, 4, 5, 6d–j, 9–15

REFLECTION

Why is understanding of the Commutative Law useful?

FLUENCY

1 **WE1** If $a = 2$, $b = 3$ and $c = 5$, evaluate the following expressions.

a $a + b$

b $c - b$

c $c - a - b$

d $c - (a - b)$

e $7a + 8b - 11c$

f $\frac{a}{2} + \frac{b}{3} + \frac{c}{5}$

g abc

h $ab(c - b)$

i $a^2 + b^2 - c^2$

j $c^2 + a$

k $-a \times b \times -c$

l $2.3a - 3.2b$

2 If $d = -6$ and $k = -5$, evaluate the following.

a $d + k$

b $d - k$

c $k - d$

d kd

e $-d(k + 1)$

f d^2

g k^3

h $\frac{k - 1}{d}$

i $3k - 5d$

3 If $x = \frac{1}{3}$ and $y = \frac{1}{4}$, evaluate the following.

a $x + y$

b $y - x$

c xy

d $\frac{x}{y}$

e x^2y^3

f $\frac{9x}{y^2}$

4 If $x = 3$, find the value of the following.

a x^2

b $-x^2$

c $(-x)^2$

d $2x^2$

e $-2x^2$

f $(-2x)^2$

5 If $x = -3$, find the value of the following.

a x^2

b $-x^2$

c $(-x)^2$

d $2x^2$

e $-2x^2$

f $(-2x)^2$

6 **WE2** Calculate the unknown variable in the following real-life mathematical formulas.

a If $c = \sqrt{a^2 + b^2}$, calculate c if $a = 8$ and $b = 15$.

b If $A = \frac{1}{2}bh$, determine the value of A if $b = 12$ and $h = 5$.

c The perimeter, P , of a rectangle is given by $P = 2L + 2W$. Calculate the perimeter, P , of a rectangle, given $L = 1.6$ and $W = 2.4$.

d If $T = \frac{C}{L}$, determine the value of T if $C = 20.4$ and $L = 5.1$.

e If $K = \frac{n + 1}{n - 1}$, determine the value of K if $n = 5$.

f Given $F = \frac{9C}{5} + 32$, calculate F if $C = 20$.

g If $v = u + at$, evaluate v if $u = 16$, $a = 5$, $t = 6$.

h The area, A , of a circle is given by the formula $A = \pi r^2$. Calculate the area of a circle, correct to 1 decimal place, if $r = 6$.

i If $E = \frac{1}{2}mv^2$, calculate m if $E = 40$, $v = 4$.

j Given $r = \sqrt{\frac{A}{\pi}}$, evaluate A to 1 decimal place if $r = 14.1$.

7 **MC** **a** If $p = -5$ and $q = 4$, then pq is equal to:

A 20

B 1

C -1

D -20

E $-\frac{5}{4}$

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Like terms

doc-5183

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Collecting like terms

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Finding the highest

common factor

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Order of operations

doc-5189

- b** If $c^2 = a^2 + b^2$, and $a = 6$ and $b = 8$, then c is equal to:
A 28 **B** 100 **C** 10 **D** 14 **E** 44
- c** Given $h = 6$ and $k = 7$, then kh^2 is equal to:
A 294 **B** 252 **C** 1764 **D** 5776 **E** 85

UNDERSTANDING

- 8** Knowing the length of two sides of a right-angled triangle, the third side can be calculated using Pythagoras' theorem. If the two shorter sides have lengths of 1.5 cm and 3.6 cm, calculate the length of the hypotenuse.
- 9** The volume of a sphere can be calculated using the formula $\frac{4}{3}\pi r^3$. What is the volume of a sphere with a radius of 2.5 cm? Give your answer correct to 2 decimal places.
- 10** A rectangular park is 200 m by 300 m. If Blake runs along the diagonal of the park, how far will he run? Give your answer to the nearest metre.



REASONING

- 11** **WE3** Determine the value of the following expressions, given the integer values $x = 1$, $y = -2$ and $z = -1$. Comment on whether the Closure Law for integers holds true for each of the expressions when these values are substituted.
- a** $x + y$ **b** $y - z$ **c** $y \times z$ **d** $x \div z$ **e** $z - x$ **f** $x \div y$
- 12** Find the value of the following expressions, given the natural number values $x = 8$, $y = 2$ and $z = 6$. Comment on whether the Closure Law for natural numbers holds true for each of the expressions.
- a** $x + y$ **b** $y - z$ **c** $y \times z$ **d** $x \div z$ **e** $z - x$ **f** $x \div y$
- 13** For each of the following, complete the relationship to illustrate the stated law. Justify your reasoning.
- a** $(a + 2b) + 4c =$ _____ Associative Law
b $(x \times 3y) \times 5c =$ _____ Associative Law
c $2p \div q \neq$ _____ Commutative Law
d $5d + q =$ _____ Commutative Law
e $3z + 0 =$ _____ Identity Law
f $2x \times$ _____ = _____ Inverse Law
g $(4x \div 3y) \div 5z \neq$ _____ Associative Law
h $3d - 4y \neq$ _____ Commutative Law

PROBLEM SOLVING

14 $s = ut + \frac{1}{2}at^2$

where t is the time in seconds, s is the displacement in metres, u is the initial velocity and a is the acceleration due to gravity.

- a** Calculate s when $u = 16.5$ m/s, $t = 2.5$ seconds and $a = 9.8$ m/s².
- b** A body has an initial velocity of 14.7 m/s and after t seconds has a displacement of 137.2 metres. Find the value of t if $a = 9.8$ m/s².
- 15** Find the value of m if $n = p\sqrt{1 + \frac{1}{m}}$, when $n = 6$ and $p = 4$.





CHALLENGE 2.1

The lowest common multiple of four terms is $24a^3bc^2d$. Three of the terms are $12a^2bc$, $8ab$ and $4a^2cd$. The fourth term contains only two pronumerals, and its coefficient is an odd prime number. What is the fourth term?



2.3 Adding and subtracting algebraic fractions

- In an algebraic fraction, the denominator, the numerator or both are algebraic expressions. For example, $\frac{x}{2}$, $\frac{3x+1}{2x-5}$ and $\frac{1}{x^2+5}$ are all **algebraic fractions**.
- As with all fractions, algebraic fractions must have a common denominator if they are to be added or subtracted, so an important step is to find the lowest common denominator (LCD).

WORKED EXAMPLE 4

TI

CASIO

Simplify the following expressions.

a $\frac{2x}{3} - \frac{x}{2}$

b $\frac{x+1}{6} + \frac{x+4}{4}$

THINK

- a
- Write the expression.
 - Rewrite each fraction as an equivalent fraction using the LCD of 3 and 2, which is 6.
 - Express as a single fraction.
 - Simplify the numerator.

WRITE

a $\frac{2x}{3} - \frac{x}{2}$

$$= \frac{2x}{3} \times \frac{2}{2} - \frac{x}{2} \times \frac{3}{3}$$

$$= \frac{4x}{6} - \frac{3x}{6}$$

$$= \frac{4x - 3x}{6}$$

$$= \frac{x}{6}$$



b 1 Write the expression.

$$\frac{x+1}{6} + \frac{x+4}{4}$$

2 Rewrite each fraction as an equivalent fraction using the LCD of 6 and 4, which is 12.

$$\begin{aligned} &= \frac{x+1}{6} \times \frac{2}{2} + \frac{x+4}{4} \times \frac{3}{3} \\ &= \frac{2(x+1)}{12} + \frac{3(x+4)}{12} \end{aligned}$$

3 Express as a single fraction.

$$= \frac{2(x+1) + 3(x+4)}{12}$$

4 Simplify the numerator by expanding brackets and collecting like terms.

$$\begin{aligned} &= \frac{2x+2+3x+12}{12} \\ &= \frac{5x+14}{12} \end{aligned}$$

Pronumerals in the denominator

- If pronumerals appear in the denominator, the process involved in adding and subtracting the fractions is to find a lowest common denominator as usual.
- When there is an algebraic expression in the denominator of each fraction, we can obtain a common denominator by writing the product of the denominators. For example, if $x+3$ and $2x-5$ are in the denominator of each fraction, then a common denominator of the two fractions will be $(x+3)(2x-5)$.

WORKED EXAMPLE 5

Simplify $\frac{2}{3x} - \frac{1}{4x}$.

THINK

1 Write the expression.

WRITE

$$\frac{2}{3x} - \frac{1}{4x}$$

2 Rewrite each fraction as an equivalent fraction using the LCD of $3x$ and $4x$, which is $12x$.
Note: $12x^2$ is not the lowest LCD.

$$\begin{aligned} &= \frac{2}{3x} \times \frac{4}{4} - \frac{1}{4x} \times \frac{3}{3} \\ &= \frac{8}{12x} - \frac{3}{12x} \end{aligned}$$

3 Express as a single fraction.

$$= \frac{8-3}{12x}$$

4 Simplify the numerator.

$$= \frac{5}{12x}$$

WORKED EXAMPLE 6

Simplify $\frac{x+1}{x+3} + \frac{2x-1}{x+2}$ by writing it first as a single fraction.

THINK

- 1 Write the expression.
- 2 Rewrite each fraction as an equivalent fraction using the LCD of $x+3$ and $x+2$, which is the product $(x+3)(x+2)$.
- 3 Express as a single fraction.
- 4 Simplify the numerator by expanding brackets and collecting like terms.
Note: The denominator is generally kept in factorised form. That is, it is not expanded.

WRITE

$$\begin{aligned}
 & \frac{x+1}{x+3} + \frac{2x-1}{x+2} \\
 &= \frac{(x+1)}{(x+3)} \times \frac{(x+2)}{(x+2)} + \frac{(2x-1)}{(x+2)} \times \frac{(x+3)}{(x+3)} \\
 &= \frac{(x+1)(x+2)}{(x+3)(x+2)} + \frac{(2x-1)(x+3)}{(x+3)(x+2)} \\
 &= \frac{(x+1)(x+2) + (2x-1)(x+3)}{(x+3)(x+2)} \\
 &= \frac{(x^2 + 2x + x + 2) + (2x^2 + 6x - x - 3)}{(x+3)(x+2)} \\
 &= \frac{(x^2 + 3x + 2 + 2x^2 + 5x - 3)}{(x+3)(x+2)} \\
 &= \frac{3x^2 + 8x - 1}{(x+3)(x+2)}
 \end{aligned}$$

WORKED EXAMPLE 7

TI

CASIO

Simplify $\frac{x+2}{x-3} + \frac{x-1}{(x-3)^2}$ by writing it first as a single fraction.

THINK

- 1 Write the expression.
- 2 Rewrite each fraction as an equivalent fraction using the LCD of $x-3$ and $(x-3)^2$, which is $(x-3)^2$.
- 3 Express as a single fraction.
- 4 Simplify the numerator.

WRITE

$$\begin{aligned}
 & \frac{x+2}{x-3} + \frac{x-1}{(x-3)^2} \\
 &= \frac{x+2}{x-3} \times \frac{x-3}{x-3} + \frac{x-1}{(x-3)^2} \\
 &= \frac{(x+2)(x-3)}{(x-3)^2} + \frac{x-1}{(x-3)^2} \\
 &= \frac{x^2 - x - 6}{(x-3)^2} + \frac{x-1}{(x-3)^2} \\
 &= \frac{x^2 - x - 6 + x - 1}{(x-3)^2} \\
 &= \frac{x^2 - 7}{(x-3)^2}
 \end{aligned}$$

assess on

Exercise 2.3 Adding and subtracting algebraic fractions

INDIVIDUAL PATHWAYS

PRACTISE

Questions:
1a-f, 2a-f, 3a-f, 4-6

CONSOLIDATE

Questions:
1d-i, 2a-f, 3a-i, 4-7, 9

MASTER

Questions:
1g-i, 2e-j, 3d-i, 4-10

Individual pathway interactivity int-4567

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REFLECTION

Why can't we just add the numerators and the denominators of fractions; for example, $\frac{a}{b} + \frac{c}{d} = \frac{a+c}{b+d}$?

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Digital docs

SkillsSHEET
Addition and subtraction of fractions
doc-5186
SkillsSHEET
Writing equivalent algebraic fractions with the lowest common denominator
doc-5190

FLUENCY

1 Simplify each of the following.

a $\frac{4}{7} + \frac{2}{3}$

b $\frac{1}{8} + \frac{5}{9}$

c $\frac{3}{5} + \frac{6}{15}$

d $\frac{4}{9} - \frac{3}{11}$

e $\frac{3}{7} - \frac{2}{5}$

f $\frac{1}{5} - \frac{x}{6}$

g $\frac{5x}{9} - \frac{4}{27}$

h $\frac{3}{8} - \frac{2x}{5}$

i $\frac{5}{x} - \frac{2}{3}$

2 **WE4** Simplify the following expressions.

a $\frac{2y}{3} - \frac{y}{4}$

b $\frac{y}{8} - \frac{y}{5}$

c $\frac{4x}{3} - \frac{x}{4}$

d $\frac{8x}{9} + \frac{2x}{3}$

e $\frac{2w}{14} - \frac{w}{28}$

f $\frac{y}{20} - \frac{y}{4}$

g $\frac{12y}{5} + \frac{y}{7}$

h $\frac{10x}{5} + \frac{2x}{15}$

i $\frac{x+1}{5} + \frac{x+3}{2}$

j $\frac{x+2}{4} + \frac{x+6}{3}$

k $\frac{2x-1}{5} - \frac{2x+1}{6}$

l $\frac{3x+1}{2} + \frac{5x+2}{3}$

3 **WE5** Simplify the following.

a $\frac{2}{4x} + \frac{1}{8x}$

b $\frac{3}{4x} - \frac{1}{3x}$

c $\frac{5}{3x} + \frac{1}{7x}$

d $\frac{12}{5x} + \frac{4}{15x}$

e $\frac{1}{6x} + \frac{1}{8x}$

f $\frac{9}{4x} - \frac{9}{5x}$

g $\frac{2}{100x} + \frac{7}{20x}$

h $\frac{1}{10x} + \frac{5}{x}$

i $\frac{4}{3x} - \frac{3}{2x}$

4 **WE6, 7** Simplify the following by writing as single fractions.

a $\frac{2}{x+4} + \frac{3x}{x-2}$

b $\frac{2x}{x+5} + \frac{5}{x-1}$

c $\frac{5}{2x+1} + \frac{x}{x-2}$

d $\frac{2x}{x+1} - \frac{3}{2x-7}$

e $\frac{4x}{x+7} + \frac{3x}{x-5}$

f $\frac{x+2}{x+1} + \frac{x-1}{x+4}$

g $\frac{x+8}{x+1} - \frac{2x+1}{x+2}$

h $\frac{x+5}{x+3} - \frac{x-1}{x-2}$

i $\frac{x+1}{x+2} - \frac{2x-5}{3x-1}$

j $\frac{2}{x-1} - \frac{3}{1-x}$

k $\frac{4}{(x+1)^2} + \frac{3}{x+1}$

l $\frac{3}{x-1} - \frac{1}{(x-1)^2}$

UNDERSTANDING

- 5 A classmate attempted to complete an algebraic fraction subtraction problem.

$$\begin{aligned}
 \frac{x}{x-1} - \frac{3}{x-2} &= \frac{x}{x-1} \times \frac{(x-2)}{(x-2)} - \frac{3}{x-2} \times \frac{(x-1)}{(x-1)} \\
 &= \frac{x(x-2) - 3(x-1)}{(x-1)(x+2)} \\
 &= \frac{x^2 - 2x - 3x - 1}{(x-1)(x+2)} \\
 &= \frac{x^2 - 5x - 1}{(x-1)(x+2)}
 \end{aligned}$$

- a What mistake did she make? b What is the correct answer?

REASONING

Adding and subtracting algebraic fractions can become more complicated if you add a third fraction into the expression.

- 6 Simplify the following.

a $\frac{1}{x+2} + \frac{2}{x+1} + \frac{1}{x+3}$

b $\frac{1}{x-1} + \frac{4}{x+2} + \frac{2}{x-4}$

c $\frac{3}{x+1} + \frac{2}{x+3} - \frac{1}{x+2}$

d $\frac{2}{x-4} - \frac{3}{x-1} + \frac{5}{x+3}$

- 7 Why is the process that involves finding the lowest common denominator important in question 6?
- 8 The reverse process of adding or subtracting algebraic fractions is quite complex. Use trial and error, or technology, to determine the value of a if

$$\frac{7x-4}{(x-8)(x+5)} = \frac{a}{x-8} + \frac{3}{x+5}.$$

PROBLEM SOLVING

9 Simplify $\frac{3}{x^2+7x+12} - \frac{1}{x^2+x-6} + \frac{2}{x^2+2x-8}$.

10 Simplify $\frac{x^2+3x-18}{x^2-x-42} - \frac{x^2-3x+2}{x^2-5x+4}$.



2.4 Multiplying and dividing algebraic fractions

Simplifying algebraic fractions

- Algebraic fractions can be simplified using the index laws and by cancelling factors common to the numerator and denominator.
- A fraction can only be simplified if:
 - there is a common factor in the numerator and the denominator
 - the numerator and denominator are both written in factorised form, that is, as the *product* of two or more factors.

$$\frac{3ab}{12a} = \frac{\overset{1}{3} \times \overset{1}{a} \times b}{\overset{4}{12} \times \overset{1}{a}} \leftarrow \text{product of factors} \quad \frac{3a+b}{12a} = \frac{3 \times a + b}{12 \times a} \leftarrow \text{not a product of factors}$$

$$= \frac{b}{4} \quad \text{Cannot be simplified}$$

Multiplying algebraic fractions

- Multiplication of algebraic fractions follows the same rules as multiplication of numerical fractions: multiply the numerators, then multiply the denominators.

WORKED EXAMPLE 8

Simplify each of the following.

a $\frac{5y}{3x} \times \frac{6z}{7y}$

b $\frac{2x}{(x+1)(2x-3)} \times \frac{x+1}{x}$

THINK

a 1 Write the expression.

2 Cancel common factors in the numerator and denominator. The y can be cancelled in the denominator and the numerator. Also, the 3 in the denominator can divide into the 6 in the numerator.

3 Multiply the numerators, then multiply the denominators.

b 1 Write the expression.

2 Cancel common factors in the numerator and the denominator. $(x+1)$ and the x are both common in the numerator and the denominator and can therefore be cancelled.

3 Multiply the numerators, then multiply the denominators.

WRITE

a $\frac{5y}{3x} \times \frac{6z}{7y}$

$$= \frac{5\cancel{y}^1}{\overset{1}{3}x} \times \frac{\overset{6}{\cancel{6}^2}z}{7\cancel{y}^1}$$

$$= \frac{5}{x} \times \frac{2z}{7}$$

$$= \frac{10z}{7x}$$

b $\frac{2x}{(x+1)(2x-3)} \times \frac{x+1}{x}$

$$= \frac{2\cancel{x}^1}{\overset{1}{(x+1)}(2x-3)} \times \frac{\cancel{x+1}^1}{\cancel{x}^1}$$

$$= \frac{2}{2x-3} \times \frac{1}{1}$$

$$= \frac{2}{2x-3}$$

Dividing algebraic fractions

- When dividing algebraic fractions, follow the same rules as for division of numerical fractions: write the division as a multiplication and invert the second fraction. This process is sometimes known as multiplying by the **reciprocal**.

WORKED EXAMPLE 9

TI

CASIO

Simplify the following expressions.

a $\frac{3xy}{2} \div \frac{4x}{9y}$

b $\frac{4}{(x+1)(3x-5)} \div \frac{x-7}{x+1}$

THINK

a 1 Write the expression.

WRITE

a $\frac{3xy}{2} \div \frac{4x}{9y}$

- 2 Change the division sign to a multiplication sign and write the second fraction as its reciprocal.

$$= \frac{3xy}{2} \times \frac{9y}{4x}$$

- 3 Cancel common factors in the numerator and denominator and cancel. The pronumeral x is common to both the numerator and denominator and can therefore be cancelled.

$$= \frac{3y}{2} \times \frac{9y}{4}$$

- 4 Multiply the numerators, then multiply the denominators.

$$= \frac{27y^2}{8}$$

- b 1 Write the expression.

$$\frac{4}{(x+1)(3x-5)} \div \frac{x-7}{x+1}$$

- 2 Change the division sign to a multiplication sign and write the second fraction as its reciprocal.

$$= \frac{4}{(x+1)(3x-5)} \times \frac{x+1}{x-7}$$

- 3 Cancel common factors in the numerator and denominator and cancel. $(x+1)$ is common to both the numerator and denominator and can therefore be cancelled.

$$= \frac{4}{3x-5} \times \frac{1}{x-7}$$

- 4 Multiply the numerators, then multiply the denominators.

$$= \frac{4}{(3x-5)(x-7)}$$

Exercise 2.4 Multiplying and dividing algebraic fractions

assess on

INDIVIDUAL PATHWAYS

PRACTISE

Questions:
1a-f, 2a-f, 3a-i, 4a-b

CONSOLIDATE

Questions:
1d-i, 2a-f, 3a-i, 4, 5, 7

MASTER

Questions:
1g-l, 2e-j, 3d-l, 4-8

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REFLECTION

How are multiplying and dividing algebraic fractions different to adding and subtracting them?

FLUENCY

- 1 WE8a Simplify each of the following.

a $\frac{x}{5} \times \frac{20}{y}$

b $\frac{x}{4} \times \frac{12}{y}$

c $\frac{y}{4} \times \frac{16}{x}$

d $\frac{x}{2} \times \frac{9}{2y}$

e $\frac{x}{10} \times \frac{-25}{2y}$

f $\frac{3w}{-14} \times \frac{-7}{x}$

g $\frac{3y}{4x} \times \frac{8z}{7y}$

h $\frac{-y}{3x} \times \frac{6z}{-7y}$

i $\frac{x}{3z} \times \frac{-9z}{2y}$

j $\frac{5y}{3x} \times \frac{x}{8y}$

k $\frac{-20y}{7x} \times \frac{-21z}{5y}$

l $\frac{y}{-3w} \times \frac{x}{2y}$

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Multiplication of fractions
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SkillSHEET
Division of fractions
doc-5188
SkillSHEET
Simplification of algebraic fractions
doc-5191

2 WE8b Simplify the following expressions.

a $\frac{2x}{(x-1)(3x-2)} \times \frac{x-1}{x}$

c $\frac{9x}{(5x+1)(x-6)} \times \frac{5x+1}{2x}$

e $\frac{2x}{x+1} \times \frac{x-1}{(x+1)(x-1)}$

g $\frac{2x}{4(a+3)} \times \frac{3a}{15x}$

i $\frac{6x^2}{20(x-2)^2} \times \frac{15(x-2)}{16x^4}$

b $\frac{5x}{(x-3)(4x+7)} \times \frac{4x+7}{x}$

d $\frac{(x+4)}{(x+1)(x+3)} \times \frac{x+1}{x+4}$

f $\frac{2}{x(2x-3)} \times \frac{x(x+1)}{4}$

h $\frac{15c}{12(d-3)} \times \frac{21d}{6c}$

j $\frac{7x^2(x-3)}{5x(x+1)} \times \frac{3(x-3)(x+1)}{14(x-3)^2(x-1)}$

3 WE9a Simplify the following expressions.

a $\frac{3}{x} \div \frac{5}{x}$

b $\frac{2}{x} \div \frac{9}{x}$

c $\frac{4}{x} \div \frac{12}{x}$

d $\frac{20}{y} \div \frac{20}{3y}$

e $\frac{1}{5w} \div \frac{5}{w}$

f $\frac{7}{2x} \div \frac{3}{5x}$

g $\frac{3xy}{7} \div \frac{3x}{4y}$

h $\frac{2xy}{5} \div \frac{5x}{y}$

i $\frac{6y}{9} \div \frac{3x}{4xy}$

j $\frac{8wx}{5} \div \frac{3w}{4y}$

k $\frac{2xy}{5} \div \frac{3xy}{5}$

l $\frac{10xy}{7} \div \frac{20x}{14y}$

4 WE9b Simplify the following expressions.

a $\frac{9}{(x-1)(3x-7)} \div \frac{x+3}{x-1}$

b $\frac{1}{(x+2)(2x-5)} \div \frac{x-9}{2x-5}$

c $\frac{12(x-3)^2}{(x+5)(x-9)} \div \frac{4(x-3)}{7(x-9)}$

d $\frac{13}{6(x-4)^2(x-1)} \div \frac{3(x+1)}{2(x-4)(x-1)}$

REASONING

5 Is $\frac{3}{x+2}$ the same as $\frac{1}{x+2} + \frac{1}{x+2} + \frac{1}{x+2}$? Explain your reasoning.

6 a Simplify $\frac{(x-4)(x+3)}{4x-x^2} \times \frac{x^2-x}{(x+3)(x-1)}$.

b Find and describe the error in the following reasoning.

$$\begin{aligned} & \frac{(x-4)(x+3)}{4x-x^2} \times \frac{x^2-x}{(x+3)(x-1)} \\ &= \frac{(x-4)(x+3)}{x(4-x)} \times \frac{x(x-1)}{(x+3)(x-1)} \\ &= 1 \end{aligned}$$

PROBLEM SOLVING

7 Simplify $\frac{x^2-2x-3}{x^4-1} \times \frac{x^2+4x-5}{x^2-5x+6} \div \frac{x^2+7x+10}{x^4-3x^2-4}$.

8 Simplify $\frac{x+1}{x-\frac{x}{a}}$ where $a = \frac{x-1}{x+1}$.

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CHALLENGE 2.2

Simplify the expression:

$$\left(1 + \frac{\frac{1}{x}}{1 + \frac{1}{x}}\right) \times \left(1 + \frac{\frac{1}{x}}{1 - \frac{1}{x}}\right).$$



2.5 Solving simple equations

- **Equations** show the equivalence of two expressions.
- Equations can be solved using inverse operations.
- When solving equations, the last operation performed on the pronumeral when building the equation is the first operation undone by applying inverse operations to both sides of the equation. For example, the equation $2x + 3 = 5$ is built from x by multiplying x by 2 and then adding 3 to give the result of 5. To solve the equation, undo the adding 3 by subtracting 3, then undo the multiplying by 2 by dividing by 2.

+ and − are inverse operations
 × and ÷ are inverse operations
² and √ are inverse operations

One-step equations

- Equations that require one step to solve are called one-step equations.

WORKED EXAMPLE 10

Solve the following equations.

a $a + 27 = 71$

b $\frac{d}{16} = 3\frac{1}{4}$

c $\sqrt{e} = 0.87$

d $f^2 = \frac{4}{25}$

THINK

- a**
- 1 Write the equation.
 - 2 27 has been added to a resulting in 71. The addition of 27 has to be reversed by subtracting 27 from both sides of the equation to obtain the solution.
- b**
- 1 Write the equation.
 - 2 Express $3\frac{1}{4}$ as an improper fraction.
 - 3 The pronumeral d has been divided by 16 resulting in $\frac{13}{4}$. Therefore the division has to be reversed by multiplying both sides of the equation by 16 to obtain d .

WRITE

a

$$a + 27 = 71$$

$$a + 27 - 27 = 71 - 27$$

$$a = 44$$

b

$$\frac{d}{16} = 3\frac{1}{4}$$

$$\frac{d}{16} = \frac{13}{4}$$

$$\frac{d}{16} \times 16 = \frac{13}{4} \times 16$$

$$d = 52$$



- c** **1** Write the equation.
- 2** The square root of e has been taken to result in 0.87. Therefore, the square root has to be reversed by squaring both sides of the equation to obtain e .
- d** **1** Write the equation.
- 2** The pronumeral f has been squared, resulting in $\frac{4}{25}$. Therefore the squaring has to be reversed by taking the square root of both sides of the equation to obtain f . Note that there are two possible solutions, one positive and one negative, since two negative numbers can also be multiplied together to produce a positive result.

$$\begin{aligned} \sqrt{e} &= 0.87 \\ (\sqrt{e})^2 &= 0.87^2 \\ e &= 0.7569 \end{aligned}$$

$$\begin{aligned} f^2 &= \frac{4}{25} \\ f &= \pm \sqrt{\frac{4}{25}} \\ f &= \pm \frac{2}{5} \end{aligned}$$

Two-step equations

- Two-step equations involve the inverse of two operations in their solutions.

WORKED EXAMPLE 11

TI

CASIO

Solve the following equations.

a $5y - 6 = 79$ **b** $\frac{4x}{9} = 5$

THINK

- a** **1** Write the equation.
- 2** Step 1: Add 6 to both sides of the equation.
- 3** Step 2: Divide both sides of the equation by 5 to obtain y .
- b** **1** Write the equation.
- 2** Step 1: Multiply both sides of the equation by 9.
- 3** Step 2: Divide both sides of the equation by 4 to obtain x .
- 4** Express the improper fraction as a mixed number.

WRITE

$$\begin{aligned} \mathbf{a} \quad 5y - 6 &= 79 \\ 5y - 6 + 6 &= 79 + 6 \\ 5y &= 85 \\ \frac{5y}{5} &= \frac{85}{5} \\ y &= 17 \end{aligned}$$

$$\begin{aligned} \mathbf{b} \quad \frac{4x}{9} &= 5 \\ \frac{4x}{9} \times 9 &= 5 \times 9 \\ 4x &= 45 \\ \frac{4x}{4} &= \frac{45}{4} \\ x &= \frac{45}{4} \\ x &= 11\frac{1}{4} \end{aligned}$$

Equations where the pronumeral appears on both sides

- In solving equations where the pronumeral appears on both sides, subtract the smaller pronumeral term so that it is eliminated from both sides of the equation.

WORKED EXAMPLE 12

Solve the following equations.

a $5h + 13 = 2h - 2$

b $14 - 4d = 27 - d$

c $2(x - 3) = 5(2x + 4)$

THINK

- a**
- 1 Write the equation.
 - 2 Eliminate the pronumeral from the right-hand side by subtracting $2h$ from both sides of the equation.
 - 3 Subtract 13 from both sides of the equation.
 - 4 Divide both sides of the equation by 3 and write your answer.
- b**
- 1 Write the equation.
 - 2 Add $4d$ to both sides of the equation.
 - 3 Subtract 27 from both sides of the equation.
 - 4 Divide both sides of the equation by 3.
 - 5 Express the improper fraction as a mixed number.
 - 6 Write your answer so that d is on the left-hand side.
- c**
- 1 Write the equation.
 - 2 Expand the brackets on both sides of the equation.
 - 3 Subtract $2x$ from both sides of the equation.
 - 4 Subtract 20 from both sides of the equation.
 - 5 Divide both sides of the equation by 8.
 - 6 Simplify and write your answer with the pronumeral on the left-hand side.

WRITE

a $5h + 13 = 2h - 2$

$$3h + 13 = -2$$

$$3h = -15$$

$$h = -5$$

b $14 - 4d = 27 - d$

$$14 = 27 + 3d$$

$$-13 = 3d$$

$$-\frac{13}{3} = d$$

$$-4\frac{1}{3} = d$$

$$d = -4\frac{1}{3}$$

c $2(x - 3) = 5(2x + 4)$

$$2x - 6 = 10x + 20$$

$$2x - 2x - 6 = 10x - 2x + 20$$

$$-6 - 20 = 8x + 20 - 20$$

$$-26 = 8x$$

$$-\frac{26}{8} = x$$

$$x = -\frac{13}{4}$$



Exercise 2.5 Solving simple equations

INDIVIDUAL PATHWAYS

REFLECTION

Describe in one sentence what it means to solve linear equations.

PRACTISE

Questions:

1a-f, 2a-e, 3a-f, 4a-b, 5a-b, 6a-b, 7a-f, 8a-d, 9a-b, 10a-b, 11a-c, 12a-b, 13a-c, 15a-d, 16a-d, 17a-d, 19-21, 25

CONSOLIDATE

Questions:

1d-i, 2d-i, 3a-f, 4, 5a-b, 6a-b, 7d-i, 8c-f, 9c-g, 10a-d, 11c-f, 12, 13d-i, 15c-f, 16c-f, 17c-f, 19-21, 23, 25, 26

MASTER

Questions:

3-6, 7d-i, 8c-f, 9e-i, 10d-f, 11d-f, 12, 14, 15g-i, 16g-i, 17g-i, 18-26

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FLUENCY

1 **WE10a** Solve the following equations.

a $a + 61 = 85$

b $k - 75 = 46$

c $g + 9.3 = 12.2$

d $r - 2.3 = 0.7$

e $h + 0.84 = 1.1$

f $i + 5 = 3$

g $t - 12 = -7$

h $q + \frac{1}{3} = \frac{1}{2}$

i $x - 2 = -2$

2 **WE10b** Solve the following equations.

a $\frac{f}{4} = 3$

b $\frac{i}{10} = -6$

c $6z = -42$

d $9v = 63$

e $6w = -32$

f $\frac{k}{12} = \frac{5}{6}$

g $4a = 1.7$

h $\frac{m}{19} = \frac{7}{8}$

i $\frac{y}{4} = 5\frac{3}{8}$

3 **WE10c, d** Solve the following equations.

a $\sqrt{t} = 10$

b $y^2 = 289$

c $\sqrt{q} = 2.5$

d $f^2 = 1.44$

e $\sqrt{h} = \frac{4}{7}$

f $p^2 = \frac{9}{64}$

g $\sqrt{g} = \frac{15}{22}$

h $j^2 = \frac{196}{961}$

i $a^2 = 2\frac{7}{9}$

4 Solve the following equations.

a $\sqrt{t} - 3 = 2$

b $5x^2 = 180$

c $3\sqrt{m} = 12$

d $-2t^2 = -18$

e $t^2 + 11 = 111$

f $\sqrt{m} + 5 = 0$

5 Solve the following equations.

a $\sqrt[3]{x} = 2$

b $x^3 = -27$

c $\sqrt[3]{m} = \frac{1}{2}$

d $x^3 = \frac{27}{64}$

e $\sqrt[3]{m} = 0.2$

f $w^3 = 15\frac{5}{8}$

6 Solve the following equations.

a $x^3 + 1 = 0$

b $3x^3 = -24$

c $\sqrt[3]{m} + 5 = 6$

d $-2 \times \sqrt[3]{w} = 16$

e $\sqrt[3]{t} - 13 = -8$

f $2x^3 - 14 = 2$

7 **WE11a** Solve the following.

a $5a + 6 = 26$

b $6b + 8 = 44$

c $8i - 9 = 15$

d $7f - 18 = 45$

e $8q + 17 = 26$

f $10r - 21 = 33$

g $6s + 46 = 75$

h $5t - 28 = 21$

i $8a + 88 = 28$

8 Solve the following.

a $\frac{f}{4} + 6 = 16$

b $\frac{g}{6} + 4 = 9$

c $\frac{r}{10} + 6 = 5$

d $\frac{m}{9} - 12 = -10$

e $\frac{n}{8} + 5 = 8.5$

f $\frac{p}{12} - 1.8 = 3.4$

9 Solve the following.

a $6(x + 8) = 56$

b $7(y - 4) = 35$

c $5(m - 3) = 7$

d $3(2k + 5) = 24$

e $5(3n - 1) = 80$

f $6(2c + 7) = 58$

g $2(x - 5) + 3(x - 7) = 19$

h $3(x + 5) - 5(x - 1) = 12$

i $3(2x - 7) - (x + 3) = -60$

10 **WE11b** Solve the following.

a $\frac{3k}{5} = 15$

b $\frac{9m}{8} = 18$

c $\frac{7p}{10} = -8$

d $\frac{8u}{11} = -3$

e $\frac{11x}{4} = 2$

f $\frac{4v}{15} = 0.8$

11 Solve the following.

a $\frac{x - 5}{3} = 7$

b $\frac{2m + 1}{3} = -3$

c $\frac{3w - 1}{4} = 6$

d $\frac{t - 5}{2} = 0$

e $\frac{6 - x}{3} = -1$

f $\frac{3n - 5}{4} = -6$

12 **MC** a The solution to the equation $\frac{p}{5} + 2 = 7$ is:

A $p = 5$

B $p = 25$

C $p = 45$

D $p = 10$

E $p = 1$

b If $5h + 8 = 53$, then h is equal to:

A $\frac{1}{5}$

B 12.2

C 225

D 10

E 9

c The exact solution to the equation $14x = 75$ is:

A $x = 5.357\ 142\ 857$

B $x = 5.357$ (to 3 decimal places)

C $x = 5\frac{5}{14}$

D $x = 5.4$

E $x = 5.5$

13 Solve the following equations.

a $-x = 5$

b $2 - d = 3$

c $5 - p = -2$

d $-7 - x = 4$

e $-5h = 10$

f $-6t = -30$

g $-\frac{v}{5} = 4$

h $-\frac{r}{12} = \frac{1}{4}$

i $-4g = 3.2$

14 Solve the following equations.

a $6 - 2x = 8$

b $10 - 3v = 7$

c $9 - 6l = -3$

d $-3 - 2g = 1$

e $-5 - 4t = -17$

f $-\frac{3e}{5} = 14$

g $-\frac{8j}{3} = 9$

h $-\frac{k}{4} - 3 = 6$

i $-\frac{4f}{7} + 1 = 8$

15 WE12a Solve the following equations.

a $6x + 5 = 5x + 7$

b $7b + 9 = 6b + 14$

c $11w + 17 = 6w + 27$

d $8f - 2 = 7f + 5$

e $10t - 11 = 5t + 4$

f $12r - 16 = 3r + 5$

g $12g - 19 = 3g - 31$

h $7h + 5 = 2h - 6$

i $5a - 2 = 3a - 2$

16 WE12b Solve the following equations.

a $5 - 2x = 6 - x$

b $10 - 3c = 8 - 2c$

c $3r + 13 = 9r - 3$

d $k - 5 = 2k - 6$

e $5y + 8 = 13y + 17$

f $17 - 3g = 3 - g$

g $14 - 5w = w + 8$

h $4m + 7 = 8 - m$

i $14 - 5p = 9 - 2p$

17 WE12c Solve the following equations.

a $3(x + 5) = 2x$

b $8(y + 3) = 3y$

c $6(t - 5) = 4(t + 3)$

d $10(u + 1) = 3(u - 3)$

e $12(f - 10) = 4(f - 5)$

f $2(4r + 3) = 3(2r + 7)$

g $5(2d + 9) = 3(3d + 13)$

h $5(h - 3) = 3(2h - 1)$

i $2(4x + 1) = 5(3 - x)$

18 MC a The solution to $8 - 4k = -2$ is:

A $k = 2\frac{1}{2}$

B $k = -2\frac{1}{2}$

C $k = 1\frac{1}{2}$

D $k = -1\frac{1}{2}$

E $k = \frac{2}{5}$

b The solution to $-\frac{6n}{5} + 3 = -7$ is:

A $n = 3\frac{1}{3}$

B $n = -3\frac{1}{3}$

C $n = \frac{1}{3}$

D $n = 8\frac{1}{3}$

E $n = -8\frac{1}{3}$

c The solution to $p - 6 = 8 - 4p$ is:

A $p = \frac{2}{5}$

B $p = 2\frac{4}{5}$

C $p = 4\frac{2}{3}$

D $p = \frac{2}{3}$

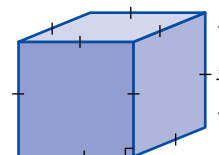
E $p = \frac{4}{5}$

UNDERSTANDING

19 If the side length of a cube is x cm, then its volume V is given by $V = x^3$. What is the side length (correct to the nearest cm) of a cube that has a volume of:

a 216 cm^3

b 2 m^3



20 The surface area of a cube with side length x cm is given by $A = 6x^2$.

Find the side length (correct to the nearest cm) of a cube that has a surface area of:

a 37.5 cm^2

b 1 m^2 .

21 A pebble is dropped down a well. In time t seconds it falls a distance of d metres, given by $d = 5t^2$.

a How far does the pebble fall in 1 s?

b How many seconds will it take the pebble to fall 40 m? (Answer correct to 1 decimal place.)

22 The surface area of a sphere is given by the formula $A = 4\pi r^2$, where r is the radius of the sphere.

a Find the surface area of a sphere that has a radius of 5 cm.

b What is the radius of a sphere that has a surface area equal to 500 cm^2 ? (Answer correct to the nearest mm.)

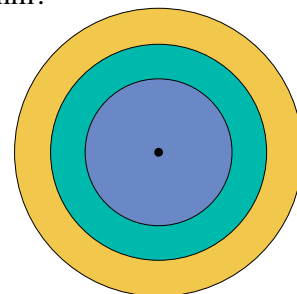


REASONING

- 23** Find the radius of a circle of area 10 cm^2 .
- 24** The volume of a sphere is given by the formula $V = \frac{4}{3}\pi r^3$, where r is the radius of the sphere. If the sphere can hold 1 litre of water, what is its radius correct to the nearest mm?

PROBLEM SOLVING

- 25** The width of a room is three-fifths of its length. When the width is increased by 2 metres and the length is decreased by 2 metres, the resultant shape is a square. Find the dimensions of the room.
- 26** A target board for a dart game has been designed as three concentric circles where each coloured region is the same area. If the radius of the purple circle is $r \text{ cm}$ and the radius of the outer circle is 10 cm , find the value of r .



2.6 Solving multi-step equations

Equations with multiple brackets

Many equations need to be simplified by expanding brackets and collecting like terms before they are solved. Doing this reduces the equation to one of the basic types covered in the previous exercise.

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WORKED EXAMPLE 13

TI

CASIO

Solve each of the following linear equations.

a $6(x + 1) - 4(x - 2) = 0$

b $7(5 - x) = 3(x + 1) - 10$

THINK

- a**
- 1 Write the equation.
 - 2 Expand all the brackets. (Be careful with the -4 .)
 - 3 Collect like terms.
 - 4 Subtract 14 from both sides of the equation.
 - 5 Divide both sides of the equation by 2 to find the value of x .
- b**
- 1 Write the equation.
 - 2 Expand all the brackets.
 - 3 Collect like terms.
 - 4 Create a single pronumeral term by adding $7x$ to both sides of the equation.
 - 5 Add 7 to both sides of the equation.
 - 6 Divide both sides of the equation by 10 to solve for x and simplify.
 - 7 Express the improper fraction as a mixed number fraction.
 - 8 Rewrite the equation so that x is on the left-hand side.

WRITE

a $6(x + 1) - 4(x - 2) = 0$

$$6x + 6 - 4x + 8 = 0$$

$$2x + 14 = 0$$

$$2x = -14$$

$$x = -7$$

b $7(5 - x) = 3(x + 1) - 10$

$$35 - 7x = 3x + 3 - 10$$

$$35 - 7x = 3x - 7$$

$$35 = 10x - 7$$

$$42 = 10x$$

$$\frac{42}{10} = x$$

$$\frac{21}{5} = x$$

$$4\frac{1}{5} = x$$

$$x = 4\frac{1}{5}$$

Equations involving algebraic fractions

- To solve an equation containing algebraic fractions, multiply both sides of the equation by the lowest common multiple (LCM) of the denominators. This gives an equivalent form of the equation without fractions.

WORKED EXAMPLE 14

Solve the equation $\frac{x-5}{3} = \frac{x+7}{4}$ and verify the solution.

THINK

- Write the equation.
- The LCM is $3 \times 4 = 12$.
Multiply both sides of the equation by 12.
- Simplify the fractions.
- Expand the brackets.
- Subtract $3x$ from both sides of the equation.
- Add 20 to both sides of the equation and write the answer.
- To verify, check that the answer $x = 41$ is true for both the left-hand side (LHS) and the right-hand side (RHS) of the equation by substitution.

Substitute $x = 41$ into the LHS.

Substitute $x = 41$ into the RHS.

WRITE

$$\frac{x-5}{3} = \frac{x+7}{4}$$

$$\frac{4\cancel{12}(x-5)}{\cancel{1}2} = \frac{3\cancel{12}(x+7)}{\cancel{1}4}$$

$$4(x-5) = 3(x+7)$$

$$4x - 20 = 3x + 21$$

$$x - 20 = 21$$

$$x = 41$$

$$\begin{aligned} \text{LHS} &= \frac{41-5}{3} \\ &= \frac{36}{3} \\ &= 12 \end{aligned}$$

$$\begin{aligned} \text{RHS} &= \frac{41+7}{4} \\ &= \frac{48}{4} \\ &= 12 \end{aligned}$$

- Write your answer.

Because the LHS = RHS, the solution $x = 41$ is correct.

WORKED EXAMPLE 15

TI

CASIO

Solve each of the following equations.

$$\text{a } \frac{5(x+3)}{6} = 4 + \frac{3(x-1)}{5}$$

$$\text{b } \frac{4}{3(x-1)} = \frac{1}{x+1}$$

THINK

- 1** Write the equation.
- 2** The lowest common denominator of 5 and 6 is 30. Write each term as an equivalent fraction with a denominator of 30.
- 3** Multiply each term by 30. This effectively removes the denominator.
- 4** Expand the brackets and collect like terms.
- 5** Subtract $18x$ from both sides of the equation.
- 6** Subtract 75 from both sides of the equation.
- 7** Divide both sides of the equation by 7 to solve for x .
- 8** Express the improper fraction as a mixed number.

- 1** Write the equation.
- 2** The lowest common denominator of 3, $x+1$ and $x-1$ is $3(x-1)(x+1)$. Write each term as an equivalent fraction with a common denominator of $3(x-1)(x+1)$.
- 3** Multiply each term by the common denominator.
- 4** Expand the brackets.
- 5** Subtract $3x$ from both sides of the equation.
- 6** Subtract 4 from both sides of the equation to solve for x .

WRITE

$$\text{a } \frac{5(x+3)}{6} = 4 + \frac{3(x-1)}{5}$$

$$\frac{25(x+3)}{30} = \frac{120}{30} + \frac{18(x-1)}{30}$$

$$25(x+3) = 120 + 18(x-1)$$

$$25x + 75 = 120 + 18x - 18$$

$$25x + 75 = 102 + 18x$$

$$7x + 75 = 102$$

$$7x = 27$$

$$x = \frac{27}{7}$$

$$x = 3\frac{6}{7}$$

$$\text{b } \frac{4}{3(x-1)} = \frac{1}{x+1}$$

$$\frac{4(x+1)}{3(x-1)(x+1)} = \frac{3(x-1)}{3(x-1)(x+1)}$$

$$4(x+1) = 3(x-1)$$

$$4x + 4 = 3x - 3$$

$$x + 4 = -3$$

$$x + 4 - 4 = -3 - 4$$

$$= -7$$



Exercise 2.6 Solving multi-step equations

INDIVIDUAL PATHWAYS

REFLECTION

Do the rules for the order of operations apply to algebraic fractions? Explain.

PRACTISE

Questions:

1, 2a-c, 3a-e, 4a-d, 5, 6, 9

CONSOLIDATE

Questions:

1, 2a-f, 3b-i, 4c-g, 6, 7, 9, 11, 12

MASTER

Questions:

1, 2c-h, 3g-l, 4g-l, 7-13

Individual pathway interactivity int-4570

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FLUENCY

1 **WE13** Solve each of the following linear equations.

a $6(4x - 3) + 7(x + 1) = 9$

b $9(3 - 2x) + 2(5x + 1) = 0$

c $8(5 - 3x) - 4(2 + 3x) = 3$

d $9(1 + x) - 8(x + 2) = 2x$

e $6(4 + 3x) = 7(x - 1) + 1$

f $10(4x + 2) = 3(8 - x) + 6$

2 **WE14** Solve each of the following equations and verify the solutions.

a $\frac{x+1}{2} = \frac{x+3}{3}$

b $\frac{x-7}{5} = \frac{x-8}{4}$

c $\frac{x-6}{4} = \frac{x-2}{2}$

d $\frac{8x+3}{5} = 2x$

e $\frac{2x-1}{5} = \frac{x-3}{4}$

f $\frac{4x+1}{3} = \frac{x+2}{4}$

g $\frac{6-x}{3} = \frac{2x-1}{5}$

h $\frac{8-x}{9} = \frac{2x+1}{3}$

3 Solve each of the following linear equations.

a $\frac{x}{3} + \frac{4x}{5} = \frac{1}{3}$

b $\frac{x}{4} - \frac{x}{5} = \frac{3}{4}$

c $\frac{x}{4} - \frac{4x}{7} = 2$

d $\frac{-3x}{5} + \frac{x}{8} = \frac{1}{4}$

e $\frac{2x}{3} - \frac{x}{6} = -\frac{3}{4}$

f $\frac{5x}{8} - 8 = \frac{2x}{3}$

g $\frac{2}{7} - \frac{x}{8} = \frac{3x}{8}$

h $\frac{4}{x} - \frac{1}{6} = \frac{2}{x}$

i $\frac{15}{x} - 4 = \frac{2}{x}$

j $\frac{1}{3} + \frac{4}{x} = \frac{5}{x}$

k $\frac{2x-4}{5} + 6 = \frac{x}{2}$

l $\frac{4x-1}{2} - \frac{2x+5}{3} = 0$

4 **WE15** Solve each of the following linear equations.

a $\frac{3(x+1)}{2} + \frac{5(x+1)}{3} = 4$

b $\frac{2(x+1)}{7} + \frac{3(2x-5)}{8} = 0$

c $\frac{2(4x+3)}{5} - \frac{6(x-2)}{2} = \frac{1}{2}$

d $\frac{8(x+3)}{5} = \frac{3(x+2)}{4}$

e $\frac{5(7-x)}{2} = \frac{2(2x-1)}{7} + 1$

f $\frac{2(6-x)}{3} = \frac{9(x+5)}{6} + \frac{1}{3}$

g $\frac{-5(x-2)}{3} - \frac{6(2x-1)}{5} = \frac{1}{3}$

h $\frac{9(2x-1)}{7} = \frac{4(x-5)}{3}$

i $\frac{1}{x-1} + \frac{3}{x+1} = \frac{8}{x+1}$

j $\frac{3}{x+1} + \frac{5}{x-4} = \frac{5}{x+1}$

k $\frac{1}{x-1} - \frac{3}{x} = \frac{-1}{x-1}$

l $\frac{4}{2x-1} - \frac{5}{x} = \frac{-1}{x}$

eBookplus

Digital doc
WorkSHEET 2.2
doc-13848

UNDERSTANDING

- 5 Last week Maya broke into her money box. She spent one-quarter of the money on a birthday present for her brother and one-third of the money on an evening out with her friends, leaving her with \$75. How much money was in her money box?
- 6 At work Keith spends one-fifth of his time in planning and buying merchandise. He spends seven-twelfths of his time in customer service and one-twentieth of his time training the staff. This leaves him ten hours to deal with the accounts. How many hours does he work each week?
- 7 Last week's school fete was a great success, raising a good deal of money. Three-eighths of the profit came from sales of food and drink, and the market stalls recorded one-fifth of the total. A third of the profit came from the major raffle, and the jumping castle raised \$1100. How much money was raised at the fete?
- 8 Lucy had half as much money as Mel, but since Grandma gave them each \$20 she now has three-fifths as much. How much money does Lucy have?



REASONING

- 9 a Which numbers smaller than 100 have exactly 3 factors (including 1 and the number itself)?
 b Which two numbers smaller than 100 have exactly 5 factors?
 c Which number smaller than 100 has exactly 7 factors?
- 10 To raise money for a charity, a Year 10 class has decided to organise a school lunch. Tickets will cost \$6 each. The students have negotiated a special deal for delivery of drinks and pizzas, and they have budgeted \$200 for drinks and \$250 for pizzas. If they raise \$1000 or more, they qualify for a special award.
 a Write an equation to represent the minimum number of tickets required to be sold to qualify for the award.
 b Solve the equation to find the number of tickets they must sell to qualify for the award. Explain your answer.
- 11 If $\frac{x+7}{(x+2)(x+3)} \equiv \frac{a}{x+2} - \frac{4}{x+3}$, explain why a must be equal to 5.
 (Note: ' $\equiv$ ' means identically equal to.)



PROBLEM SOLVING

- 12 Solve for x :

$$\frac{2}{9}(x-1) - \frac{5}{8}(x-2) = \frac{2}{5}(x-4) - \frac{7}{12}.$$
- 13 If $\frac{2(4x+3)}{(x-3)(x+7)} \equiv \frac{a}{x-3} + \frac{b}{x+7}$, find the values of a and b .

2.7 Literal equations

- **Literal equations** are equations that include several pronumerals or variables. Solving literal equations involves changing the subject of the equation to a particular pronumeral.
- A formula is a literal equation that records an interesting or important real-life relationship.

WORKED EXAMPLE 16

Solve the following literal equations for x .

a $ax^2 + bd = c$

b $ax = cx + b$

THINK

- a**
- 1 Write the equation.
 - 2 Subtract bd from both sides of the equation.
 - 3 Divide both sides by a .
 - 4 To solve for x , take the square root of both sides. This gives both a positive and negative result for x .
- b**
- 1 Write the equation.
 - 2 Subtract cx from both sides.
 - 3 Factorise by taking x as a common factor.
 - 4 To solve for x , divide both sides by $a - c$.

WRITE

a $ax^2 + bd = c$

$$ax^2 = c - bd$$

$$x^2 = \frac{c - bd}{a}$$

$$x = \pm \sqrt{\frac{c - bd}{a}}$$

b $ax = cx + b$

$$ax - cx = b$$

$$x(a - c) = b$$

$$x = \frac{b}{a - c}$$

WORKED EXAMPLE 17

Make b the subject of the formula $D = \sqrt{b^2 - 4ac}$.

THINK

- 1 Write the formula.
- 2 Square both sides.
- 3 Add $4ac$ to both sides of the equation.
- 4 Take the square root of both sides.
- 5 Make b the subject of the formula by solving for b .

WRITE

$$D = \sqrt{b^2 - 4ac}$$

$$D^2 = b^2 - 4ac$$

$$D^2 + 4ac = b^2$$

$$\pm \sqrt{D^2 + 4ac} = b$$

$$b = \pm \sqrt{D^2 + 4ac}$$

Restrictions on variables

- Some variables may have implicit restrictions on the values that they may be assigned in an equation or formula. For example:
 - if $V = \frac{d}{t}$, then t cannot equal zero, otherwise the value of V would be undefined.
 - if $d = \sqrt{x - 9}$, then:
 - the value of d will be restricted to positive values or 0
 - the value of $x - 9$ must be greater than or equal to zero because the square root of a negative number cannot be found.

$$x - 9 \geq 0$$

$x \geq 9$ (Hence x must be greater than or equal to 9.)

- Other restrictions may arise once a formula is rearranged. For example, if we look at the formula $V = ls^2$, there are no restrictions on the values that the variables l and s can be assigned. (However, the sign of V must always be the same as the sign of l because s^2 is always positive.) If the formula is transposed to make s the subject, then:

$$\begin{aligned} V &= ls^2 \\ \frac{V}{l} &= s^2 \\ \text{or } s &= \pm \sqrt{\frac{V}{l}} \end{aligned}$$



This shows the restrictions that $l \neq 0$ and $\frac{V}{l} \geq 0$.

If the formula $V = ls^2$ represents the volume of the rectangular prism shown, additional restrictions become evident: the variables l and s represent a length and must be positive

numbers. Hence, when we make s the subject we get $s = \sqrt{\frac{V}{l}}$.

Exercise 2.7 Literal equations

assessment

INDIVIDUAL PATHWAYS

PRACTISE

Questions:

1a-f, 2a-d, 3a-d, 4a-d, 5, 7

CONSOLIDATE

Questions:

1a-g, 2a-f, 3a-f, 4a-f, 5, 7, 8

MASTER

Questions:

1d-i, 2c-j, 3c-h, 4-8

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REFLECTION

Why is it important to consider restrictions on variables when solving literal equations?

FLUENCY

- 1 **WE16** Solve the following literal equations for x .

a $\frac{ax}{bc} = d$

b $\frac{x}{a} - bc = d$

c $\sqrt{x} + n = m$

d $acx^2 = w$

e $\frac{a}{x} = \frac{b}{y}$

f $\frac{x+m}{n} = w$

g $ab(x+b) = c$

h $\frac{a}{x} = \frac{b}{c} + m$

i $mx = ay - bx$

j $\frac{x}{m} + a = \frac{c}{d}$

- 2 **WE17** Rearrange each of the following literal equations to make the variable in brackets the subject.

a $V = lbh$

[l]

b $P = 2l + 2b$

[b]

c $A = \frac{1}{2}bh$

[h]

d $c = \sqrt{a^2 + b^2}$

[a]

e $F = \frac{9C}{5} + 32$

[C]

f $A = \pi r^2$

[r]

g $v = u + at$

[a]

h $I = \frac{PRN}{100}$

[N]

i $E = \frac{1}{2}mv^2$

[m]

j $E = \frac{1}{2}mv^2$

[v]

k $v^2 = u^2 + 2as$

[a]

l $v^2 = u^2 + 2as$

[u]

m $\frac{1}{x} = \frac{1}{a} + \frac{1}{b}$

[a]

n $x = \frac{nx_1 + mx_2}{m + n}$

[x_1]

- 3 Complete the following.

a If $c = \sqrt{a^2 + b^2}$, calculate a if $c = 13$ and $b = 5$.

b If $A = \frac{1}{2}bh$, find the value of h if $A = 56$ and $b = 16$.

- c** If $F = \frac{9C}{5} + 32$, find the value of C if $F = 86$.
- d** If $v = u + at$, find the value of a if $v = 83.6$, $u = 15$ and $t = 7$.
- e** If $V = ls^2$, find the value of s if $V = 2028$ and $l = 12$.
- f** If $v^2 = u^2 + 2as$, find the value of u if $v = 16$, $a = 10$ and $s = 6.75$.
- g** If $A = \frac{1}{2}h(a + b)$, find the value of a if $A = 360$, $b = 15$ and $h = 18$.
- h** If $x = \frac{nx_1 + mx_2}{m + n}$, find the value of x_2 if $x = 10$, $m = 2$, $n = 1$ and $x_1 = 4$.

UNDERSTANDING

4 For the following equations:

- list any restrictions on the variables in the equation.
- rearrange the equation to make the variable in brackets the subject.
- list any new restrictions on the variables in the equation formed in part **ii**.

- | | | | |
|---|-------|--------------------------------------|-------|
| a $y = x^2 + 4$ | $[x]$ | b $y = \frac{2}{x - 3}$ | $[x]$ |
| c $v = u + at$ | $[t]$ | d $c = \sqrt{a^2 + b^2}$ | $[b]$ |
| e $s = \frac{a}{1 - r}$ | $[r]$ | f $m = \frac{pb + qa}{p + q}$ | $[b]$ |
| g $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ | $[c]$ | h $m = \frac{pb + qa}{p + q}$ | $[p]$ |

REASONING

5 The volume of a cylinder is given by $V = \pi r^2 h$, where r is the radius and h is the height of the cylinder.

- State any restrictions on the values of the variables in this formula.
- Make r the subject of the formula.
- List any new restrictions on the variables in the formula.

6 T is the period of a pendulum whose length is l and g is the acceleration due to gravity. The formula relating these variables is $T = 2\pi\sqrt{\frac{l}{g}}$.

- What restrictions are applied to the variables T and l ?
- Make l the subject of the equation.
- Do the restrictions stated in part **a** still apply?
- Find the length of a pendulum that has a period of 3 seconds, given that $g = 9.8 \text{ m/s}^2$. Give your answer correct to 1 decimal place.



PROBLEM SOLVING

7 $F = 32 + \frac{9}{5}C$ is the formula relating degrees Celsius (C) to degrees Fahrenheit (F).

- Transform the equation to make C the subject.
- Find the temperature when degrees Celsius is equal to degrees Fahrenheit.

8 Jing Jing and Pieter live on the same main road but Jing Jing lives a kilometres to the east of Pieter. Both Jing Jing and Pieter set off on their bicycles at exactly the same time and both ride in a westerly direction. Jing Jing rides at j kilometres per hour and Pieter rides at p kilometres per hour. It is known that $j > p$. Find an equation in terms of a , j and p for the distance Jing Jing has ridden in order to catch up with Pieter.

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2.8 Review

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The Maths Quest Review is available in a customisable format for students to demonstrate their knowledge of this topic.

The Review contains:

- **Fluency** questions — allowing students to demonstrate the skills they have developed to efficiently answer questions using the most appropriate methods
- **Problem Solving** questions — allowing students to demonstrate their ability to make smart choices, to model and investigate problems, and to communicate solutions effectively.

A summary of the key points covered and a concept map summary of this topic are available as digital documents.

Review questions

Download the Review questions document from the links found in your eBookPLUS.

eBookplus

Interactivities

Word search
int-2829



Crossword
int-2830



Sudoku
int-3589



Language

It is important to learn and be able to use correct mathematical language in order to communicate effectively. Create a summary of the topic using the key terms below. You can present your summary in writing or using a concept map, a poster or technology.

algebra
denominator
equation
evaluate
expression
factor

formula
literal
multiple
numerator
pronumeral
restriction

simplify
substitution
undefined
variable

Link to assessON for questions to test your readiness **FOR** learning, your progress **AS** you learn and your levels **OF** achievement.

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The story of mathematics

is an exclusive Jacaranda video series that explores the history of mathematics and how it helped shape the world we live in today.

Al-Khwarizmi (eles-1841) tells the story of Muhammad Al-Khwarizmi, a Persian mathematician of the 9th century who wrote a famous mathematical text known as the *Compendious Book* and subsequently became known as the father of algebra.



RICH TASK

Checking for data entry errors



When entering numbers into an electronic device, or even writing numbers down, errors frequently occur. A common type of error is a transposition error, which occurs when two digits are written in the reverse order. Take the number 2869, for example. With this type of error, it could be written as 8269, 2689 or 2896. A common rule for checking these errors is as follows.

If the difference between the correct number and the recorded number is a multiple of 9, a transposition error has occurred.

We can use algebraic expressions to check this rule. Let the digit in the thousands position be represented by a , the digit in the hundreds position by b , the digit in the tens position by c and the digit in the ones position by d . So the real number can be represented as $1000a + 100b + 10c + d$.

- 1 If the digits in the ones position and the tens position were written in the reverse order, the number would be $1000a + 100b + 10d + c$. The difference between the correct number and the incorrect one would then be: $1000a + 100b + 10c + d - (1000a + 100b + 10d + c)$.
 - a Simplify this expression.
 - b Is the expression a multiple of 9? Explain.
- 2 If a transposition error had occurred in the tens and hundreds position, the incorrect number would be $1000a + 100c + 10b + d$. Perform the procedure shown in question 1 to determine whether the difference between the correct number and the incorrect one is a multiple of 9.
- 3 Consider, lastly, a transposition error in the thousands and hundreds positions. Is the difference between the two numbers a multiple of 9?
- 4 Comment on the checking rule for transposition errors.



CODE PUZZLE

In which country was the first practical ice-making machine and refrigerator produced in 1856?



Match each statement on the left with the equivalent statement on the right. The numbers and letters can be used to solve the code below.

1 Simplify	$\frac{3}{5x} - \frac{5}{3x}$	E	5
2 Simplify	$\frac{3}{x-1} + \frac{1}{x-2}$	B	$6x$
3 Simplify	$\frac{2}{x-2} + \frac{4}{x-1}$	U	6
4 Simplify	$\frac{24}{5x^2} \div \frac{4}{5x^3}$	W	$\frac{15(x-2)}{x-1}$
5 Simplify	$\frac{15}{3(x-1)} \div \frac{3}{9(x-2)}$	T	$4\frac{2}{3}$
6 Simplify	$\frac{2x}{(x+3)(x-1)} \div \frac{1}{(x+3)}$	K	$\frac{(4x-7)}{(x-1)(x-2)}$
7 Simplify	$\frac{3(2x-1)}{4} \times \frac{8}{2x-1}$	R	12
8 Solve	$\frac{x}{5} - \frac{3}{7} = \frac{2x}{7}$	C	$\frac{2x}{x-1}$
9 Solve	$\frac{x}{3} - \frac{2x}{5} = \frac{-1}{3}$	A	2
10 Solve	$\frac{5}{3x} = 1 - \frac{3}{x}$	I	$\frac{6x-10}{(x-1)(x-2)}$
11 Solve	$\frac{1}{x-1} - \frac{3}{x} = \frac{-1}{x-1}$	L	3
12 Solve	$\frac{6}{x} - \frac{1}{4} = \frac{3}{x}$	D	$\frac{-16}{15x}$
13 Solve	$-\frac{1}{21} = \frac{x}{7} - \frac{x}{6}$	S	-5

12	9	13	1					
4	13	6	2	5	13	12	1	8
13	3	11	13	12	10	8	7	13

Activities

2.1 Overview

Video

- The story of mathematics (eles-1841)

2.2 Substitution

Digital docs

- SkillSHEET (doc-5183): Like terms
- SkillSHEET (doc-5184): Collecting like terms
- SkillSHEET (doc-5185): Finding the highest common factor
- SkillSHEET (doc-5189): Order of operations

Interactivity

- IP interactivity 2.2 (int-4566): Substitution

2.3 Adding and subtracting algebraic fractions

Digital docs

- SkillSHEET (doc-5186): Addition and subtraction of fractions
- SkillSHEET (doc-5190): Writing equivalent algebraic fractions with the lowest common denominator

Interactivity

- IP interactivity 2.3 (int-4567): Adding and subtracting algebraic fractions

2.4 Multiplying and dividing algebraic fractions

Digital docs

- SkillSHEET (doc-5187): Multiplication of fractions
- SkillSHEET (doc-5188): Division of fractions
- SkillSHEET (doc-5191): Simplification of algebraic fractions
- WorkSHEET 2.1 (doc-13847): Algebraic fractions

Interactivity

- IP interactivity 2.4 (int-4568): Multiplying and dividing algebraic fractions

2.5 Solving simple equations

Interactivity

- IP interactivity 2.5 (int-4569): Solving simple equations

2.6 Solving multi-step equations

Interactivities

- Solving equations (int-2778)
- IP interactivity 2.6 (int-4570): Solving multi-step equations

Digital doc

- WorkSHEET 2.2 (doc-13848): Solving equations with fractions

2.7 Literal equations

Interactivity

- IP interactivity 2.7 (int-4571): Literal equations

2.8 Review

Interactivities

- Word search (int-2829)
- Crossword (int-2830)
- Sudoku (int-3589)

Digital docs

- Topic summary (doc-13803)
- Concept map (doc-13706)

To access eBookPLUS activities, log on to



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Answers

TOPIC 2 Algebra and equations

Exercise 2.2 — Substitution

- 1 a 5 d 6 g 30 j 27
2 a -11 d 30 g -125
3 a $\frac{7}{12}$ d $1\frac{1}{3}$
4 a 9 d 18
5 a 9 d 18
6 a 17 d 4 g 46 j 624.6
7 a D b C c B
8 3.9 cm
9 65.45 cm³
10 361 m
11 a -1 — in this case, addition is closed on integers.
b -1 — in this case, subtraction is closed on integers.
c 2 — in this case, multiplication is closed on integers.
d -1 — in this case, division is closed on integers.
e -2 — in this case, subtraction is closed on integers.
f $-\frac{1}{2}$ — in this case, division is not closed on integers.
12 a 10 — in this case, addition is closed on natural numbers.
b -4 — in this case, subtraction is not closed on natural numbers.
c 12 — in this case, multiplication is closed on natural numbers.
d $\frac{4}{3}$ — in this case, division is not closed on natural numbers.
e -2 — in this case, subtraction is not closed on natural numbers.
f 4 — in this case, division is closed on natural numbers.
13 a $(a + 2b) + 4c = a + (2b + 4c)$
b $(x \times 3y) \times 5c = x \times (3y \times 5c)$
c $2p \div q \neq q \div 2p$
d $5d + q = q + 5d$
e $3z + 0 = 0 + 3z = 3z$
f $2x \times \frac{1}{2x} = \frac{1}{2x} \times 2x = 1$
g $(4x \div 3y) \div 5z \neq 4x \div (3y \div 5z)$
h $3d - 4y \neq 4y - 3d$
14 a $s = 71.875$ metres b $t = 4$ seconds
15 $m = \frac{4}{5}$

Challenge 2.1

$$3a^3c^2$$

Exercise 2.3 — Adding and subtracting algebraic fractions

- 1 a $\frac{26}{21}$ or $1\frac{5}{21}$ b $\frac{49}{72}$ c 1
d $\frac{17}{99}$ e $\frac{1}{35}$ f $\frac{6-5x}{30}$
g $\frac{15x-4}{27}$ h $\frac{15-16x}{40}$ i $\frac{15-2x}{3x}$

- 2 a $\frac{5y}{12}$ b $-\frac{3y}{40}$ c $\frac{13x}{12}$
d $\frac{14x}{9}$ e $\frac{3w}{28}$ f $-\frac{y}{5}$
g $\frac{89y}{35}$ h $\frac{32x}{15}$ i $\frac{7x+17}{10}$
j $\frac{7x+30}{12}$ k $\frac{2x-11}{30}$ l $\frac{19x+7}{6}$
3 a $\frac{5}{8x}$ b $\frac{5}{12x}$ c $\frac{38}{21x}$
d $\frac{8}{3x}$ e $\frac{7}{24x}$ f $\frac{9}{20x}$
g $\frac{37}{100x}$ h $\frac{51}{10x}$ i $-\frac{1}{6x}$
4 a $\frac{3x^2+14x-4}{(x+4)(x-2)}$ b $\frac{2x^2+3x+25}{(x+5)(x-1)}$ c $\frac{2x^2+6x-10}{(2x+1)(x-2)}$
d $\frac{4x^2-17x-3}{(x+1)(2x-7)}$ e $\frac{7x^2+x}{(x+7)(x-5)}$ f $\frac{2x^2+6x+7}{(x+1)(x+4)}$
g $\frac{-x^2+7x+15}{(x+1)(x+2)}$ h $\frac{x-7}{(x+3)(x-2)}$ i $\frac{x^2+3x+9}{(x+2)(3x-1)}$
j $\frac{5-5x}{(x-1)(1-x)} = \frac{5}{x-1}$
k $\frac{3x+7}{(x+1)^2}$ l $\frac{3x-4}{(x-1)^2}$
5 a The student transcribed the denominator incorrectly and wrote $(x+2)$ instead of $(x-2)$ in line 2. Also, the student forgot that multiplying a negative number by a negative number gives a positive number. Line 3 should have +3 in the numerator, not -1. They didn't multiply.
b $\frac{x^2-5x+3}{(x-1)(x-2)}$
6 a $\frac{4x^2+17x+17}{(x+2)(x+1)(x+3)}$ b $\frac{7x^2-20x+4}{(x-1)(x+2)(x-4)}$
c $\frac{4x^2+17x+19}{(x+1)(x+3)(x+2)}$ d $\frac{2(2x^2-9x+25)}{(x-4)(x-1)(x+3)}$
7 The lowest common denominator may not always be the product of the denominators. Each fraction must be multiplied by the correct multiple.
8 $a = 4$
9 $\frac{4(x-1)}{(x+3)(x+4)(x-2)}$
10 $\frac{2(x-1)}{(x-7)(x-4)}$

Exercise 2.4 — Multiplying and dividing algebraic fractions

- 1 a $\frac{4x}{y}$ b $\frac{3x}{y}$ c $\frac{4y}{x}$
d $\frac{9x}{4y}$ e $\frac{-5x}{4y}$ f $\frac{3w}{2x}$

g $\frac{6z}{7x}$	h $\frac{2z}{7x}$	i $\frac{-3x}{2y}$	7 a $a = 4$	b $b = 6$	c $i = 3$
j $\frac{5}{24}$	k $\frac{12z}{x}$	l $\frac{-x}{6w}$	d $f = 9$	e $q = 1\frac{1}{8}$	f $r = 5\frac{2}{5}$
2 a $\frac{2}{3x-2}$	b $\frac{5}{x-3}$	c $\frac{9}{2(x-6)}$	g $s = 4\frac{5}{6}$	h $t = 9\frac{4}{5}$	i $a = -7\frac{1}{2}$
d $\frac{1}{x+3}$	e $\frac{2x}{(x+1)^2}$	f $\frac{x+1}{2(2x-3)}$	8 a $f = 40$	b $g = 30$	c $r = -10$
g $\frac{a}{10(a+3)}$	h $\frac{35d}{8(d-3)}$	i $\frac{9}{32x^2(x-2)}$	d $m = 18$	e $n = 28$	f $p = 62.4$
j $\frac{3x}{10(x-1)}$			9 a $x = 1\frac{1}{3}$	b $y = 9$	c $m = 4\frac{2}{5}$
3 a $\frac{3}{5}$	b $\frac{2}{9}$	c $\frac{1}{3}$	d $k = 1\frac{1}{2}$	e $n = 5\frac{2}{3}$	f $c = 1\frac{1}{3}$
d 3	e $\frac{1}{25}$	f $\frac{35}{6}$ or $5\frac{5}{6}$	g $x = 10$	h $x = 4$	i $x = -7\frac{1}{5}$
g $\frac{4y^2}{7}$	h $\frac{2y^2}{25}$	i $\frac{8y^2}{9}$	10 a $k = 25$	b $m = 16$	c $p = -11\frac{3}{7}$
j $\frac{32xy}{15}$	k $\frac{2}{3}$	l y^2	d $u = -4\frac{1}{8}$	e $x = \frac{8}{11}$	f $v = 3$
4 a $\frac{9}{(3x-7)(x+3)}$	b $\frac{1}{(x+2)(x-9)}$		11 a $x = 26$	b $m = -5$	c $w = \frac{25}{3}$
c $\frac{21(x-3)}{x+5}$	d $\frac{13}{9(x-4)(x+1)}$		d $t = 5$	e $x = 9$	f $n = -\frac{19}{3}$

5 Yes, because all of the fractions have the same denominator and therefore can be added together.

6 a -1

b $4 - x$ considered to be the same as $x - 4$

7 1

8 $\frac{(x+1)^2}{x^2+1}$

Challenge 2.2

$\frac{x(x+2)}{(x-1)(x+1)}$ or $\frac{x(x+2)}{x^2-1}$

Exercise 2.5 — Solving simple equations

1 a $a = 24$	b $k = 121$	c $g = 2.9$
d $r = 3$	e $h = 0.26$	f $i = -2$
g $t = 5$	h $q = \frac{1}{6}$	i $x = 0$
2 a $f = 12$	b $i = -60$	c $z = -7$
d $v = 7$	e $w = -5\frac{1}{3}$	f $k = 10$
g $a = 0.425$	h $m = 16\frac{5}{8}$	i $y = 21\frac{1}{2}$
3 a $t = 100$	b $y = \pm 17$	c $q = 6.25$
d $f = \pm 1.2$	e $h = \frac{16}{49}$	f $p = \pm \frac{3}{8}$
g $g = \frac{225}{484}$	h $j = \pm \frac{14}{31}$	i $a = \pm 1\frac{2}{3}$
4 a $t = 25$	b $x = \pm 6$	c $m = 16$
d $t = \pm 3$	e $t = \pm 10$	f $m = 25$
5 a $x = 8$	b $x = -3$	c $m = \frac{1}{8}$
d $x = \frac{3}{4}$	e $m = 0.008$	f $w = 2\frac{1}{2}$
6 a $x = -1$	b $x = -2$	c $m = 1$
d $w = -512$	e $t = 125$	f $x = 2$

7 a $a = 4$	b $b = 6$	c $i = 3$
d $f = 9$	e $q = 1\frac{1}{8}$	f $r = 5\frac{2}{5}$
g $s = 4\frac{5}{6}$	h $t = 9\frac{4}{5}$	i $a = -7\frac{1}{2}$
8 a $f = 40$	b $g = 30$	c $r = -10$
d $m = 18$	e $n = 28$	f $p = 62.4$
9 a $x = 1\frac{1}{3}$	b $y = 9$	c $m = 4\frac{2}{5}$
d $k = 1\frac{1}{2}$	e $n = 5\frac{2}{3}$	f $c = 1\frac{1}{3}$
g $x = 10$	h $x = 4$	i $x = -7\frac{1}{5}$
10 a $k = 25$	b $m = 16$	c $p = -11\frac{3}{7}$
d $u = -4\frac{1}{8}$	e $x = \frac{8}{11}$	f $v = 3$
11 a $x = 26$	b $m = -5$	c $w = \frac{25}{3}$
d $t = 5$	e $x = 9$	f $n = -\frac{19}{3}$
12 a B	b E	c C
13 a $x = -5$	b $d = -1$	c $p = 7$
d $x = -11$	e $h = -2$	f $t = 5$
g $v = -20$	h $r = -3$	i $g = -0.8$
14 a $x = -1$	b $v = 1$	c $l = 2$
d $g = -2$	e $t = 3$	f $e = -23\frac{1}{3}$
g $j = -3\frac{3}{8}$	h $k = -36$	i $f = -12\frac{1}{4}$
15 a $x = 2$	b $b = 5$	c $w = 2$
d $f = 7$	e $t = 3$	f $r = 2\frac{1}{3}$
g $g = -1\frac{1}{3}$	h $h = -2\frac{1}{5}$	i $a = 0$
16 a $x = -1$	b $c = 2$	c $r = 2\frac{2}{3}$
d $k = 1$	e $y = -1\frac{1}{8}$	f $g = 7$
g $w = 1$	h $m = \frac{1}{5}$	i $p = 1\frac{2}{3}$
17 a $x = -15$	b $y = -4\frac{4}{5}$	c $t = 21$
d $u = -2\frac{5}{7}$	e $f = 12\frac{1}{2}$	f $r = 7\frac{1}{2}$
g $d = -6$	h $h = -12$	i $x = 1$
18 a A	b D	c B
19 a 6 cm	b 1.26 cm	
20 a 2.5 cm	b 41 cm	
21 a 5 m	b 2.8 s	
22 a 314 cm	b 6.3 cm	
23 1.8 cm		
24 6.2 cm		
25 Dimensions are 10 m by 6 m .		
26 $\frac{10\sqrt{3}}{3} \text{ cm}$		

Exercise 2.6 — Solving multi-step equations

1 a $x = \frac{20}{31}$	b $x = 3\frac{5}{8}$	c $x = \frac{29}{36}$
d $x = -7$	e $x = -2\frac{8}{11}$	f $x = \frac{10}{43}$
2 a $x = 3$	b $x = 12$	c $x = -2$
d $x = \frac{3}{2}$	e $x = -\frac{11}{3}$ or $x = -3\frac{2}{3}$	
f $x = \frac{2}{13}$	g $x = 3$	h $x = \frac{5}{7}$

- 3 a $x = \frac{5}{17}$ b $x = 15$ c $x = -6\frac{2}{9}$
 d $x = -\frac{10}{19}$ e $x = -1\frac{1}{2}$ f $x = -192$
 g $x = \frac{4}{7}$ h $x = 12$ i $x = 3\frac{1}{4}$
 j $x = 3$ k $x = 52$ l $x = 1\frac{5}{8}$
 4 a $x = \frac{5}{19}$ b $x = 1\frac{31}{58}$ c $x = 4\frac{11}{14}$
 d $x = -3\frac{15}{17}$ e $x = 5\frac{20}{43}$ f $x = -1\frac{10}{13}$
 g $x = 1\frac{2}{61}$ h $x = -4\frac{9}{26}$ i $x = 1.5$
 j $x = -4\frac{1}{3}$ k $x = 3$ l $x = 1$
 5 \$180 6 60 hours 7 \$12 000
 8 \$60
 9 a 4, 9, 25, 49 b 16, 81 c 64
 10 a $6x - 450 = 1000$

b $241\frac{1}{3}$ tickets. This means they need to sell 242 tickets to qualify, as the number of tickets must be a whole number.

11 Teacher to check

12 4

13 $a = 3, b = 5$

Exercise 2.7 — Literal equations

- 1 a $x = \frac{bcd}{a}$ b $x = a(d + bc)$ c $x = (m - n)^2$
 d $x = \pm\sqrt{\frac{w}{ac}}$ e $x = \frac{ay}{b}$ f $x = nw - m$
 g $x = \frac{c}{ab} - b$ h $x = \frac{ac}{b + mc}$ i $x = \frac{ay}{m + b}$
 j $x = \frac{mc + amd}{d}$
 2 a $l = \frac{V}{bh}$ b $b = \frac{P - 2l}{2}$ c $h = \frac{2A}{b}$
 d $a = \pm\sqrt{c^2 - b^2}$ e $C = \frac{5}{9}(F - 32)$ f $r = \pm\sqrt{\frac{A}{\pi}}$
 g $a = \frac{v - u}{t}$ h $N = \frac{100I}{PR}$ i $m = \frac{2E}{v^2}$
 j $v = \pm\sqrt{\frac{2E}{m}}$ k $a = \frac{v^2 - u^2}{2s}$
 l $u = \pm\sqrt{v^2 - 2as}$ m $a = \frac{xb}{b - x}$
 n $x_1 = \frac{x(m + n) - mx_2}{n}$
 3 a $a = \pm 12$ b $h = 7$ c $C = 30$ d $a = 9.8$
 e $s = \pm 13$ f $u = \pm 11$ g $a = 25$ h $x_2 = 13$

- 4 a i No restrictions on x ii $x = \pm\sqrt{y - 4}$ iii $y \geq 4$
 b i $x \neq 3$ ii $x = \frac{2}{y} + 3$ iii $y \neq 0$
 c i No restrictions ii $t = \frac{v - u}{a}$ iii $a \neq 0$
 d i $c \geq 0$ ii $b = \pm\sqrt{c^2 - a^2}$ iii $|c| \geq |a|$
 e i $r \neq 1$ ii $r = \frac{s - a}{s}$ iii $s \neq 0$
 f i $p \neq -q$ ii $b = \frac{m(p + q) - qa}{p}$ iii $p \neq 0$
 g i $a \neq 0, b^2 \geq 4ac$ ii $c = \frac{b^2 - (2ax + b)^2}{4a}$ or $c = -ax^2 - bx$
 iii No new restrictions
 h i $p \neq -q$ ii $p = \frac{q(a - m)}{m - b}$ iii $m \neq b$
 5 a No restrictions, all values must be positive for a cylinder to exist.
 b $r = \sqrt{\frac{V}{\pi h}}$ c $h \neq 0$, no new restrictions
 6 a T and l must be greater than zero.
 b $l = \frac{T^2 g}{4\pi^2}$
 c The restrictions still hold.
 d 2.2 m
 7 a $C = \frac{5}{9}(F - 32)$
 b -40°
 8 Distance Jing has ridden is $\frac{ja}{j - p}$ kilometres.

Investigation — Rich task

- 1 a $9(c - d)$
 b Yes, this is a multiple of 9 as the number that multiplies the brackets is 9.
 2 $90(b - c)$; 90 is a multiple of 9 so the difference between the correct and incorrect one is a multiple of 9.
 3 $900(a - b)$; again 900 is a multiple of 9.
 4 If two adjacent digits are transposed, the difference between the correct number and the transposed number is a multiple of 9.

Code puzzle

Australia

