



The Mathematical Association of Victoria

MATHS METHODS (CAS)

2007 WRITTEN EXAMINATIONS

SOLUTIONS

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**2007 Mathematical Methods
Written Examination 1
Suggested answers and solutions**

Question 1

$$f(x) = \frac{x^3}{\sin(x)}$$

Using the Quotient Rule:
$$f'(x) = \frac{\sin(x) \cdot 3x^2 - x^3 \cdot \cos(x)}{\sin^2(x)}$$
$$= \frac{3x^2 \sin(x) - x^3 \cos(x)}{\sin^2(x)}$$

Question 2

a. $\log_e(3x+5) + \log_e(2) = 2$

$$\log_e 2(3x+5) = \log_e(e^2)$$

$$2(3x+5) = e^2 \text{ and so } 3x+5 = \frac{e^2}{2}$$

$$\text{Therefore } x = \frac{1}{3} \left(\frac{1}{2} e^2 - 5 \right) \text{ or } \frac{e^2 - 10}{6}.$$

b. $g(x) = \log_e(\tan(x))$

Let $u = \tan(x)$ and so $\frac{du}{dx} = \sec^2(x)$ which is the same as $\frac{1}{\cos^2(x)}$

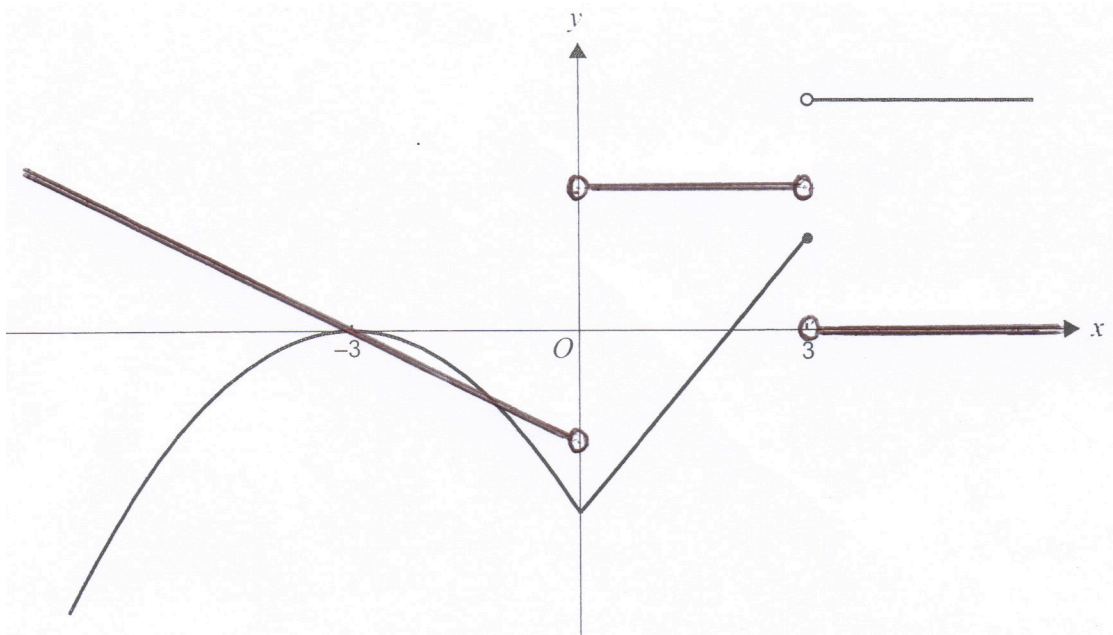
If $y = \log_e(u)$ then $\frac{dy}{du} = \frac{1}{u} = \frac{1}{\tan(x)}$

Now $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$ (Chain Rule)
$$= \frac{\sec^2(x)}{\tan(x)}$$

At $x = \frac{\pi}{4}$ we know that $\sec^2(\frac{\pi}{4}) = 2$ and $\tan(\frac{\pi}{4}) = 1$ and so $\frac{dy}{dx} = 2$.

Question 3

a.



b. The derivative will not exist at a discontinuous point (such as $x = 3$) or at a “sharp” point (such as $x = 0$).

The domain of the derivative function is $R / \{0, 3\}$

Alternatively using interval notation $(-\infty, 0) \cup (0, 3) \cup (3, \infty)$

Question 4

$$\frac{dV}{dt} = 8 \quad \text{and} \quad \frac{dV}{dx} = 6x^{\frac{1}{2}} = 6\sqrt{x}$$

$$\begin{aligned} \text{Now } \frac{dx}{dt} &= \frac{dx}{dV} \times \frac{dV}{dt} \\ &= \frac{8}{6\sqrt{x}} \end{aligned}$$

At $x = 4$ this value is $\frac{8}{12}$.

So the rate of change of the depth of the wine glass is $\frac{2}{3}$ cm/s.

Question 5

$\Pr(\text{More than two customers}) = \Pr(3 \text{ customers}) + \Pr(4 \text{ customers})$

The binomial distribution for this is $\left(\frac{1}{2} + \frac{1}{2}\right)^4$.

$$\begin{aligned}\Pr(3) + \Pr(4) &= {}^4C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^4 \\ &= \frac{4}{16} + \frac{1}{16}\end{aligned}$$

The probability that more than two customers will order a cup of coffee is $\frac{5}{16}$ or 0.3125.

Question 6

a. Consider the probability table (Karnaugh mapping)

	Pr(B)	Pr(B')	
Pr(A)	Pr(A ∩ B) $\frac{1}{8}$	Pr(A ∩ B')	$\frac{1}{5}$
Pr(A')	Pr(A' ∩ B) ?	Pr(A' ∩ B')	$\frac{4}{5}$
	$\frac{1}{3}$	$\frac{2}{3}$	1

Using column $\Pr(B)$: $\Pr(A' \cap B) + \frac{1}{8} = \frac{1}{3}$ and so $\Pr(A' \cap B) = \frac{1}{3} - \frac{1}{8}$ or $\frac{5}{24}$

b. If A and B are mutually exclusive then $\Pr(A \cap B) = 0$. The table now is:

	Pr(B)	Pr(B')	
Pr(A)	0		$\frac{1}{5}$
Pr(A')	?		$\frac{4}{5}$
	$\frac{1}{3}$	$\frac{2}{3}$	1

Using column $\Pr(B)$: $\Pr(A' \cap B) + 0 = \frac{1}{3}$ and so $\Pr(A' \cap B) = \frac{1}{3}$

N.B. Each of the above could have been done using Venn diagrams.

Question 7

Using the Product Rule: $\frac{d}{dx}(x \cos(3x)) = x \frac{d}{dx}(\cos(3x)) + \cos(3x) \frac{d}{dx}(x)$
 $= -3x \sin(3x) + \cos(3x)$

Taking the anti-derivative of both sides of this equation:

$$\int \left(\frac{d}{dx}(x \cos(3x)) \right) dx = \int -3x \sin(3x) dx + \int \cos(3x) dx$$

The integral of the derivative cancels out the calculus operations (back to the original function). Therefore $x \cos(3x) = \int -3x \sin(3x) dx + \left(\frac{1}{3} \sin(3x) \right)$

Re-arranging: $3 \int x \sin(3x) dx = \frac{1}{3} \sin(3x) - x \cos(3x)$

Dividing both sides by 3 gives: $\int x \sin(3x) dx = \frac{1}{9} \sin(3x) - \frac{1}{3} x \cos(3x)$

Note: No constant of integration (c) is necessary since the question asks for *an* anti-derivative. It is not wrong to add this if you wish.

Question 8

a. The reference angle for $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$ is $\frac{\pi}{3}$. If $0 \leq x \leq 3$ then $0 \leq \frac{2\pi x}{3} \leq 2\pi$.

$\sin\left(\frac{2\pi x}{3}\right) = -\frac{\sqrt{3}}{2}$ indicates that sine is negative and so the angle $\frac{2\pi x}{3}$ is in the 3rd and 4th quadrants.

$$\frac{2\pi x}{3} = \pi + \frac{\pi}{3} \text{ and } 2\pi - \frac{\pi}{3}$$

$$\frac{2\pi x}{3} = \frac{4\pi}{3} \text{ and } \frac{5\pi}{3}$$

Multiplying through by 3 gives: $2\pi x = 4\pi$ and 5π

Dividing through by 2π gives the answer: $x = 2$ and $\frac{5}{2}$.

b. $g(x) = 3f(x-1) + 2$

$$3f(x-1) + 2 = 3 \sin\left(\frac{2\pi}{3}(x-1)\right) + 2$$

Now sine is maximum when the angle is $\frac{\pi}{2}$.

$$\frac{2\pi}{3}(x-1) = \frac{\pi}{2} \text{ and so } \frac{2}{3}(x-1) = \frac{1}{2}$$

Answer: $x = \frac{7}{4}$

Question 9

- a. If $f(x) = e^{\frac{x}{2}} + 1$ then $f'(x) = 0.5e^{\frac{x}{2}}$

At $x = 0$, $f'(0) = 0.5$ and $f(0) = 2$

Hence the gradient of the normal is -2 ($m_1 m_2 = -1$ if two lines are perpendicular).

Equation of normal: $y - 2 = -2(x - 0)$

Answer: $y = -2x + 2$

b.

Method 1

Using the area of the triangle which lies below the shaded area.

$$\begin{aligned} \text{Area} &= \int_0^1 (e^{\frac{x}{2}} + 1) dx - \frac{1}{2} \times 2 \times 1 \\ &= \left[2e^{\frac{x}{2}} + x \right]_0^1 - 1 \\ &= (2e^{0.5} + 1) - (2e^0 + 0) - 1 \\ &= 2e^{0.5} + 1 - 2 - 1 \\ &= 2e^{0.5} - 2 \end{aligned}$$

Method 2

Area = $\int$ (upper curve - lower curve) dx

$$\begin{aligned} &= \int_0^1 ((e^{\frac{x}{2}} + 1) - (-2x + 2)) dx \\ &= \int_0^1 (e^{\frac{x}{2}} + 2x - 1) dx \\ &= \left[2e^{\frac{x}{2}} + x^2 - x \right]_0^1 \\ &= (2e^{0.5} + 1 - 1) - (2e^0 + 0 - 0) \\ &= 2e^{0.5} - 2 \end{aligned}$$

Question 10

As k is a positive constant, the graph of $y = kx^{0.5}$ is entirely above the x -axis for $x > 0$.

Therefore $27 = \int_0^9 kx^{0.5} dx$

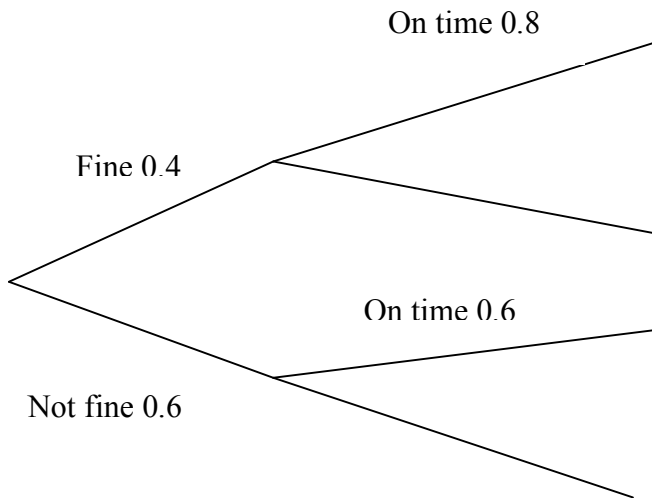
$$27 = \left[\frac{2}{3} kx^{1.5} \right]_0^9$$

$$27 = \frac{2}{3} k(9^{\frac{3}{2}} - 0)$$

$$27 = \frac{2}{3} k \times 3^3 \text{ or } 18k \text{ and so } k = 1.5$$

Question 11

a.



$$\begin{aligned}\text{Pr}(\text{on time}) &= 0.4 \times 0.8 + 0.6 \times 0.6 \\ &= 0.32 + 0.36 \\ &= 0.68\end{aligned}$$

b. $\frac{0.32}{0.68} = \frac{32}{68}$ or $\frac{8}{17}$

Question 12

Method 1

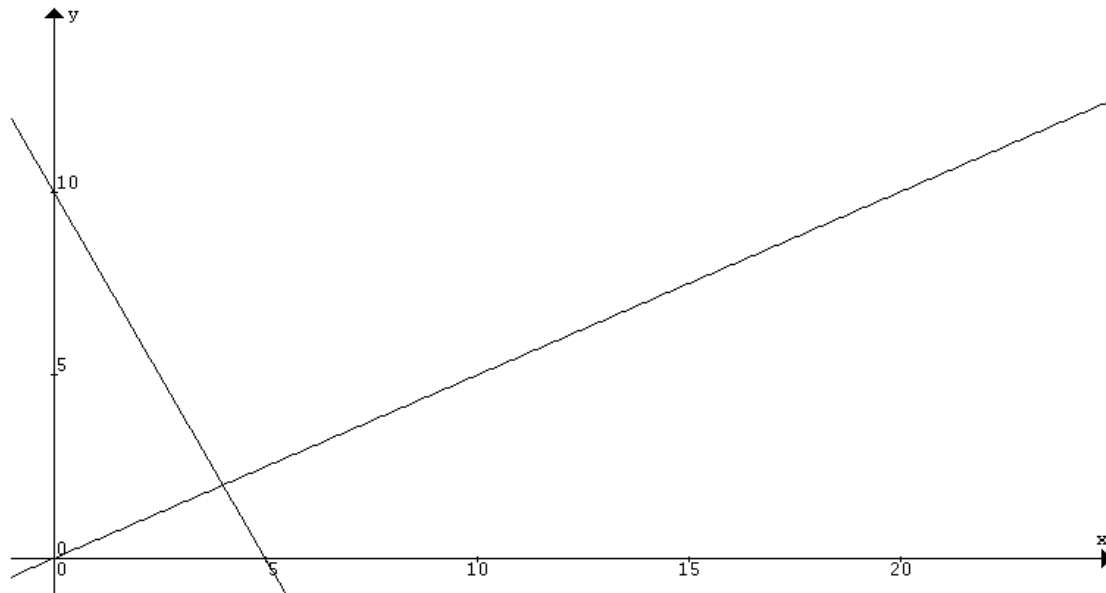
The gradient of line $2x + y - 10 = 0$ is -2 . The shortest line from the origin to this line will be perpendicular to it and so have a gradient of $\frac{1}{2}$. Its equation is $y = \frac{x}{2}$.

Solve $2x + y - 10 = 0$ and $y = \frac{x}{2}$ simultaneously to find the intersection point.

Therefore $2x + \frac{x}{2} - 10 = 0$ and so $x = 4$.

Since $x = 4$ and $y = \frac{x}{2}$ then $y = 2$.

The coordinates of P are $(4, 2)$ and the distance from the origin to P is $\sqrt{20}$ or $2\sqrt{5}$.

Method 2

Let the coordinates of the point P be $(x, 10 - 2x)$.

$$\begin{aligned}\text{The length of the line OP} &= \sqrt{(x-0)^2 + (10-2x-0)^2} \\ &= \sqrt{x^2 + (10-2x)^2} \\ &= \sqrt{5x^2 - 40x + 100}\end{aligned}$$

$$\text{If } L = \sqrt{5x^2 - 40x + 100} \text{ then } \frac{dL}{dx} = \frac{10x - 40}{2\sqrt{5x^2 - 40x + 100}}$$

$$\frac{dL}{dx} = 0 \text{ if } 5x - 20 = 0$$

Therefore $x = 4$ and the coordinates of P are $(4, 2)$.

Let $x < 4$ (say 3)	Let $x = 4$	Let $x > 4$ (say 5)
$\frac{dL}{dx} = -\frac{10}{10} = -1$	$\frac{dL}{dx} = 0$	$\frac{dL}{dx} = \frac{10}{10} = 1$
Slope : Negative	Slope : Zero	Slope : Positive

Shape of a minimum turning point.

The minimum distance is $\sqrt{20}$ or $2\sqrt{5}$.

