



The Mathematical Association of Victoria

SPECIALIST MATHEMATICS

2007 WRITTEN EXAMINATIONS

SOLUTIONS

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Question 1

$$2\sqrt{3} + 2i = rcis\theta$$

$$r = \sqrt{12 + 4}$$

$$= \sqrt{16}$$

$$= 4$$

$$\theta = \tan^{-1}\left(\frac{2}{2\sqrt{3}}\right)$$

$$= \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

$$= \frac{\pi}{6}$$

$$2\sqrt{3} + 2i = 4cis\left(\frac{\pi}{6}\right)$$

$$1 - \sqrt{3}i = rcis\theta$$

$$r = \sqrt{1 + 3}$$

$$= \sqrt{4}$$

$$= 2$$

$$\theta = \tan^{-1}(-\sqrt{3})$$

$$= -\frac{\pi}{3}$$

$$1 - \sqrt{3}i = 2cis\left(-\frac{\pi}{3}\right)$$

$$\frac{2\sqrt{3} + 2i}{1 - \sqrt{3}i} = \frac{4cis\left(\frac{\pi}{6}\right)}{2cis\left(-\frac{\pi}{3}\right)}$$

$$= 2cis\left(\frac{\pi}{6} + \frac{\pi}{3}\right)$$

$$= 2cis\left(\frac{\pi}{2}\right)$$

Question 2**a.**

$$0 = z^3 - (\sqrt{5} - i)z^2 + 4z - 4\sqrt{5} + 4i$$

$$= z^2(z - (\sqrt{5} - i)) + 4(z - (\sqrt{5} - i))$$

$$= (z^2 + 4)(z - (\sqrt{5} - i))$$

$$\therefore z - (\sqrt{5} - i) = 0$$

$$\Rightarrow z = \sqrt{5} - i$$

Thus $\sqrt{5} - i$ is a solution of

$$z^3 - (\sqrt{5} - i)z^2 + 4z - 4\sqrt{5} + 4i = 0$$

b.

From Question 2a

$$z^2 + 4 = 0$$

$$\therefore z = \pm 2i$$

Question 3

$$x^3 - 2x^2y + 2y^2 = 2$$

By implicit differentiation

$$3x^2 - 4xy - 2x^2 \frac{dy}{dx} + 4y \frac{dy}{dx} = 0$$

$$3x^2 - 4xy = 2x^2 \frac{dy}{dx} - 4y \frac{dy}{dx}$$

$$3x^2 - 4xy = \frac{dy}{dx}(2x^2 - 4y)$$

$$\frac{dy}{dx} = \frac{3x^2 - 4xy}{2x^2 - 4y}$$

At the point (2, 3)

$$\frac{dy}{dx} = \frac{12 - 24}{8 - 12} = 3$$

Equation of Tangent

$$y - y_1 = \frac{dy}{dx}(x - x_1)$$

$$y - 3 = 3(x - 2)$$

$$y = 3x - 3$$

Question 4

$$V = \pi \int_{-\frac{1}{2}}^0 y^2 dx$$

$$= \pi \int_{-\frac{1}{2}}^0 \left(\frac{1}{\sqrt{1-x^2}} \right)^2 dx$$

$$= \pi \int_{-\frac{1}{2}}^0 \frac{1}{1-x^2} dx$$

$$\begin{aligned} \frac{1}{1-x^2} &= \frac{A}{1-x} + \frac{B}{1+x} \\ &= \frac{A(1+x) + B(1-x)}{1-x^2} \end{aligned}$$

$$1 \equiv A(1+x) + B(1-x)$$

$$\text{at } x = -1, B = \frac{1}{2}$$

$$\text{at } x = 1, A = \frac{1}{2}$$

$$\begin{aligned} V &= \frac{\pi}{2} \int_{-\frac{1}{2}}^0 \frac{1}{1-x} + \frac{1}{1+x} dx \\ &= \frac{\pi}{2} \left[-\log_e |(1-x)| + \log_e |(1+x)| \right]_{-\frac{1}{2}}^0 \\ &= \frac{\pi}{2} \left[\log_e \left| \frac{1+x}{1-x} \right| \right]_{-\frac{1}{2}}^0 \\ &= \frac{\pi}{2} \left[\log_e 1 - \log_e \left(\frac{1-\frac{1}{2}}{1+\frac{1}{2}} \right) \right] \\ &= \frac{\pi}{2} \left(-\log_e \left(\frac{1}{3} \right) \right) \\ &= \frac{\pi}{2} \log_e 3 \end{aligned}$$

Question 5**a.**

Assuming constant acceleration

$$\text{Using } v^2 = u^2 - 2as$$

$$v = 0$$

$$u = 4$$

$$s = 3$$

$$\therefore 0^2 = 4^2 - 6a$$

$$a = -\frac{16}{6} = -\frac{8}{3} \text{ m/s}^2$$

b.

From Question 5 a, we know that

Friction (F)

$$F = ma$$

$$F = \frac{-8}{3} \times 6 = -16$$

$$F = \mu N = -6g\mu$$

$$-6g\mu = -16$$

$$\mu = \frac{16}{6g}$$

$$= \frac{8}{3g}$$

Question 6**a.**

$$\begin{aligned} \tilde{r}(t) &= \int \tilde{v}(t) dt \\ &= \int -4 \sin(2t) \tilde{i} + 6 \cos(2t) \tilde{j} dt \\ &= 2 \cos(2t) \tilde{i} + 3 \sin(2t) \tilde{j} + c \end{aligned}$$

$$\text{We are told that } \tilde{r}(0) = 2 \tilde{i}$$

$$2 \tilde{i} = 2 \cos(0) \tilde{i} + 3 \sin(0) \tilde{j} + c$$

$$c = 0$$

$$\therefore \tilde{r}(t) = 2 \cos(2t) \tilde{i} + 3 \sin(2t) \tilde{j}$$

b.

$$x = 2 \cos(2t)$$

$$y = 3 \sin(2t)$$

$$x^2 = 4 \cos^2(2t)$$

$$y^2 = 9 \sin^2(2t)$$

$$\frac{x^2}{4} = \cos^2(2t)$$

$$\frac{y^2}{9} = \sin^2(2t)$$

$$\frac{x^2}{4} + \frac{y^2}{9} = \cos^2(2t) + \sin^2(2t)$$

$$\frac{x^2}{4} + \frac{y^2}{9} = 1$$

We need to consider the domain

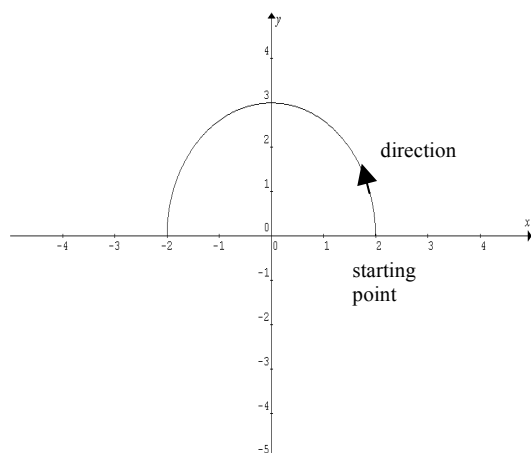
$$0 \leq t \leq \frac{\pi}{2}$$

This means that $-2 \leq x \leq 2$ and $0 \leq y \leq 3$

When $t = 0$, $x = 2$ and $y = 0$

When $t = \frac{\pi}{2}$, $x = -2$ and $y = 0$

When $t = \frac{\pi}{4}$, $x = 0$ and $y = 3$

c.**Question 7****a.**

$$y_{n+1} = y_n + h \frac{dy}{dx}$$

$$y_{n+1} = y_n + 0.1 \times \frac{1}{x_n}$$

$$x_0 = 1$$

$$y_0 = 1$$

$$x_1 = x_0 + 0.1 = 1.1$$

$$y_1 = y_0 + 0.1 \times \frac{1}{x_0} = 1 + 0.1 = 1.1$$

$$x_2 = x_1 + 0.1 = 1.1 + 0.1 = 1.2$$

$$y_2 = y_1 + 0.1 \times \frac{1}{x_1} = 1.1 + 0.1 \times \frac{1}{1.1} = \frac{1.31}{1.1} = \frac{131}{110}$$

b.

$$\frac{dy}{dx} = \frac{1}{x}$$

$$dy = \frac{dx}{x}$$

$$y = \int \frac{dx}{x}$$

$$= \log_e x + c$$

When $x = 1$, $y = 1$

$$y = \log_e x + c$$

$$1 = \log_e 1 + c$$

$$c = 1$$

$$y = \log_e x + 1$$

When $x = 1$

$$y = \log_e(1.2) + 1$$

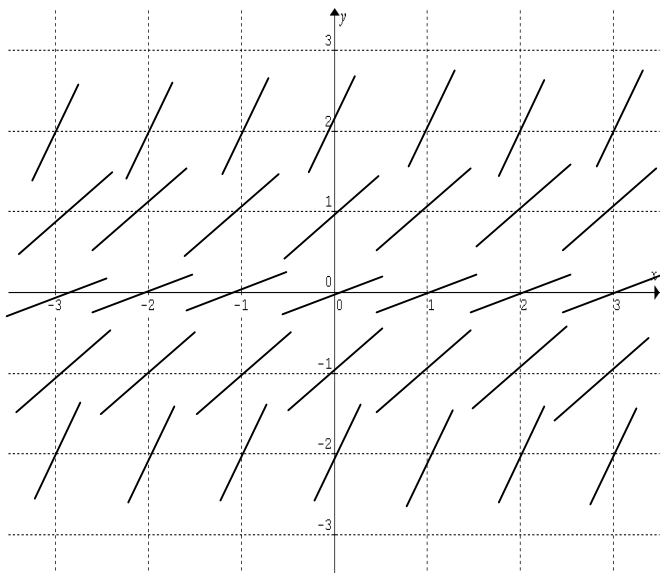
Question 8**a.**

Considering each of the y-values individually beginning with $y = -2$

$$\frac{dy}{dx} = \frac{1+y^2}{2} = \frac{1+(-2)^2}{2} = \frac{5}{2} = 2.5$$

Continuing this process for each value, we would gain the following table.

y	-2	-1	0	1	2
x	2.5	1	0.5	1	2.5

**b.**

$$\frac{dy}{dx} = \frac{1+y^2}{2}$$

$$\frac{dx}{dy} = \frac{2}{1+y^2}$$

$$x = 2 \int \frac{1}{1+y^2} dy$$

$$= 2 \tan^{-1} y + c$$

We are told that $y = -1$ when $x = 0$

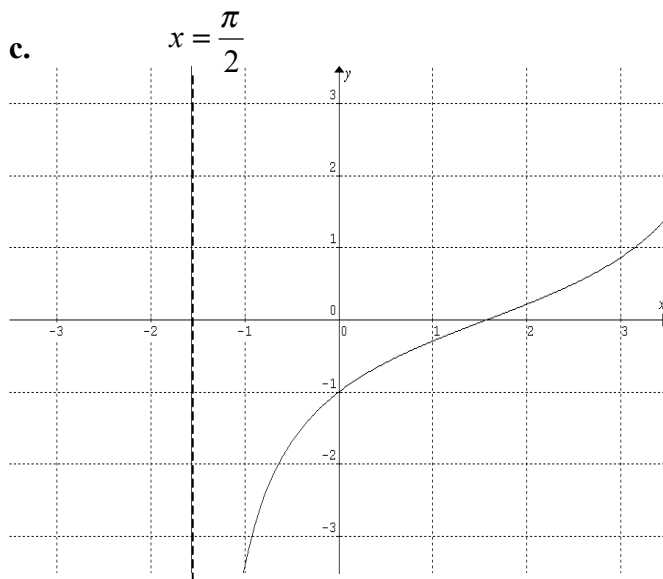
$$x = 2 \tan^{-1} y + c$$

$$0 = 2 \tan^{-1}(-1) + c$$

$$c = \frac{\pi}{2}$$

$$x = 2 \tan^{-1} y + \frac{\pi}{2}$$

$$y = \tan \frac{1}{2} \left(x - \frac{\pi}{2} \right)$$

**Question 9**

$$\tilde{v} = \frac{d\tilde{r}}{dt} = \frac{dx}{dt} \tilde{i} + \frac{dy}{dt} \tilde{j} = -y \tilde{i} + x \tilde{j}$$

$$\Rightarrow \frac{dx}{dt} = -y \quad \frac{dy}{dt} = x$$

$$\frac{d\tilde{v}}{dt} = -\frac{dy}{dt} \tilde{i} + \frac{dx}{dt} \tilde{j}$$

$$= -x \tilde{i} - y \tilde{j}$$

$$= -(x \tilde{i} + y \tilde{j})$$

$$= -\tilde{r}$$

Question 10

$$1 + \tan^2 2x = \sec^2 2x$$

$$1 + \left(\frac{4\sqrt{2}}{7}\right)^2 = \sec^2 2x$$

$$\frac{49 + 32}{49} = \frac{1}{\cos^2 2x}$$

$$\cos 2x = \frac{7}{9}$$

$$\cos 2x = 1 - 2\sin^2 x$$

$$\frac{7}{9} = 1 - 2\sin^2 x$$

$$2\sin^2 x = \frac{2}{9}$$

$$\sin^2 x = \frac{1}{9}$$

$$\sin x = \pm \frac{1}{3}$$

$$\text{Given } x \in [0, \frac{\pi}{4})$$

$$\sin x = \frac{1}{3}$$

$$z = -1 \pm \sqrt{2} \operatorname{cis}\left(\frac{\pi}{6}\right)$$

$$= -1 \pm \sqrt{2} \left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right)$$

$$= -1 \pm \left(\frac{\sqrt{6}}{2} + \frac{\sqrt{2}i}{2}\right)$$

$$= \frac{-2 + \sqrt{6}}{2} + \frac{\sqrt{2}i}{2}, \frac{-2 - \sqrt{6}}{2} - \frac{\sqrt{2}i}{2}$$