



The Mathematical Association of Victoria

FURTHER MATHEMATICS

2007 WRITTEN EXAMINATIONS

SOLUTIONS

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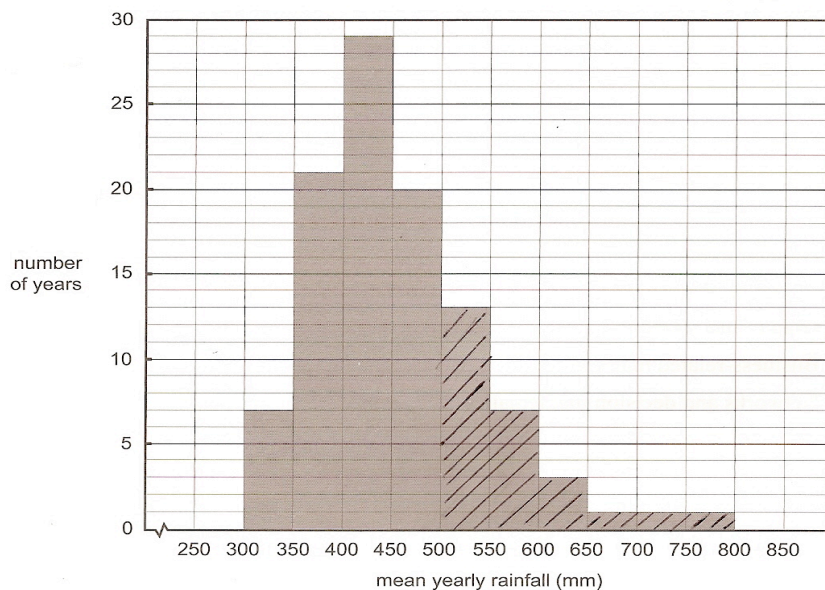
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**2007 Further Mathematics
Written Examination 2 (Analysis task)
Suggested answers and solutions**

Core

Question 1

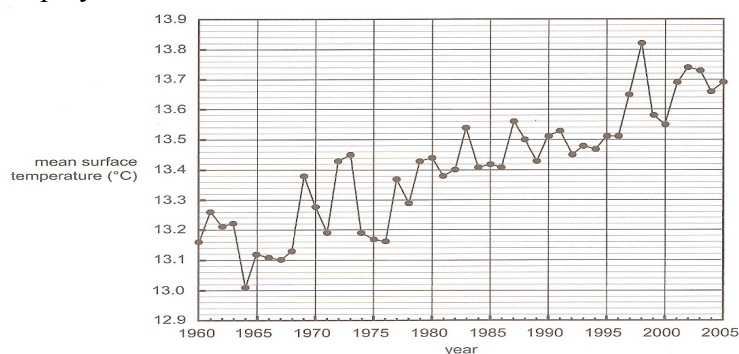
The histogram shows the distribution of mean yearly rainfall (mm) in Australia over 103 years.



- a. The histogram is positively skewed.
- b.
 - i. The yearly rainfall was 500mm or more in $(13 + 7 + 3 + 3) = 26$ years.
 - ii. The percentage of years in which the yearly rainfall was between 500mm and 600mm is $\frac{20}{103} \times 100\% = 19.4\%$ (correct to one decimal place)

Question 2

The mean surface temperature (in $^{\circ}\text{C}$) of Australia for the period 1960 to 2005 is displayed in the time series below.



- a. From the time series plot, the lowest mean surface temperature recorded was in 1964.
- b. The least squares regression line for this time series plot is:
 $\text{Mean surface temperature} = -12.361 + 0.013 \times \text{year}$
- i. In 2010,
 $\text{Mean surface temperature} = -12.361 + 0.013 \times 2010 = 13.77^\circ\text{C}$ (correct to two decimal places)

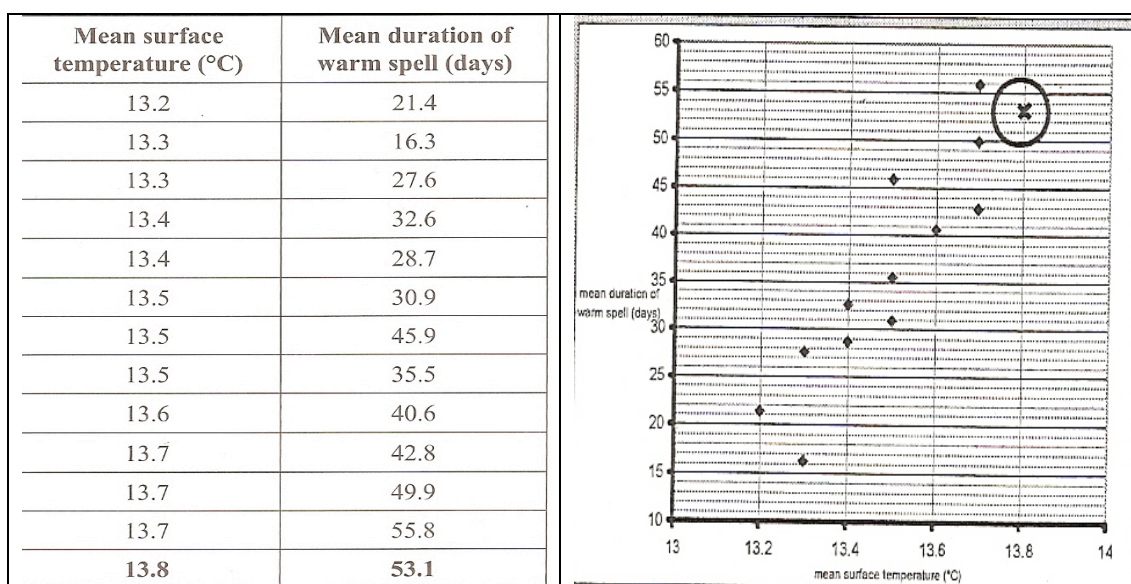
The actual mean surface temperature in the year 2000 was 13.55°C .

- ii. $\text{Residual} = y_{\text{actual}} - y_{\text{estimate}} = 13.55 - (-12.361 + 0.013 \times 2000) = -0.09$
 The residual value in year 2000 is -0.09°C (correct to two decimal places).
- iii. Each year, Australia's mean surface temperature will rise 0.013°C (correct to three decimal places).

Question 3

The table below displays the mean surface temperature ($^\circ\text{C}$) and the mean duration of warm spells (in days) in Australia for 13 years selected at random for the period 1960 – 2005.

- a. The point (13.8, 53.1) is plotted on the scatterplot with a $\times$.



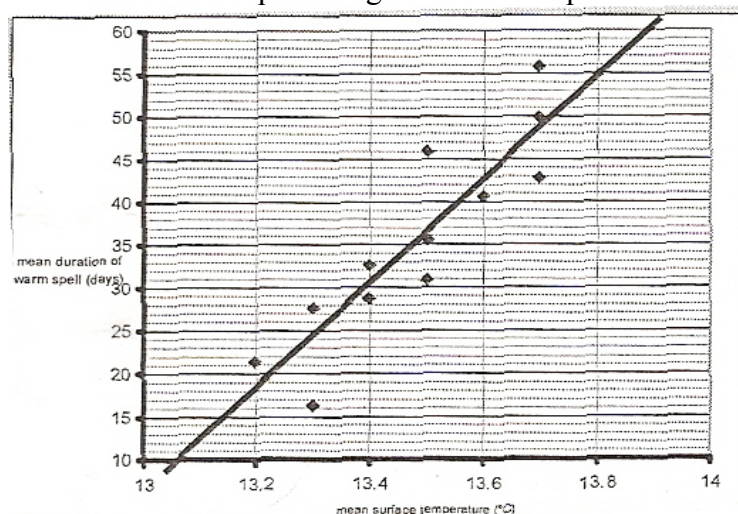
- b.
- i. To determine the least squares regression line for the data in the table above:
 Input data into 2 lists, mean surface temperature (L_1) and mean duration of warm spells (L_2) and perform a LinReg using the following steps:
 STAT
 CALC
 8: LinReg ($a+bx$) L_1, L_2

L1	L2	L3	3	LinReg
13.2	21.4			$y=a+bx$
13.3	16.3			$a=-776.9323308$
13.3	27.6			$b=60.2575188$
13.4	32.6			$r^2=.8288225052$
13.4	28.7			$r=.9103968943$
13.5	30.9			
13.5	45.9			
L3(1)=				

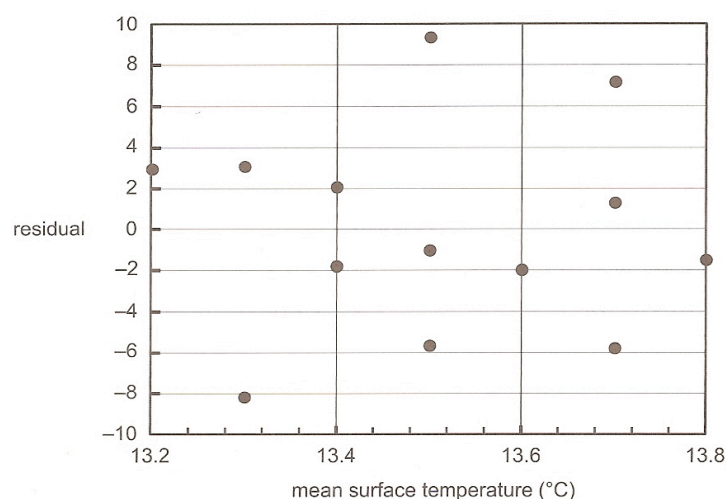
The least squares regression line is:

Mean Duration of Warm Spells = $-776.9 + 60.3$ Mean Surface Temperature.
(correct to one decimal place.)

ii. The least squares regression line is plotted on the scatterplot below.



c.



The residual plot supports the assumption of linearity for this distribution as it is random.

d. The coefficient of determination $r^2 = 0.8288$.
So 83% of variation in the mean duration of a warm spell is explained by the variation in the mean surface temperature.

e. The relationship between the mean duration of a warm spell and the mean surface temperature (referring to the scatterplot in questions 3b i. & ii.) can be described as:

Strength – Strong ($r = 0.9104$)

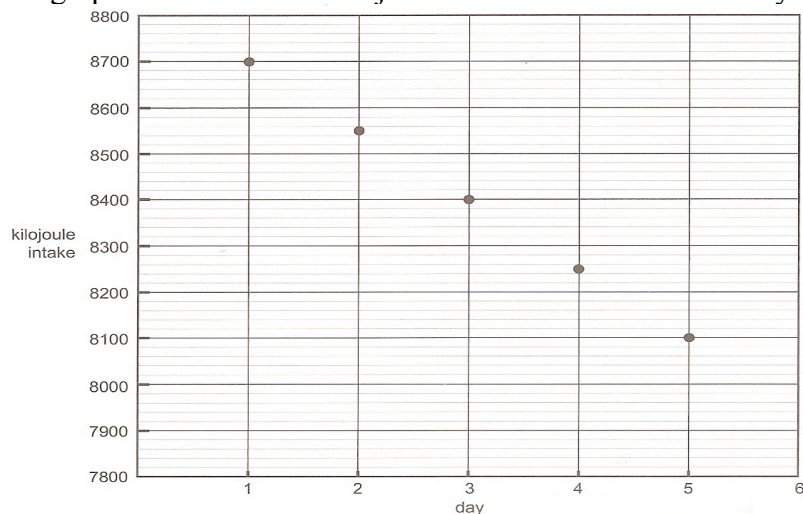
Direction – Positive (gradient)

Form – Linear.

Module 1 – Number Patterns

Question 1

The graph shows Maria's kilojoule intake for the first five days.



- a. Maria's kilojoule intake on day 1 is 8700.
- b. Maria's kilojoule intake forms the sequence 8700, 8550, 8400, 8250, 8100.
The common difference is $8550 - 8700 = -150$.
So Maria's kilojoule intake on day 6 = $8100 - 150 = 7950$

- c. On the n th day, Maria's kilojoule intake is given by the equation

$$K_n = a - 150 \times n$$

$$\text{When } n = 1, K_1 = 8700,$$

$$8700 = a - 150 \times 1$$

$$a = 8700 + 150 = 8850$$

- d.

$$6750 = 8850 - 150n$$

$$150n = 8850 - 6750$$

$$n = \frac{2100}{150} = 14$$

Maria's daily kilojoule intake is 6750 on the 14th day.

Question 2

Maria's brother, Rupert, believes he will benefit by reducing his daily kilojoule intake. His kilojoule intake over 14 days will follow a geometric sequence with a common ratio of 0.95.

On day 1, Rupert's kilojoule intake was 12000.

- a. Rupert's kilojoule intake is reduced by $(1 - 0.95) = 0.05 = 5\%$ each day.
- b. For this geometric sequence, $a = 12000$ and $r = 0.95$
 $t_3 = ar^2 = 12000 \times 0.95^2 = 10830$

On day 3, Rupert's kilojoule intake is 10830.

c. Rupert's kilojoule intake on the n th day is $R_n = 12000 \times 0.95^{n-1}$

d. $t_9 = 12000 \times 0.95^8 = 7961$

$$t_{10} = 12000 \times 0.95^9 = 7563$$

The difference between Rupert's kilojoule intake on days 9 and 10 is $7961 - 7563 = 398$.

e. $S_{14} = \frac{12000(1 - 0.95^{14})}{1 - 0.95} = 122958$

$$S_7 = \frac{12000(1 - 0.95^7)}{1 - 0.95} = 72399$$

Rupert's total kilojoule intake from day 8 to 14 inclusive is $122958 - 72399 = 50559$.

Question 3

Maria decides to improve her fitness level by cycling each day.

The time in minutes M_n that Maria cycles on the n th day is modelled by the difference equation $M_{n+1} = 0.75M_n + 8$ where $M_2 = 20$

a. $M_3 = 0.75 \times 20 + 8 = 23$

$$M_4 = 0.75 \times 23 + 8 = 25.25$$

On day 4, Maria cycles for 25.25 minutes or 25 minutes 15 seconds.

b. The sequence for Maria's cycling is given by 20, 23, 25.25....

$$M_3 - M_2 = 23 - 20 = 3 \quad M_4 - M_3 = 25.25 - 23 = 2.25$$

So the sequence is not arithmetic.

$$\frac{M_3}{M_2} = \frac{23}{20} = 1.15 \quad \frac{M_4}{M_3} = \frac{25.25}{23} = 1.097$$

So the sequence is not geometric.

c.

$$M_2 = 0.75M_1 + 8$$

$$20 = 0.75M_1 + 8$$

$$12 = 0.75M_1$$

$$M_1 = \frac{12}{0.75} = 16$$

Maria cycles 16 minutes on day 1.

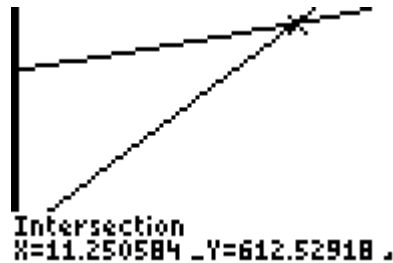
Question 4

Rupert decides to include both swimming and running in his exercise plan.

The swimming is an arithmetic sequence with n th term $t_n = 100 + (n - 1) \times 50$

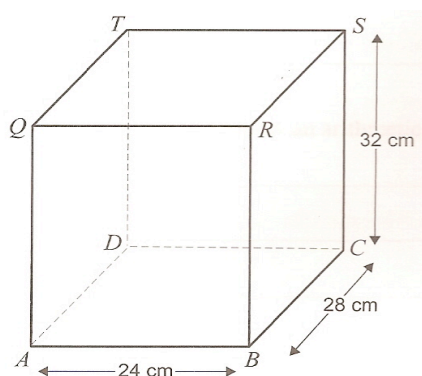
The running is a geometric sequence with the n th term $t_n = 500 \times (1 + 0.02)^{(n-1)}$

To find on which day Rupert swims further than he runs, find the intersection of two graphs representing the n th terms.



During the 12th day, Rupert swims further than he runs.

Module 2 – Geometry and Trigonometry



Question 1

a.

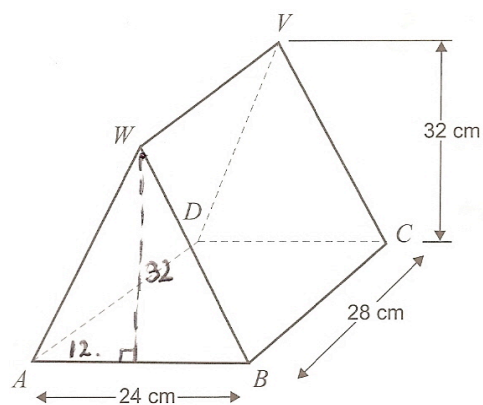
	$QW = \frac{QR}{2} = \frac{24}{2} = 12 \text{ cm}$
--	--

b. $\angle WAQ = \tan^{-1}\left(\frac{12}{32}\right) = 20.6^\circ$ (correct to one decimal place)

c. $\angle AWB = 2 \times \angle WAQ = 2 \times 20.6^\circ = 41.2^\circ$ (correct to one decimal place)

d. From the diagram in question 1a., it can be seen that the area of triangle AWB represents 50% of the area of rectangle $ABRQ$.

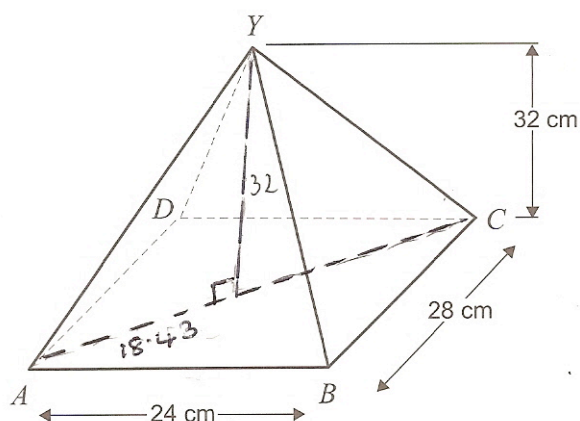
Question 2



a. $AW = \sqrt{12^2 + 32^2} = \sqrt{1168} = 34.176 \approx 34$ as required.

b. Total Surface Area of triangular prism $ABCDWV$
 $= A_{ABCD} + 2 \times A_{BWVC} + 2 \times A_{AWB}$
 $= (24 \times 28) + 2 \times (34 \times 28) + 2 \times \left(\frac{1}{2} \times 24 \times 32\right)$
 $= 672 + 1904 + 768$
 $= 3344$
 Total Surface Area of $ABCDWV$ is 3344 cm^2

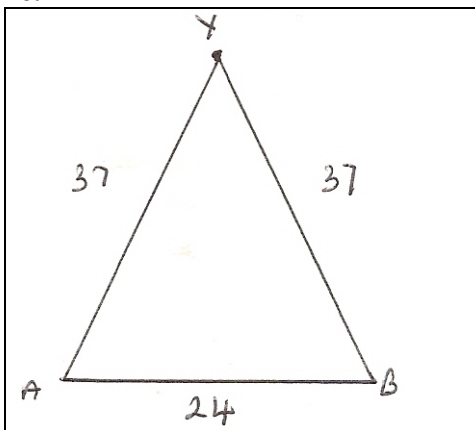
Question 3



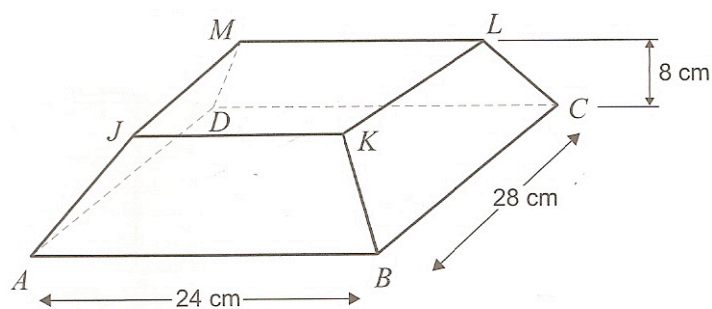
a. $V_{\text{pyramid}} = \frac{1}{3} \times A_{\text{base}} \times \text{height}$
 $V_{ABCDY} = \frac{1}{3} \times (24 \times 28) \times 32 = 7168$
 Volume of pyramid ABCDY is 7168 cm^3 .

b. $AC = \sqrt{24^2 + 28^2} \approx 36.88$
 $AY = \sqrt{\left(\frac{36.88}{2}\right)^2 + 32^2} = \sqrt{18.43^2 + 32^2} = \sqrt{1363.66} = 36.93 \approx 37$
 as required.

c.

	Using Herron's Rule $s = \frac{37 + 37 + 24}{2} = 49$ $A_{\text{triangle}} = \sqrt{s(s-a)(s-b)(s-c)}$ $A_{YAB} = \sqrt{49(49-37)(49-37)(49-24)}$ $= \sqrt{49 \times 12 \times 12 \times 25} = \sqrt{176400} = 420$ The area of triangle YAB is 420 cm^2.
---	---

Question 4



- a. The fraction of height of the pyramid which Tessa removed is $24:32$ or $3:4$

$\frac{3}{4}$ of the pyramid has been removed.

- b. The fraction of the volume of the pyramid which remains can be found by:

$$1 - \left(\frac{3}{4}\right)^3 = 0.578125 = \frac{37}{64}$$

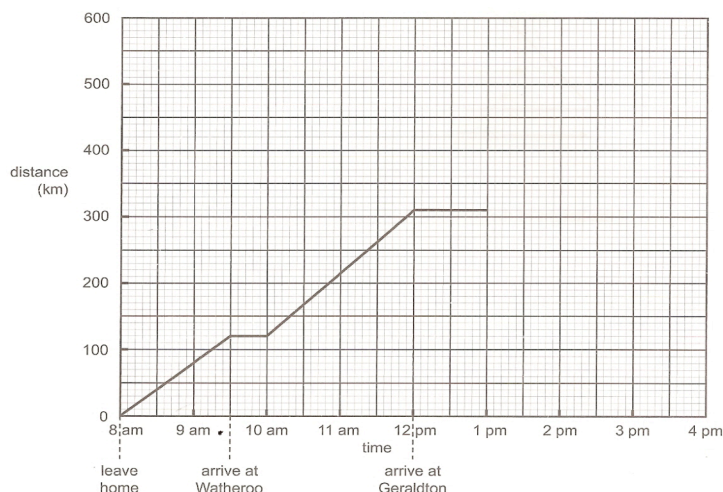
$\frac{37}{64}$ of the pyramid remains.

Module 3 – Graphs and Relations

Question 1

The Goldsmith family are going on a driving holiday in Western Australia. On the first day, they leave home at 8 am and drive to Watheroo then Geraldton.

The distance-time graph below shows their journey to Geraldton.



At 9.30 am the Goldsmiths arrive in Watheroo.

- a. They stopped for 30 minutes in Watheroo.

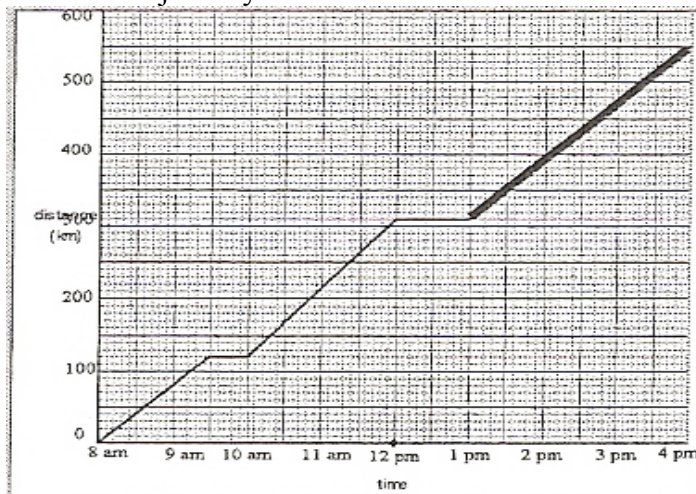
The Goldsmiths arrive in Geraldton at Noon.

- b. The distance between Watheroo and Geraldton = $310 - 120 = 190$ km.

- c. The average speed when travelling between Watheroo and Geraldton

$$= \frac{190}{2} \text{ or } 95 \text{ km/h.}$$

- d. The journey from Geraldton to Hamelin.

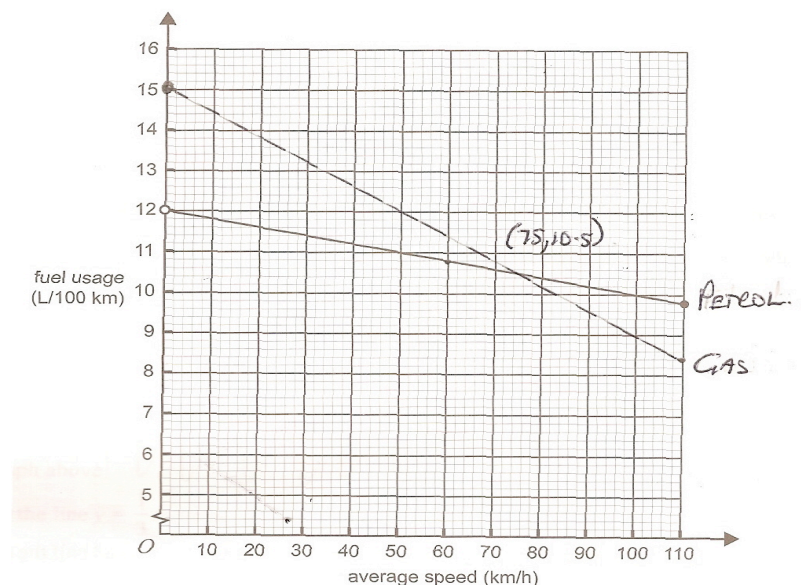


Question 2

The Goldsmiths' car can use either petrol or gas.

The following equation models the fuel usage of petrol, P ($L/100$ km) when the car is travelling at an average speed of s km/h.

$$P = 12 - 0.02s$$



- a. From the graph above, the car will use $10.8 L/100$ km at an average speed of 60 km/h.

The equation which models the gas usage is $G = 15 - 0.06s$.

- b. Refer to the graph above for the line $G = 15 - 0.06s$
- c. From the graph it can be seen that the usage of petrol and gas is the same at 75 km/h. Gas usage will be less than petrol usage for speeds above 75 km/h.

The Goldsmiths' car travels at an average speed of 85 km/h, using gas.

- c. When travelling 100 km. $G = 15 - (0.06 \times 85) = 9.9$ litres.
- d. As gas costs 80 cents per litre, the cost of gas used to travel 100 km is $9.9 \times \$0.80 = \7.92 .

Question 3

Gas is generally cheaper than petrol, but the car must run on petrol some of the time.

x = number of hours driving using gas

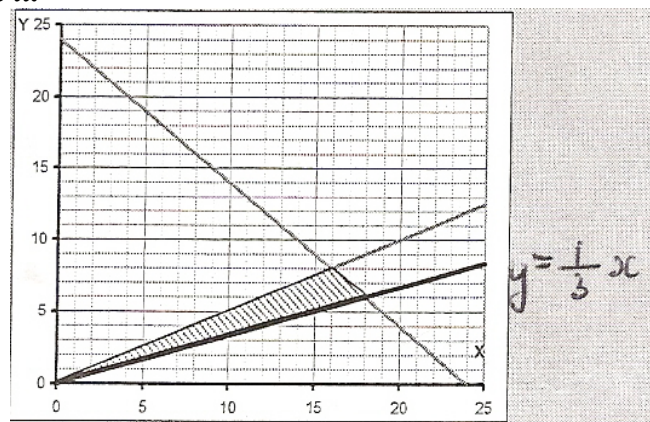
y = number of hours driving using petrol

The first four inequalities are: $x \geq 0$, $y \geq 0$, $y \leq \frac{1}{2}x$, $y \geq \frac{1}{3}x$

- a. Inequality 5 states that $x + y \leq 24$

This means that the total number of hours driving using either petrol or gas cannot exceed 24 hours.

b. i. & ii.

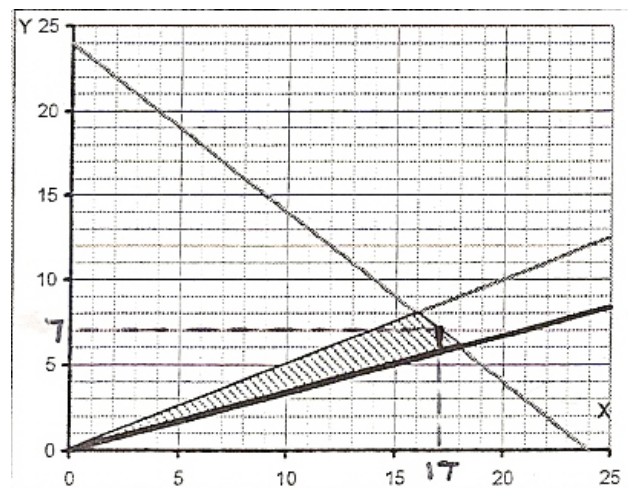


(Feasible region is shaded).

On a particular day, the Goldsmiths plan to drive for 15 hours. They use gas for 10 hours and 5 hours of petrol.

c. Yes, the Goldsmiths do comply with all constraints as (10, 5) is within the feasible region.

On another day, the Goldsmiths plan to drive for 24 hours. Their car carries enough fuel to drive for 20 hours using gas and 7 hours using petrol.



d. From the graph above, it can be seen that the maximum number of hours driving using gas is 18 and the minimum is 17, whilst satisfying the constraints.

Module 4 – Business-related mathematics**Question 1**

Khan wants to buy some office furniture that is valued at \$7000.

a.

i. $\text{Deposit} = 7000 \times 0.25 = \1750

ii. The balance is to be paid in 24 equal monthly instalments. No interest is charged.

$$\text{Monthly instalment} = \frac{7000 - 1750}{24} = \frac{5250}{24} = \$218.75$$

b.

i. Another store offers the same \$7000 office furniture for \$500 deposit and 36 monthly instalments of \$220.

$$\text{Total paid} = 500 + (36 \times 220) = \$8420$$

ii. $\text{Interest paid} = 8420 - 7000 = \1420

$$P = 7000 - 500 = \$6500, \quad t = 3 \text{ years}$$

$$r_f = \frac{100I}{Pt} = \frac{100 \times 1420}{6500 \times 3} = 7.3\%$$

The flat rate of interest is 7.3%.

c. $\text{Cash price paid} = 7000 \times 0.85 = \5950

Question 2

Khan decides to extend his home office and borrows \$30 000 for building costs. Interest is charged at a rate of 9% per annum compounding monthly.

a. $P = 30000, \quad r = \frac{9}{12} \text{ per month}, \quad t = 1$

$$A = P\left(1 + \frac{r}{100}\right)^n = 30000 \times \left(1 + \frac{9}{1200}\right)^{12} = 30000 \times 1.0075 = 30225$$

$$\text{Interest paid each month} = 30225 - 30000 = \$225$$

b. If the interest rate remains at 9% per annum compounding monthly and Khan pays \$400 each month for five years then by using TVM solver:

```

N=60
I%=9
PV=30000
PMT=-400
FV=-16800.77604
P/Y=12
C/Y=12
PMT: [END] BEGIN
  
```

Khan owes \$16801 after five years.

- c. If Khan decides to repay the \$30000 loan fully in equal monthly instalments over five years with the interest rate remaining unchanged, then using TVM solver:

```

N=60
I%=9
PV=30000
PMT=622.75065...
FV=0
P/Y=12
C/Y=12
PMT: [END] BEGIN

```

Khan will need to pay \$622.75 each month to pay the loan fully in five years.

Question 3

- a. Khan paid \$900 for a fax machine. This price included 10% GST. The cost of the fax machine before GST can be found by solving:

$$1.1x = 900$$

$$x = \frac{900}{1.1} = \$818.18$$

- b. i. Khan decided to depreciate his \$900 fax machine for taxation purposes using **flat rate depreciation**. The fax machine is valued at \$300 after five years.

To calculate annual depreciation solve

$$300 = 900 - 5n$$

$$5n = 600$$

$$n = \$120$$

Annual depreciation is \$120.

- ii. The \$900 fax machine is depreciated 46 cents for each fax that Khan sends. He sends 250 faxes per year. Using **Unit cost depreciation**, the value of the fax machine after five years is $900 - 0.46 \times (5 \times 250) = \325

Question 4

- a. After five years, Khan's books which were originally valued at \$10 000 are depreciated at 12% per annum using the **Reducing Balance** method. .

$$\text{Book Value} = 10000 \times (0.88)^5 = \$5277$$

- b. If Khan believed his books should be valued at \$4000 after five years, then using TVM solver,

```

N=5
I%=-16.74467926
PV=10000
PMT=0
FV=-4000
P/Y=1
C/Y=1
PMT:[END] BEGIN
    
```

The annual reducing balance depreciation rate of 16.7% will give a book value of \$4000 after five years.

Alternate Method

$$4000 = 10000\left(1 - \frac{r}{100}\right)^5$$

$$\left(1 - \frac{r}{100}\right)^5 = 0.4$$

Solve $\frac{r}{100} = 1 - \sqrt[5]{0.4}$

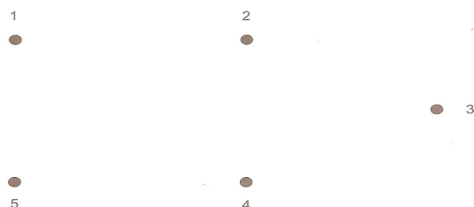
$$r = 100 \times 0.1674$$

$$r = 16.7\%$$

Module 5 – Networks and decision mathematics

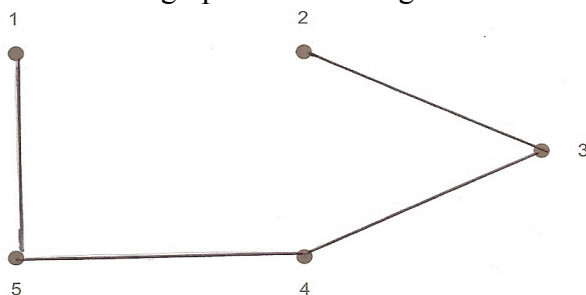
Question 1

A new housing estate is being developed. There are five houses under construction in one location. The houses are numbered as points 1 to 5 below.



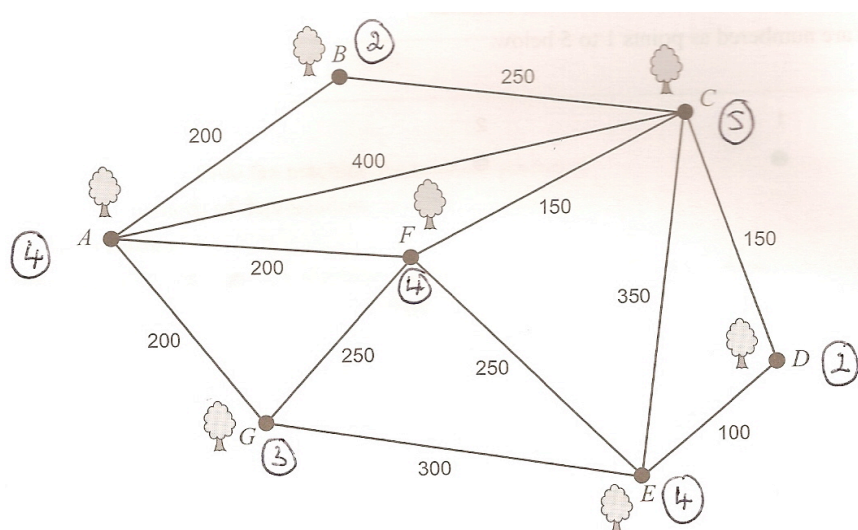
- a. The builders require the five houses to be connected by electrical cables to enable the workers to have a supply of power on each side. The minimum number of edges needed to connect the five houses is four.

- b. A connected graph with four edges could look like:



Question 2

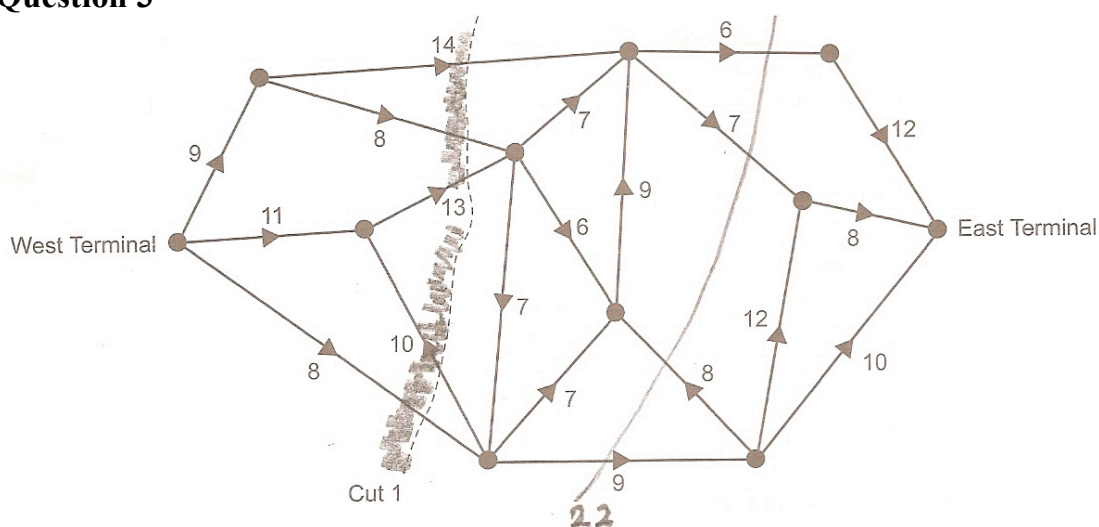
The estate has large open parklands that contain seven large trees. The trees are denoted as vertices A to G on the network below. Walking paths link the trees shown.



- a. The sum of the degrees of the vertices of this network
 $= 4 + 3 + 4 + 2 + 5 + 2 + 4 = 24$

- b. i.** If Jamie decides to go for a walk that will take him along each of the paths between the trees as well as walking the minimum distance possible, then he needs to find an Euler path, which only exists if there is a maximum of two vertices of odd degree.
Jamie should begin his walk at vertex C or G as they are both of odd degree.
- ii.** A possible Euler path for Jamie could be GEDCEFGAFCABC. The length is:
 $300 + 100 + 150 + 350 + 250 + 250 + 200 + 200 + 150 + 400 + 200 + 250 = 2800$ m
- c.** If Michelle wishes to follow a route that is the shortest Hamiltonian circuit and she begins at F then she could take either of the following routes:
FGABCDEF or FEDCBAGF.

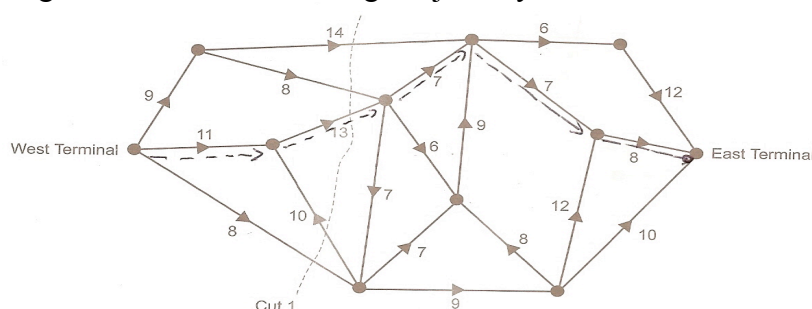
Question 3



As an attraction for young children, a miniature railway runs through the new housing estate. The train travels through stations that are represented by nodes on the directed network diagram above.

- a.** The capacity of Cut 1 = $14 + 8 + 13 + 8 = 43$
- b.** The maximum number of seats available for children for a journey from the West to the East Terminal is the minimum cut of 22 seats.

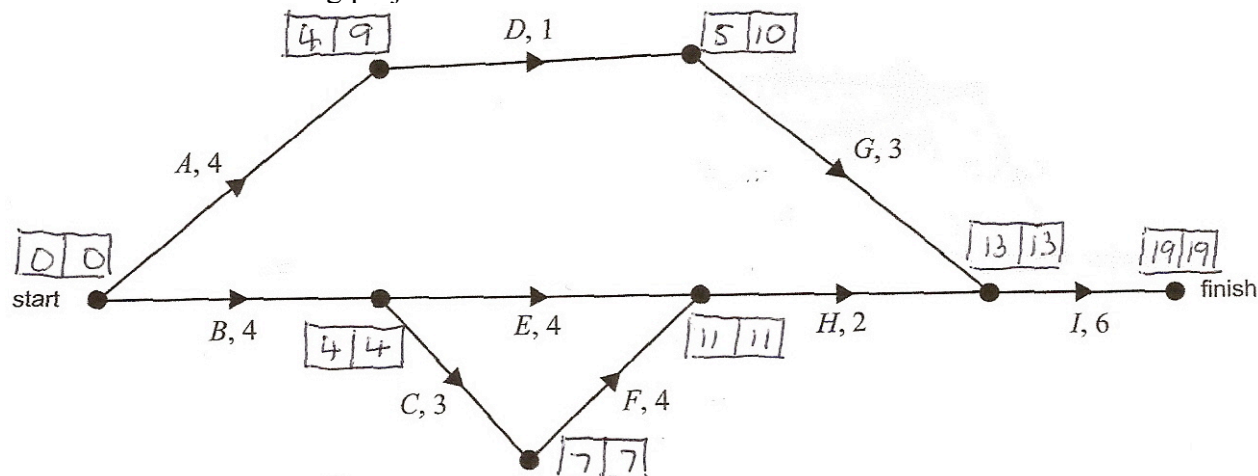
On a particular train, 10 children set out from the West Terminal and no new passengers board the train during the journey.



From the diagram, it can be seen that the maximum number of children who can reach the East Terminal is seven.

Question 4

A community centre is to be built on the new housing estate. Nine activities have been identified for this building project.



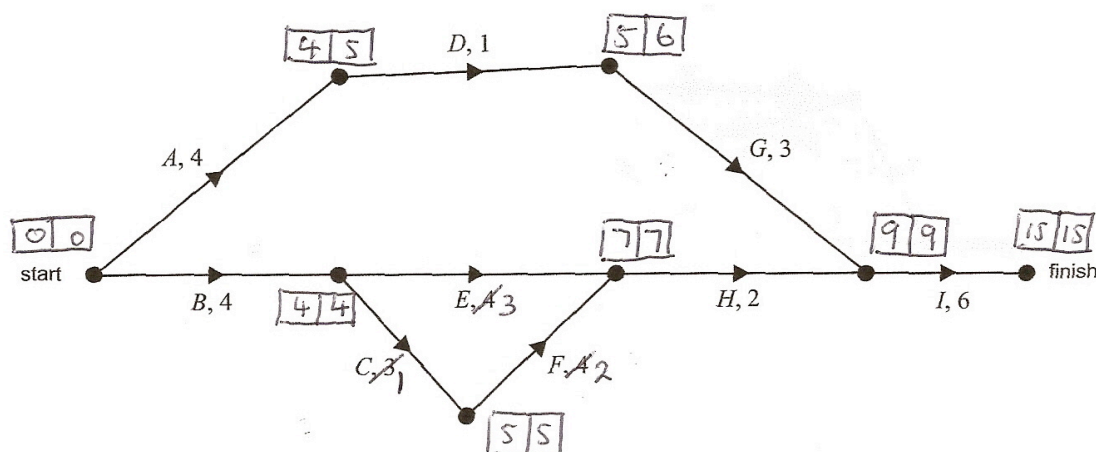
a. The minimum time to complete the project is 19 weeks.

b. The slack time for activity D is $10 - 1 - 4 = 5$ weeks.

The builders of the community centre are able to speed up the project. Some activities can be reduced in time at an additional cost. Activities A, C, E, F and G can be reduced.

c. The critical path for the network is *B C F H I*. As A, E and G are not on the critical path reducing these activities would not result in an earlier project completion.

The owner of the estate is prepared to pay the additional cost to achieve early completion. The cost of reducing each activity is \$5000 per week. The maximum reduction for each activity is 2 weeks.



d. The project can now be completed in 15 weeks.

e. The additional cost of completing the project in 15 weeks is $5 \times 5000 = \$25000$

Module 6 – Matrices**Question 1**

The table below displays the energy content and amounts of fat, carbohydrate and protein contained in a serve of four foods: bread, margarine, peanut butter and honey.

Food	Energy content (kilojoules/serve)	Fat (grams/serve)	Carbohydrate (grams/serve)	Protein (grams/serve)
Bread	531	1.2	20.1	4.2
Margarine	41	6.7	0.4	0.6
Peanut butter	534	10.7	3.5	4.6
Honey	212	0	12.5	0.1

- a. The 2×3 matrix that displays the fat, carbohydrate and protein (in columns) of bread and margarine is $\begin{bmatrix} 1.2 & 20.1 & 4.2 \\ 6.7 & 0.4 & 0.6 \end{bmatrix}$.

b. i. $AB = \begin{bmatrix} 2 & 2 & 1 & 1 \end{bmatrix} \times \begin{bmatrix} 531 \\ 41 \\ 534 \\ 212 \end{bmatrix} = \begin{bmatrix} 1890 \end{bmatrix}$

- ii. The order of the matrix product $BA = ((4 \times 1) \times (1 \times 4)) = 4 \times 4$ matrix

- iii. Matrix product AB provides the total energy content in the peanut butter and honey sandwich.

- c. The number of serves of bread, margarine, peanut butter and honey can be determined by solving the following matrix:

$$\begin{bmatrix} 1.2 & 6.7 & 10.7 & 0 \\ 20.1 & 0.4 & 3.5 & 12.5 \\ 4.2 & 0.6 & 4.6 & 0.1 \\ 531 & 41 & 534 & 212 \end{bmatrix} \begin{bmatrix} b \\ m \\ p \\ h \end{bmatrix} = \begin{bmatrix} 53 \\ 101.5 \\ 28.5 \\ 3568 \end{bmatrix}$$

To solve for b, m, p and h , use $A^{-1}AX = A^{-1}B$ using your graphics calculator. To do this, type in the 4×4 matrix A and then find its inverse A^{-1} using x^{-1} .

Then pre-multiply this by the column matrix shown.

$$\begin{bmatrix} b \\ m \\ p \\ h \end{bmatrix} = \begin{bmatrix} 4 \\ 4 \\ 2 \\ 1 \end{bmatrix} \text{ and so } b = 4, m = 4, p = 2 \text{ and } h = 1.$$

Question 2

a. If the initial state matrix of this population is $S_0 = \begin{bmatrix} 400 \\ 200 \\ 100 \\ 0 \end{bmatrix} \begin{matrix} E \\ J \\ A \\ D \end{matrix}$

then the total number of insects in the population (including eggs) at the beginning is $400 + 200 + 100 = 700$.

- b. The proportion of eggs which turn into juveniles each week is 0.5 given by the row 2, column 1 element in the following transition matrix

$$T = \begin{bmatrix} 0.4 & 0 & 0 & 0 \\ 0.5 & 0.4 & 0 & 0 \\ 0 & 0.5 & 0.8 & 0 \\ 0.1 & 0.1 & 0.2 & 1 \end{bmatrix} \begin{matrix} E \\ J \\ A \\ D \end{matrix}$$

- c. i.

$$S_1 = TS_0 = \begin{bmatrix} 0.4 & 0 & 0 & 0 \\ 0.5 & 0.4 & 0 & 0 \\ 0 & 0.5 & 0.8 & 0 \\ 0.1 & 0.1 & 0.2 & 1 \end{bmatrix} \begin{bmatrix} 400 \\ 200 \\ 100 \\ 0 \end{bmatrix} = \begin{bmatrix} 160 \\ 280 \\ 180 \\ 80 \end{bmatrix} \begin{matrix} E \\ J \\ A \\ D \end{matrix}$$

- ii. The number of juveniles in the population after one week is 280.

- iii. After four weeks,

$$S_4 = T^4 S_0 = \begin{bmatrix} 0.4 & 0 & 0 & 0 \\ 0.5 & 0.4 & 0 & 0 \\ 0 & 0.5 & 0.8 & 0 \\ 0.1 & 0.1 & 0.2 & 1 \end{bmatrix}^4 \begin{bmatrix} 400 \\ 200 \\ 100 \\ 0 \end{bmatrix} = \begin{bmatrix} 10 \\ 56 \\ 313 \\ 320 \end{bmatrix} \begin{matrix} E \\ J \\ A \\ D \end{matrix}$$

and so there are 56 live juveniles in the population.

$$\text{iv. } S_6 = T^6 S_0 = \begin{bmatrix} 0.4 & 0 & 0 & 0 \\ 0.5 & 0.4 & 0 & 0 \\ 0 & 0.5 & 0.8 & 0 \\ 0.1 & 0.1 & 0.2 & 1 \end{bmatrix}^6 \begin{bmatrix} 400 \\ 200 \\ 100 \\ 0 \end{bmatrix} = \begin{bmatrix} 1.6384 \\ 13.1072 \\ 236.6464 \\ 448.608 \end{bmatrix} \begin{matrix} E \\ J \\ A \\ D \end{matrix}$$

$$S_7 = T^7 S_0 = \begin{bmatrix} 0.4 & 0 & 0 & 0 \\ 0.5 & 0.4 & 0 & 0 \\ 0 & 0.5 & 0.8 & 0 \\ 0.1 & 0.1 & 0.2 & 1 \end{bmatrix}^7 \begin{bmatrix} 400 \\ 200 \\ 100 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.6554 \\ 6.0621 \\ 195.8707 \\ 497.4118 \end{bmatrix} \begin{matrix} E \\ J \\ A \\ D \end{matrix}$$

There are no live eggs left in the population after seven weeks.

v. The exact steady state matrix for this population is

$$S_{\text{steady state}} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 700 \end{bmatrix} \begin{matrix} E \\ J \\ A \\ D \end{matrix}$$

$$\text{d. i. } S_1 = TS_0 + BS_0 = \begin{bmatrix} 0.4 & 0 & 0 & 0 \\ 0.5 & 0.4 & 0 & 0 \\ 0 & 0.5 & 0.8 & 0 \\ 0.1 & 0.1 & 0.2 & 1 \end{bmatrix} \begin{bmatrix} 400 \\ 200 \\ 100 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0.3 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 400 \\ 200 \\ 100 \\ 0 \end{bmatrix} = \begin{bmatrix} 190 \\ 280 \\ 180 \\ 80 \end{bmatrix} \begin{matrix} E \\ J \\ A \\ D \end{matrix}$$

ii. After two weeks

$$S_2 = TS_1 + BS_1 = \begin{bmatrix} 0.4 & 0 & 0 & 0 \\ 0.5 & 0.4 & 0 & 0 \\ 0 & 0.5 & 0.8 & 0 \\ 0.1 & 0.1 & 0.2 & 1 \end{bmatrix} \begin{bmatrix} 190 \\ 280 \\ 180 \\ 80 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0.3 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 190 \\ 280 \\ 180 \\ 80 \end{bmatrix} = \begin{bmatrix} 130 \\ 207 \\ 284 \\ 163 \end{bmatrix} \begin{matrix} E \\ J \\ A \\ D \end{matrix}$$

and so the number of live eggs after two weeks is 130.