

Student Name: _____

MATHEMATICAL METHODS (CAS)

Unit 2 – Written examination 2



2007 Trial Examination

Reading Time: 15 minutes

Writing Time: 2 hours

QUESTION AND ANSWER BOOK

Structure of book

<i>Section</i>	<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
1	25	25	25
2	4	4	55
			Total 80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, and rulers, a protractor, set-squares, aids for curve sketching, one bound reference, one approved CAS calculator (memory DOES NOT need to be cleared) and if desired, one scientific calculator. For approved computer based CAS, their full functionality may be used.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer book of 18 pages.

Instructions

- Print your name in the space provided on the top of this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

SECTION 1

Instructions for Section 1

Choose the response that is **correct** for the question.

A correct answer scores 1, an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

No marks will be given if more than one answer is completed for any question.

Question 1

Steve ran 15 km due East, then 5 km due North. What is his true bearing from his starting point?

- A. $60^{\circ}32'$
- B. $19^{\circ}28'$
- C. $71^{\circ}34'$
- D. $18^{\circ}25'$
- E. $31^{\circ}10'$

Question 2

When you convert 1.5894 radians into degrees, what is the value of the minutes?

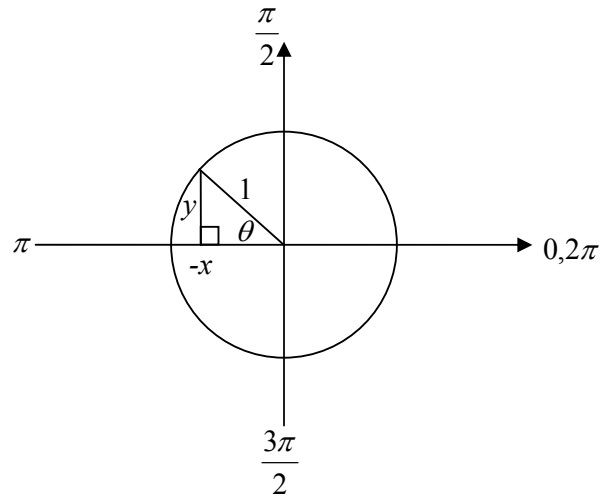
- A. 3
- B. 57
- C. 91
- D. 3.95
- E. 5

Question 3

The number of solutions to $\frac{7}{2}\tan(2x) = -1$ for $0 \leq x \leq 4\pi$ is

- A. 8
- B. 7
- C. 6
- D. 5
- E. 4

SECTION 1- continued

Question 4

If θ is an angle on the unit circle above, where $\frac{\pi}{2} \leq \theta \leq \pi$, and $\sin \theta = y$, then the value of $\cos \theta$ is

- A. x
- B. $-\frac{y}{x}$
- C. $\frac{y}{x}$
- D. $-x$
- E. $-y$

Question 5

A graph of the form $y = a \times \sin(bx) + k$ has an amplitude of 4, a period of π , and is centred along the line $y = -2$. The actual equation of the graph is

- A. $y = -2\sin(2x) + 4$
- B. $y = 4\sin(2x) - 2$
- C. $y = 4\sin(2x) + 2$
- D. $y = 4\sin(\pi x) - 2$
- E. $y = -2\sin(2x) + 2$

SECTION 1- continued
TURN OVER

Question 6

Solving the equation $4^{\frac{n}{2}} \times 3^n - 1 \times 6^{1-n} = 0$ yields

- A. 6
- B. -6
- C. 2
- D. $-\frac{1}{2}$
- E. $\frac{1}{2}$

Question 7

Two points along the function $f(x) = 2x^2$ are $(1, f(1))$ and $(1+2h, f(1+2h))$. The gradient of the secant between these two points, in terms of h , is

- A. $4(h+1)$
- B. $-4h+2$
- C. $4(h-1)$
- D. $4h+2$
- E. $-4(h+1)$

Question 8

The simplified form of $\frac{(xz^2)^4}{2x^3} \div \frac{xy}{2z^2}$ is

- A. $\frac{z^6}{4y}$
- B. $\frac{x^6 z^{10}}{y}$
- C. $\frac{z^{10}}{y}$
- D. $\frac{z^6}{y}$
- E. $\frac{z^{18}}{x^3 y}$

Question 9

Which of the following is **not** correct about the graphs of $y_1 = 2^x + 1$ and $y_2 = 3^x + 1$?

- A. Both of their domains are all positive real numbers.
- B. For all negative numbers, y_2 is below y_1 .
- C. They each have the point $(0,2)$ in common.
- D. The graph of y_2 has a steeper gradient as the value of x increases, for all values of x .
- E. For all positive numbers, y_1 is below y_2 .

Question 10

Simplifying $\log_{10} 1000 - (\log_{10} x + \log_{10} (y^2))$ yields

- A. $\log_{10} \left(\frac{2y}{1000x} \right)$
- B. $3 - \log_{10} (xy^2)$
- C. $\frac{3}{\log_{10} (xy^2)}$
- D. $\log_{10} \left(\frac{1000x}{y^2} \right)$
- E. $\log_{10} \left(\frac{y^2}{1000x} \right)$

Question 11

Simplify $2\log_5 125$ to give

- A. $\frac{2}{3}$
- B. 9
- C. 1.5
- D. 6
- E. 5

Question 12

Which of the following is correct about the graphs of $y_1 = 2\log_{10} x$ and $y_2 = \frac{2}{5}\log_{10} (x^2)$?

- A. Both of their domains are all positive real numbers.
- B. Both have an asymptote at $y = 0$
- C. They both intersect the y -axis at $(0,-1)$
- D. The graph of y_2 has a steeper gradient as the value of x increases.
- E. They each have the point $(1,0)$ in common.

SECTION 1- continued
TURN OVER

Question 13

A population of tunicates ('sea squirts') undergoes exponential population growth at certain times of the year. If the population can double every 40 hours, and it initially has 4000 members, then the equation that can model this growth is given by

- A. $P(t) = 4000 \times 2^{40t}$
- B. $P(t) = 4000 \times 40^{2t}$
- C. $P(t) = 4000 \times 2^{0.025t}$
- D. $P(t) = 4000 \times 40^{0.025t}$
- E. $P(t) = 4000 \times 40^{40t}$

Question 14

Which of the following statements is correct about $\lim_{x \rightarrow 0} \left(\frac{2}{x} \right)$?

- A. The hyperbola approaches 0 from the left and the right
- B. As you approach from the left, the limit becomes ∞
- C. The limit is 0
- D. As you approach from the right, the limit becomes $-\infty$
- E. The limit is undefined at $x = 0$

Question 15

The gradient function of $\sqrt{3}x^3 - 7x^2 + \frac{1}{2}x - 1$ is

- A. $3\sqrt{3}x^2 - 14x + \frac{1}{2}$
- B. $\sqrt{3}x^2 - 7x + \frac{1}{2}$
- C. $3\sqrt{3}x^3 - 14x^2 + \frac{1}{2}x - 1$
- D. 0
- E. $3\sqrt{3}x^2 + 14x - \frac{1}{2}$

Question 16

$\frac{dV}{dr}$ of $V = \frac{1}{3}\pi r^3$ is

- A. $\frac{1}{9}\pi r^2$
- B. πr^3
- C. $3\pi r^2$
- D. $\frac{1}{3}\pi r^2$
- E. πr^2

Question 17

If $A = 1 + \frac{2}{3}\pi r^2$, then $A'(5)$ is

- A. $\frac{100}{3}\pi$
- B. $\frac{75}{3}\pi$
- C. $\frac{20}{3}\pi$
- D. $\frac{20}{3}$
- E. $\frac{75}{3}$

Question 18

The function $g(x) = 3x^3 - 2x^2 - 4$ has stationary points when the value of x is

- A. 0 and $\frac{9}{4}$
- B. 0 and $\frac{4}{9}$
- C. -4 and 0
- D. -9 and 0
- E. 4 and 9

**SECTION 1- continued
TURN OVER**

Question 19

Two positive real numbers have a sum of 40. The largest possible product of these numbers is

- A. 20
- B. 800
- C. 40
- D. 400
- E. 80

Question 20

If $f'(x) = 3x^2 - 2x + 7$, and $f(1) = 2$ then $f(x)$ equals

- A. $x^3 - x^2 + 7x - 5$
- B. $x^3 - x^2 + 7x - 2$
- C. $x^3 - x^2 + 7x$
- D. $x^3 - x^2 + 7x + 5$
- E. $x^3 - x^2 + 7x + 2$

Question 21

Simplified, $3 \times \frac{5!}{4!} - 3!$ becomes

- A. 20
- B. $-\frac{15}{2}$
- C. -1
- D. 9
- E. $-\frac{9}{4}$

Question 22

How many ways can you arrange 5 books from a set of 8?

- A. 120
- B. 6720
- C. 40320
- D. 56
- E. 40

Question 23

How many ways can you select 5 balls from a bag of 10, each a different colour?

- A. 252
- B. 510
- C. 120
- D. 210
- E. 30240

SECTION 1- continued

Question 24

A group has to elect four members to a Committee. What is the probability that out of a group of 8 women and 5 men, exactly 3 women are chosen (to two decimal places)?

- A. 0.08
- B. 0.39
- C. 0.63
- D. 0.10
- E. 0.24

Question 25

There are six trophies of various heights on a mantelpiece over Jack's fireplace. He wants to rearrange them, but only wants to do this once. If he randomly repositions the new trophies, what is the probability that they will be in order from largest to smallest (to three decimal places)?

- A. 0.167
- B. 0.008
- C. 0.139
- D. 0.028
- E. 0.001

**END OF SECTION 1
TURN OVER**

SECTION 2**Instructions for Section 2**

A decimal approximation will not be accepted if an **exact** answer is required to a question.

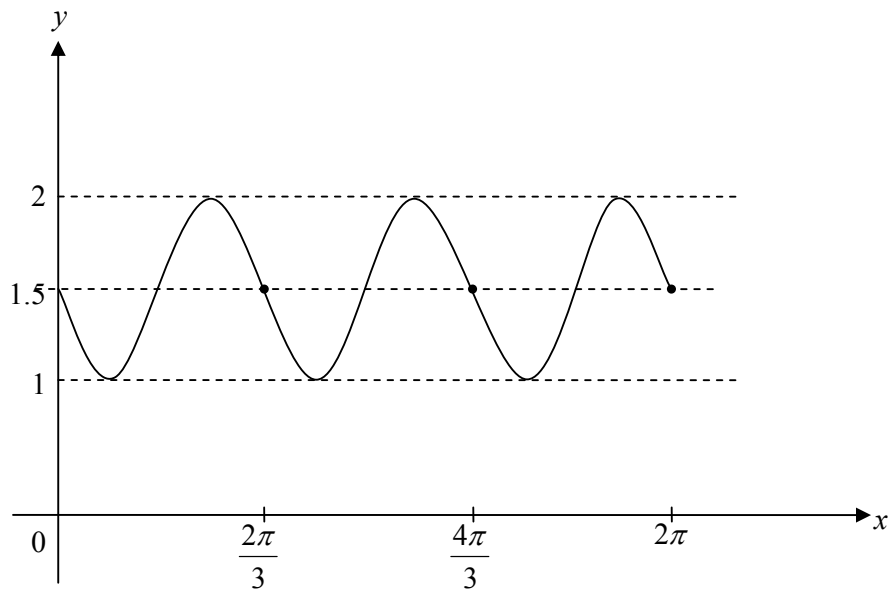
In questions worth more than one mark, appropriate working **must** be shown.

Marks are given as specified for each question.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

The following function has equation of the form $y = a \sin(bx) + c$.



- a. Determine the values of a , b and c . Hence, state the equation of the function.

5 marks

SECTION 2- Question 2- continued

b. Explain how each of the values affected the sine function in this case:

i. a : _____

ii. b : _____

iii. k : _____

1 + 1 + 1 = 3 marks

These kinds of functions do have uses in real life. For example, the height of water above the floor of a small bay can be modelled by the function $y = 2\cos\left(\frac{\pi t}{12}\right) + 5$, where $t = 0$ is midnight.

c. Determine the range of this new function.

2 marks

d. When does the water height first reach 4 metres?

1 mark

Total 11 marks

TURN OVER
SECTION 2- continued

Question 2

The amount of spores produced by a particular type of fungus depends upon how many days the fungus has spent in the reproductive cycle of its life. Studies have shown that the fungus starts by producing 2000 spores, but by the end of the week, it produces a massive 7.5×10^{10} spores. Assume that the increase in spores can be modelled by the equation $P = A \times 10^{kt}$, where P is the number of spores after t days.

- a. Determine the value of k to three decimal places. Show your working.

3 marks

- b. Calculate how many days and hours it will take this fungus to first be capable of producing 1 million spores.

2 marks

- c. How many spores will be present after 10 days? Use scientific notation to three decimal places to help approximate your answer.

2 marks

SECTION 2- Question 3- continued

A second species of fungus also shows an exponential increase in the number of spores. It starts out producing 200 spores, but by the end of a week produces 1.5×10^5 spores. It can be modelled by the equation $P = A \times b^{3.5t}$

- d. Determine the value of b to three decimal places. Show your working.

2 marks

- e. Calculate how many days and hours it will take this new fungus to produce 10 million spores.

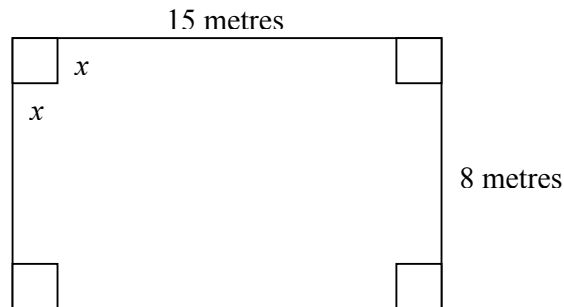
2 marks

Total 11 marks

SECTION 2- continued
TURN OVER

Question 3

A shipping company has received a large supply of steel sheeting that can be cut and made into boxes of various sizes. Each sheet measures 8 metres by 15 metres. To create a box, workers cut out squares at each of the corners which are x metres in size (see the diagram below):



- a. Determine the equation for the volume of the box for any size squares, x metres, cut from each corner. Call this V .

2 marks

- b. Showing your working at each step, use calculus to determine the maximum volume possible, to the nearest tenth of a cubic metre.

5 marks

SECTION 2- Question 4- continued

- c. Verify that the turning point you used is a maximum by using the first derivative test.

2 marks

- d. State the dimensions required to achieve the volume found in **part b** to the nearest tenth of a metre.

3 marks

Total 12 marks

SECTION 2- continued
TURN OVER

Question 4

A rock is thrown vertically upwards, with a velocity of $v(t) = 20 - 9.8t$ m/s. The only force acting on the rock is gravity at 9.8 m/s^2 , which is included in the velocity function. Assume that the rock started at ground level.

- a. Determine the height function, $h(t)$, of the rock.

2 marks

- b. Find the maximum height that the rock achieves, to the nearest tenth of a metre.

3 marks

- c. Determine how long the rock is in the air, to the nearest tenth of a second.

2 marks

- d. Find the final velocity of the rock (when the rock returns to ground level).

2 marks

Total 9 marks

SECTION 2- continued

Question 5

Nelson Automotive Company is in crisis. They are being forced to downsize their workforce due to economic pressures. The board has decided to rely on two main models of Nelson cars: the sporty Athena and the family-oriented Aries. The marketing department has come up with projected sales figures over the next three years, based on stable economic conditions (after the down-sizing), and a low number of warranty claims. The board is very interested in the figures regarding people whom lease cars, rather than buying them. In fact, there is a significantly high proportion of people who lease a different car every year. Research has projected that of those who lease an Athena in the first year, 74% of them lease a new one the following year (26% convert to an Aries). Also, of those who lease an Aries in the first year, 92% of them lease a new one the following year (8% convert to an Athena).

- a. Write down the transition matrix, T that can be used from the information given.

2 marks

At the start of this year, 2450 Athenas and 7400 Aries cars were leased.

- b. Write down the matrix showing the initial state, S_0 .

1 mark

- c. Determine how many of each car will be leased as new models after the first year. Clearly show your working.

3 marks

SECTION 2- Question 5- continued
TURN OVER

The board are interested in the long term, so they have asked for projections after 5 years.

- d. Determine how many of each car will be leased as new models after the first five years.
Hence, find S_5 .

6 marks
Total 12 marks

END OF QUESTION AND ANSWER BOOK