



The Mathematical Association of Victoria

FURTHER MATHEMATICS

2007 WRITTEN EXAMINATIONS

SOLUTIONS

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2007 Further Mathematics
Written Examination 1 (Facts, skills and applications)
Suggested answers and solutions

SECTION A (Multiple-Choice)

Answers

Core

1. **A** 2. **B** 3. **A** 4. **D** 5. **C**
 6. **C** 7. **C** 8. **D** 9. **D** 10. **E**
 11. **E** 12. **C** 13. **E**

SECTION B (Multiple-Choice)

Answers

Module 1 Number patterns and applications

1. **B** 2. **C** 3. **B** 4. **C** 5. **E**
 6. **B** 7. **E** 8. **D** 9. **A**

Module 2 Geometry and trigonometry

1. **A** 2. **B** 3. **B** 4. **A** 5. **B**
 6. **D** 7. **C** 8. **D** 9. **E**

Module 3 Graphs and relations

1. **D** 2. **C** 3. **E** 4. **B** 5. **D**
 6. **E** 7. **C** 8. **B** 9. **A**

Module 4 Business-related mathematics

1. **A** 2. **C** 3. **C** 4. **D** 5. **E**
 6. **D** 7. **E** 8. **D** 9. **A**

Module 5 Networks and decision mathematics

1. **E** 2. **C** 3. **A** 4. **B** 5. **D or E**
 6. **E** 7. **C** 8. **B** 9. **B**

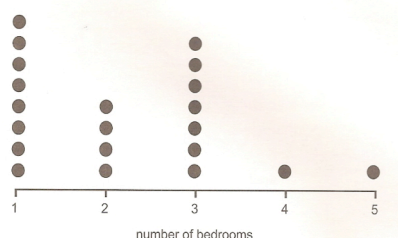
Module 6 Matrices

1. **A** 2. **D** 3. **B** 4. **B** 5. **A**
 6. **D** 7. **E** 8. **B** 9. **C**

Core

Question 1

[A]



The mode for the number of bedrooms is one as this occurs most often in this dot plot. This is at 1.

Question 2

[B]

The median point lies at 50%.

For this dot plot of 21 apartments, the median is the 11th observation from the beginning, so the median equals 2.

Question 3

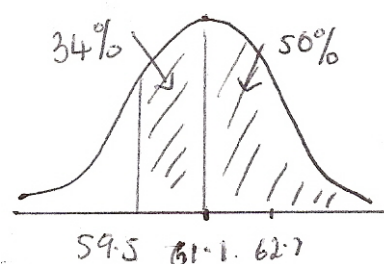
[A]

The student's standard z-score is closest to:

$$z = \frac{x - \mu}{\sigma} = \frac{56 - 67}{10.2} = -1.08$$

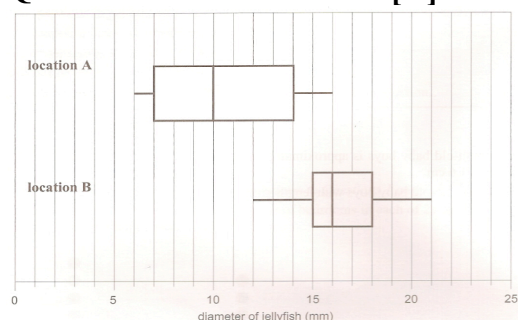
Question 4

[D]



From the diagram, the percentage of 3 month old boys with length greater than 59.5cm ($-1S_x$)

$$\approx 34\% + 50\% = 84\%$$

Question 5 [C]

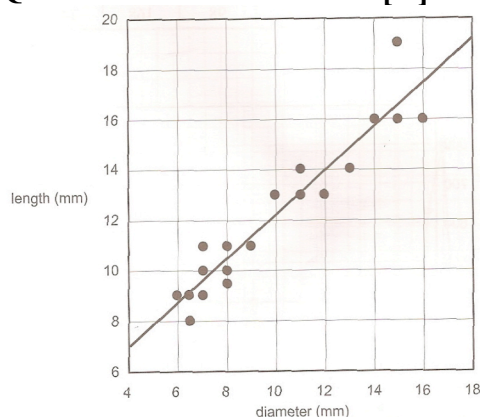
25 % of jellyfish taken from location A have a diameter greater than 14mm as the Q_3 (75% point) occurs here.

Question 6 [C]

Location A range = $16 - 6 = 10$

Location B range = $21 - 12 = 9$

It can be concluded that the diameters of jellyfish taken from Location A are less than the diameters of jellyfish in Location B and more variable.

Question 7 [C]

If the correlation coefficient r is 0.9034, then the coefficient of determination is $r^2 = (0.9034)^2 = 0.8161 = 81.6\%$

Question 8 [D]

The equation of the least squares regression line is

$$\text{length} = 3.5 + 0.87 \text{ diameter}$$

It can be concluded that there is a 0.87mm increase in *length* for each 1mm increase in *diameter*.

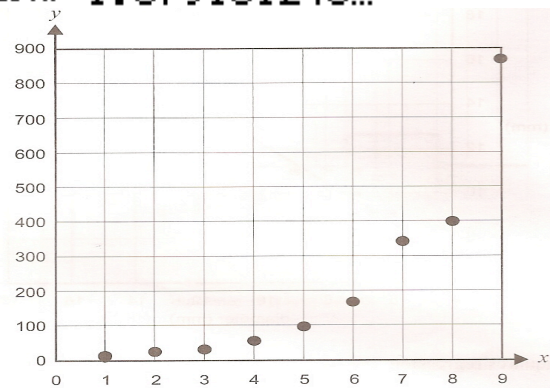
Question 9 [D]

Input data into 2 lists.

x into L_1 and y into L_2 . $L_3 = \log_{10} L_2$

L1	L2	L3
1	12	1.079181246
2	25	1.397940009
3	33	1.51851391
4	58	1.76342796
5	98	1.99122604
6	168	2.22530914
7	345	2.53767174

$L3(1) = 1.079181246...$



To find the least squares regression line for the transformed data:

STAT

CALC

8: LinReg ($a+bx$) L_1, L_3

```

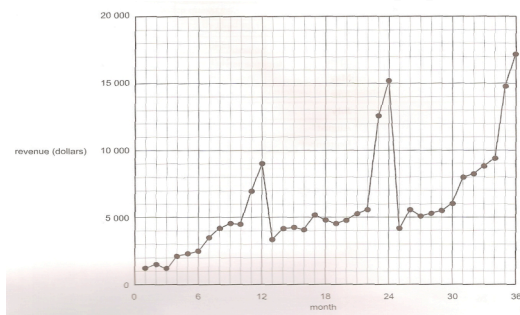
LinReg
y=a+bx
a=.8771588903
b=.2257066202
r^2=.9929580607
r=.9964728098

```

The least squares regression line is $\log y = 0.88 + 0.23x$

Question 10 [E]

As *size of car* and *salary level* are both categories of data, the most appropriate method of displaying these is a percentaged segmented bar chart.

Question 11**[E]**

This time series plot indicates that, over a three year period, revenue from sales each month showed an increasing trend with seasonal variation due to peaks in 12th, 24th and 36th months.

Question 12**[C]**

The upper median for third year (months 25-36) is:

$$\left(\frac{31+30}{2}, \frac{6000+8000}{2}\right) = (30.5, 7000)$$

The lower median for year 1 (months 1-12) is:

$$\left(\frac{6+7}{2}, \frac{2500+3500}{2}\right) = (6.5, 3000)$$

The slope m , can be calculated by

$$m = \frac{y_u - y_l}{x_u - x_l} = \frac{7000 - 3000}{30.5 - 6.5} = 166.67 \approx 167$$

Question 13**[E]**

Total Revenue for year 1 = \$43872

The Seasonal Index for September

$$= \frac{4597}{\left(\frac{43872}{12}\right)} = \frac{4597}{3656} = 1.26$$

Module 1 Number patterns**Question 1****[B]**

For the geometric sequence, 24, 6, 1.5... the common ratio is:

$$r = \frac{t_2}{t_1} = \frac{6}{24} = 0.25$$

$$r = \frac{t_3}{t_2} = \frac{1.5}{6} = 0.25$$

Question 2**[C]**

The yearly membership follows an arithmetic sequence.

$a = 15$ and $a + 2d = 29$ so the common difference $d = 7$

The membership in the fourth year

$$t_4 = a + 3d = 15 + 3 \times 7 = 36$$

Question 3**[B]**

$t_{n+1} = at_n + 6$ where $t_1 = 5$ generates the sequence 5, 21, 69, 213...

$$t_1 = 5$$

$$t_2 = at_1 + 6 \text{ so } 21 = a \times 5 + 6$$

$$5a = 15$$

$$a = 3$$

Question 4**[C]**

The number of waterfowl decreases by 4% per year since 2003.

$$r = 0.96 \text{ and } t_1 = 680$$

At the start of 2008 (t_6), the number of waterfowl is closest to:

$$t_6 = ar^5 = 680 \times (0.96)^5 \approx 554$$

Question 5**[E]**

As the sequence is geometric, the difference equation for the waterfowl is given by:

$$W_{n+1} = 0.96W_n, \quad W_1 = 680$$

Question 6**[B]**

The sequence for the first three layers is given by 20, 19, 18, so forming an arithmetic sequence.

$$a = 20 \text{ and } d = -1$$

The maximum number of cans for 20 layers is:

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{20} = \frac{20}{2}[40 + (20-1) \times -1] = 210$$

Question 7 [E]

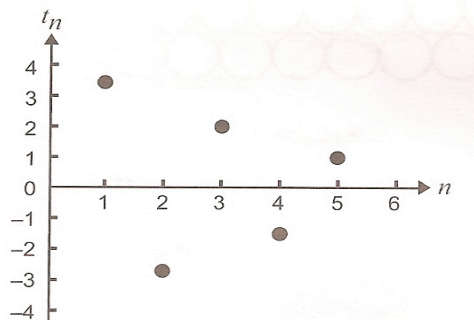
$t_1 = a$ is positive.

As the common ratio r is negative,

$t_2 = ar$ is negative.

$t_3 = ar^2$ is positive.

So the graph could be represented by:

**Question 8 [D]**

The first four terms of a sequence are 12, 18, 30 and 54.

As $18 - 12 = 6$ and $30 - 18 = 12$

The sequence is not arithmetic.

$$\frac{18}{12} = 1.5 \text{ and } \frac{30}{18} = \frac{15}{9} = 1.67$$

So the sequence is not geometric.

The difference equation is a combination of arithmetic and geometric:

$$t_{n+1} = 2t_n - 6, t_1 = 12$$

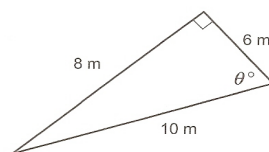
Question 9 [A]

$$a = 15 \quad r = \frac{13.5}{15} = \frac{12.15}{13.5} = 0.9$$

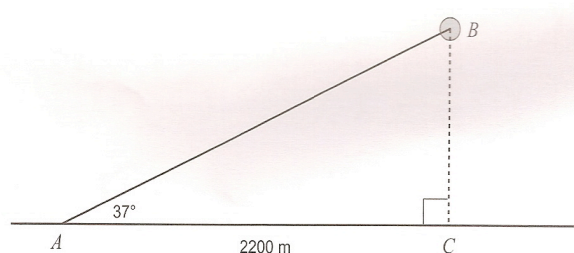
It is a geometric sequence.

$$S_{14} = \frac{15(1 - 0.9^{14})}{1 - 0.9} = 150 \times 0.7712 \approx 116$$

The total amount of forest destroyed by the volcanic eruption at the end of the fourteenth day is closest to 116 km^2

Module 2 Geometry and trigonometry**Question 1 [A]**

$$\cos \theta^\circ = \frac{6}{10}$$

Question 2 [B]

$$\tan 37^\circ = \frac{BC}{2200}$$

$$x = 2200 \tan 37^\circ = 1657.82 \approx 1658$$

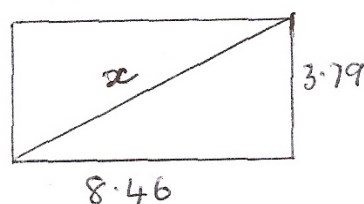
The height of the weather balloon above the ground is approximately 1658 metres.

Question 3 [B]

A rectangle is 3.79m wide and has a perimeter of 24.50m.

$$P = 2(l + w)$$

$$\frac{24.5}{2} = l + 3.79, \text{ so } l = 8.46$$



Using Pythagoras,

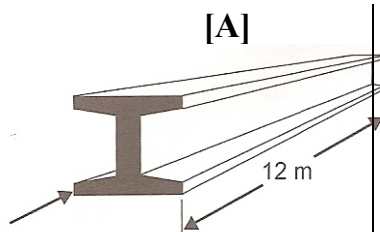
$$x = \sqrt{8.46^2 + 3.79^2} = 9.27 \approx 9.3$$

The diagonal (x) of this rectangle is 9.3 m (correct to one decimal place).

Question 4

[A]

area of cross section 0.048 m^2



Volume = Area $\times$ length

$$= 0.048 \times 12 = 0.576 \text{ m}^3$$

Question 5

[B]

$$10 \text{ cm}^2 : 4000 \text{ m}^2$$

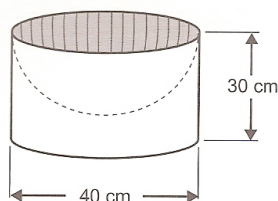
$$1 \text{ cm}^2 : 400 \text{ m}^2$$

$$1 \text{ cm} : \sqrt{400} = 20 \text{ m}$$

1 cm would represent an actual distance of 20 m.

Question 6

[D]



Total Surface Area =

$$\pi r^2 + 2\pi rh + 2\pi r^2$$

= base + curved surface area + hemisphere

$$= 3\pi r^2 + 2\pi rh$$

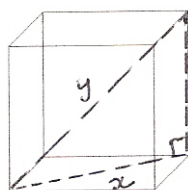
$$= 3\pi(20^2) + 2\pi(20 \times 30)$$

$$= 1200\pi + 1200\pi$$

$$\approx 7540 \text{ cm}^2$$

Question 7

[C]



Using Pythagoras,

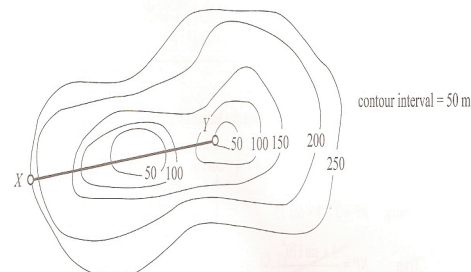
$$x = \sqrt{36^2 + 36^2} = 36\sqrt{2}$$

$$y = \sqrt{(36\sqrt{2})^2 + 36^2} = 62.354 \approx 62$$

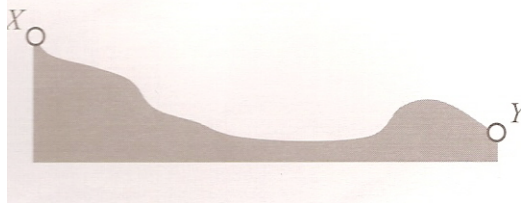
The maximum possible length of the rod is closest to 62 cm.

Question 8

[D]

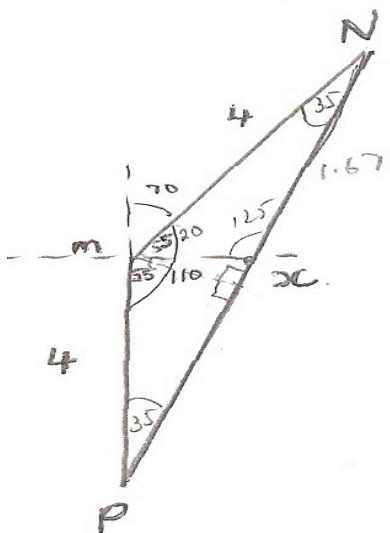


The cross section along the line segment XY could be:



Question 9

[E]



From the diagram, the Cosine Rule can be used.

$$NP = \sqrt{16 + 16 - 2 \times 4 \times 4 \cos 110^\circ}$$

So Graeme is correct.

The Sine Rule can be used:

$$\frac{4}{\sin 35^\circ} = \frac{NP}{\sin 110^\circ}$$

$$NP = \frac{4 \sin 110^\circ}{\sin 35^\circ}$$

So Tran is correct.

The non right angle triangle can be divided into 2 right angle triangles.

$$\cos 35^\circ = \frac{PX}{4}$$

$$PX = 4 \cos 35^\circ$$

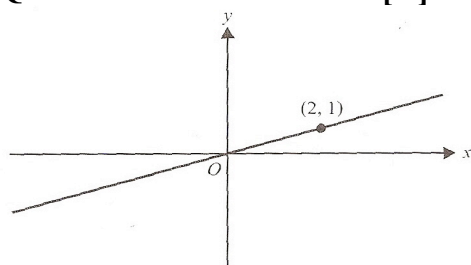
$$NP = 2 \times PX = 2 \times 4 \times \cos 35^\circ$$

So Shelley is also correct

All three gave correct calculations.

Module 3 Graphs and relations

Question 1 [D]



$$\text{Slope (m)} = \frac{\text{rise}}{\text{run}} = \frac{1}{2}$$

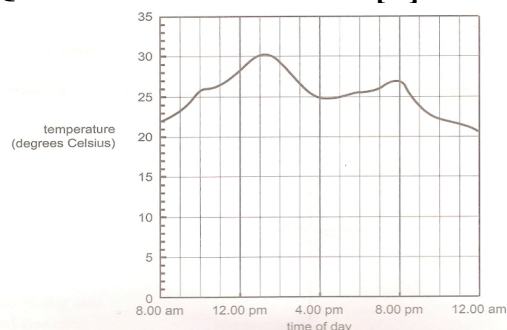
Question 2 [C]

The builder's fee, \$C, is found using the rule, $C = 60 + 55n$

$$\text{When } n = 8, C = 60 + 55 \times 8 = 500$$

The builder's fee will be \$500 for 8 hours work.

Question 3 [E]



During this period, the minimum temperature occurred at 12.00am.

Question 4 [B]

The cost for making rulers is

$$C = 60 + 0.2n$$

The rule for break even when selling rulers is:

$$S = n$$

For break even, $C = S$

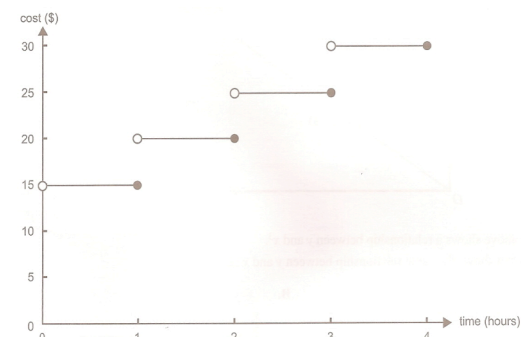
$$60 + 0.2n = n$$

$$n = \frac{60}{0.8} = 75$$

Paul sold 75 rulers last week.

Question 5 [D]

The graph represents hiring one motorbike for 4 hours.



Hiring 2 motorbikes for a total of \$45 can be done with a combination of: 1.5 hours (\$20) and 3 hours (\$25)

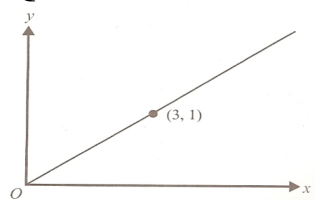
Question 6 [E]

x = number of bottles of red wine

y = number of bottles of white wine

He plans to make at least twice as many bottles of red than white can be represented by the inequality: $x \geq 2y$

Question 7 [C]



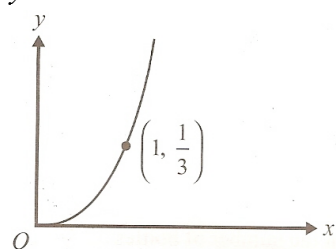
$$(ax^3, y) = (3, 1)$$

$$a = \frac{1}{3}$$

$$y = \frac{1}{3}x^3$$

When $x = 1, y = \frac{1}{3}$

So the same relationship between x and y is:



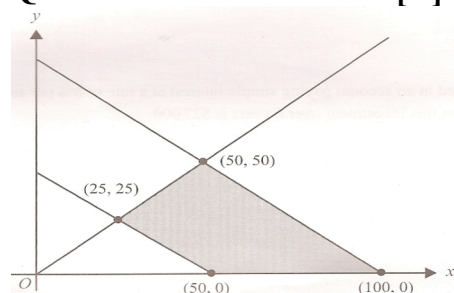
Question 8 [B]

$$2x - y = 1$$

$$2(2x - y) = 3$$

Both these lines have the same gradient so they are parallel to each other and thus no intersection or solution.

Question 9 [A]



The objective function is $Z = 2x + ky$. The maximum value of Z occurs **only** at $(100,0)$ so $Z = 2 \times 100 + 0 = 200$. Consider point $(50,50)$. If $k = 2$ then $Z = 2 \times 50 + 2 \times 50 = 200$. But this is not permitted. So $k < 2$, therefore $k = 1$

Module 4 Business-related mathematics

Question 1 [A]

An agent charged \$20 commission for selling a rare book for \$500.

$$\frac{20}{500} \times 100\% = 4\%$$

The commission is 4% of the selling price.

Question 2 [C]

A car is valued at \$30 000 new and depreciates to \$8000.

Unit cost depreciation is 25 cents for each kilometre travelled.

$$8000 = 30000 - 0.25k$$

$$0.25k = 22000$$

$$k = 88000$$

The car has travelled 88 000 km.

Question 3 [C]

$$r = 8 \quad t = 6 \quad I = 27000$$

The sum of money invested (P) is:

$$P = \frac{100I}{rt} = \frac{100 \times 27000}{8 \times 6} = \$56250$$

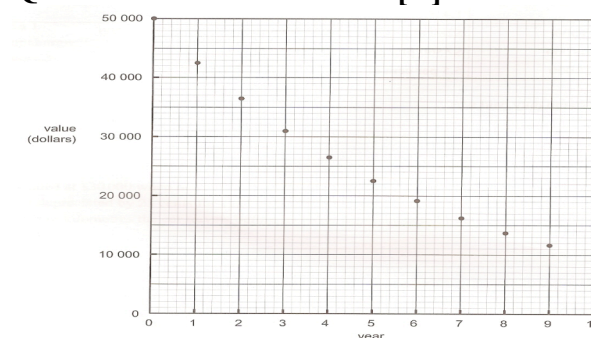
Question 4 [D]

The price of a property in 2006 was \$200000 and so it is in the price range \$1150001– \$868000.

Stamp Duty

$$= 2560 + \left(\frac{6}{100} \times 85000\right) = \$7660$$

Question 5 [E]



From the graph it can be seen that the amount of depreciation each year decreases over time.

Question 6 [D]

\$10000 is invested at a rate of 10% per annum compounding half yearly.

The investment after five years is given by:

$$A = 10000 \times (1.05)^{10}$$

Question 7 [E]

Joe's current salary is \$42 000.

Inflation last year was 2%

Last year Joe's salary was

$$\frac{42000}{1.02} = \$41176$$

The previous year, inflation was 3%.

$$\text{Joe's salary was } \frac{\$41176}{1.03} \approx \$39977$$

Question 8 [D]

A washing machine costs \$720.

To buy under hire purchase, a deposit of \$180 is required.

$$\text{So } P = 720 - 180 = 540$$

$$r = 12\% \quad t = 2 \text{ (24 months)}$$

$$I = \frac{Pr t}{100} = \frac{540 \times 12 \times 2}{100} = \$129.60$$

The monthly repayment is

$$\frac{540 + 129.60}{24} = \$27.90$$

Question 9 [A]

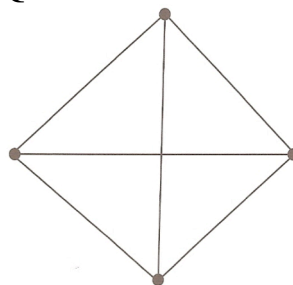
To calculate the monthly repayment

$N=240$
 $I\%=7$
 $PV=250000$
 $\blacksquare \text{ PMT} = -1938.2473...$
 $FV=0$
 $P/Y=12$
 $C/Y=12$
 $\text{PMT: } \blacksquare \text{ BEGIN}$

The total amount of interest paid on the loan is:

$$(240 \times 1938.25) - 250000$$

$$= \$215180 \approx \$215000$$

Module 5 Networks and decision mathematics**Question 1 [E]**

A tree is a connected subgraph that does not contain loops, cycles or parallel edges

This graph could not be described as a tree.

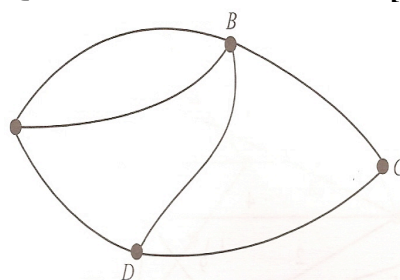
Question 2 [C]

A connected planar graph has 12 edges. Using Euler's Formula for planar graphs:

$$v - e + f = 2$$

Substituting $v = 6$ and $f = 8$

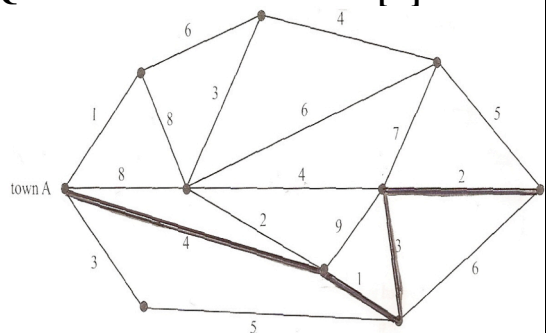
$$6 - 12 + 8 = 2$$

Question 3 [A]

An adjacency matrix to represent this graph could be

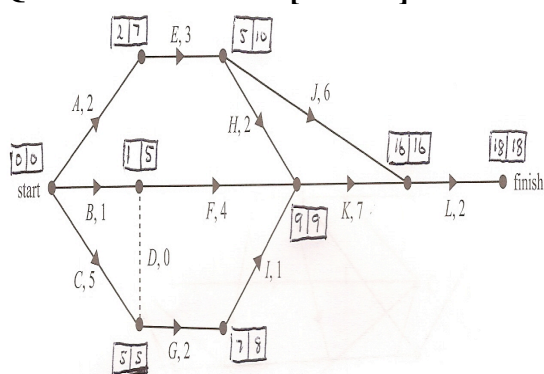
$$\begin{bmatrix} 0 & 2 & 0 & 1 \\ 2 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

Question 4 [B]



The shortest distance from town A to town B is $4 + 1 + 3 + 2 = 10$ km

Question 5 [D or E]



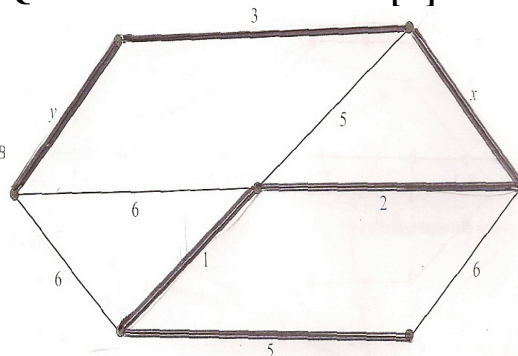
Two statements are incorrect.

D. The shortest path gives the minimum time for project completion. As the dummy path does not show its direction, it can be assumed that it can go either way. Consequently,
E. Activity L **must** be on the critical path. This is incorrect also.

Question 6 [E]

From the directed network in question 5, the earliest start time for activity L is 16 hours.

Question 7 [C]



The minimal spanning tree (using Prim's Algorithm) is shown on the diagram.

$$y + 3 + x + 1 + 2 + 5 = 19$$

$$x + y = 19 - 11 = 8$$

$$\text{So } x = 3 \text{ and } y = 5$$

Question 8 [B]

The one step matrix can be represented by the matrix :

$$[A] = \begin{bmatrix} 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

The two step dominances can be represented by the matrix:

$$[A]^2 = \begin{bmatrix} 0 & 1 & 2 & 0 & 1 \\ 0 & 0 & 3 & 1 & 2 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

So team B has the highest number of two step dominances.

Question 9 [B]

In 2007, there are 5 teams.

The total number of games played is:

$$\frac{n(n-1)}{2} = \frac{5 \times 4}{2} = 10$$

In 2008, there are 7 teams.

The total number of games played is:

$$\frac{n(n-1)}{2} = \frac{7 \times 6}{2} = 21$$

There are 11 extra games played in 2008.

Module 6 Matrices

Question 1 [A]

$$\begin{bmatrix} 0 & -4 \\ 2 & 5 \end{bmatrix} + \begin{bmatrix} 5 & 4 \\ -2 & 2 \end{bmatrix} = \begin{bmatrix} 0+5 & -4+4 \\ 2-2 & 5+2 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 0 & 7 \end{bmatrix}$$

Question 2 [D]

	2004	2005	2006
Oldtown	975	1002	1390
Newtown	2105	1081	1228
Twixtown	610	1095	1380

The number of tourists visiting the three towns in 2005 can be shown as:

$$\begin{bmatrix} 1002 \\ 1081 \\ 1095 \end{bmatrix}$$

Question 3 [B]

$$A = \begin{bmatrix} 8 & 4 \\ 5 & 3 \end{bmatrix} \text{ and } AX = \begin{bmatrix} 5 & 6 \\ 8 & 10 \end{bmatrix}$$

$$A^{-1}AX = A^{-1} \times \begin{bmatrix} 5 & 6 \\ 8 & 10 \end{bmatrix}$$

$$X = \begin{bmatrix} 0.75 & -1 \\ -1.25 & 2 \end{bmatrix} \times \begin{bmatrix} 5 & 6 \\ 8 & 10 \end{bmatrix}$$

$$= \begin{bmatrix} -4.25 & -5.5 \\ 9.75 & 12.5 \end{bmatrix}$$

Question 4 [B]

$$2x + z = 5$$

$$x - 2y = 0$$

$$y - z = -1$$

This system of three simultaneous equations can be written as:

$$\begin{bmatrix} 2 & 0 & 1 \\ 1 & -2 & 0 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \\ -1 \end{bmatrix}$$

Question 5 [A]

Table 1 – Number of students awarded prizes

Achievement level	Junior	Intermediate	Senior
Gold	21	12	10
Silver	16	18	14
Bronze	21	26	24

Table 2 shows the value, in dollars, of each prize.

Table 2 – Value of prizes (\$)

Achievement level	Junior	Intermediate	Senior
Gold	75	100	200
Silver	25	40	80
Bronze	10	15	20

A matrix that gives the total value of all the Silver prizes awarded is:

$$\begin{bmatrix} 25 & 40 & 80 \end{bmatrix} \begin{bmatrix} 16 \\ 18 \\ 14 \end{bmatrix}$$

Question 6 [D]

The number of bats on each location is

Location	Number of bats
A	1568
B	1105
C	894

The bats change feeding location according to the transition matrix: this night

$$T = \begin{bmatrix} 0.8 & 0.1 & 0.2 \\ 0.1 & 0.6 & 0.1 \\ 0.1 & 0.3 & 0.7 \end{bmatrix} \begin{matrix} A \\ B \\ C \end{matrix} \text{ next night}$$

In the long term, the steady state matrix is:

$$T^{52} = \begin{bmatrix} 0.8 & 0.1 & 0.2 \\ 0.1 & 0.6 & 0.1 \\ 0.1 & 0.3 & 0.7 \end{bmatrix}^{52} \begin{bmatrix} 1568 \\ 1105 \\ 894 \end{bmatrix}$$

$$= \begin{bmatrix} 1605.15 \\ 713.4 \\ 1248.45 \end{bmatrix}$$

$$T^{100} = \begin{bmatrix} 0.8 & 0.1 & 0.2 \\ 0.1 & 0.6 & 0.1 \\ 0.1 & 0.3 & 0.7 \end{bmatrix}^{100} \begin{bmatrix} 1568 \\ 1105 \\ 894 \end{bmatrix}$$

$$= \begin{bmatrix} 1605.15 \\ 713.4 \\ 1248.45 \end{bmatrix}$$

The number of bats feeding at location A in the long term will be 1605.

Question 7 [E]

Each year, a family goes on holiday to one of three places: Portland (P), Quambatook (Q) or Rochester (R), but they never go to the same place two years in a row.

This transition matrix could be used to model this situation:

		This year		
		P	Q	R
Next year	P	0	0	0.1
	Q	0.5	0	0.9
	R	0.5	1	0

If the family goes to P this year, next year they go to either Q or R. Note that the sum of the elements in each of the columns is 1. If the family goes to Q this year, they go to R next year. If the family goes to R this year, they go to either P or Q next year.

Question 8 [B]

Kerry sat for a multiple-choice test consisting of six questions.

Option D was chosen first and then he used the transition matrix below to answer the remaining five questions.

		This question			
		A	B	C	D
Next question	A	1	0	1	0
	B	0	0	0	1
	C	0	1	0	0
	D	0	0	0	0

The answers to the six questions are:

Question	1	2	3	4	5	6
Answer	D	B	C	A	A	A

Question 9 [C]

$M (NP)$.

Firstly, NP must be able to be done.

Its order is $(m \times n) \times (5 \times ?)$.

Therefore $n = 5$ as the number of rows of the second matrix must equal the number of columns of the first. The order of this answer is $m \times ?$.

Order of $M \times (NP)$

$= (3 \times 4) \times (m \times ?)$

Therefore $4 = m$ for the same reason as above.

N is a 4×5 matrix.