

MATHEMATICAL METHODS (CAS)

Unit 2 – Written examination 2



2007 Trial Examination

SOLUTIONS

SECTION 1: Multiple-choice questions (1 mark each)

Question 1

Answer: C

Explanation:

This is a right triangle, so use SOHCAHTOA:

$$\tan \theta = \frac{5}{15} \Rightarrow \theta = \tan^{-1}\left(\frac{1}{3}\right) \approx 18.43^\circ. \text{ Now } 90 - 18.43^\circ = 71^\circ 34' \text{ True.}$$

Question 2

Answer: D

Explanation:

$$1.5894 \times \frac{180}{\pi} \approx 91^\circ 3.95'$$

Question 3

Answer: A

Explanation:

Using CAS technology, there are 8 solutions in the specified domain.

Question 4

Answer: D

Explanation:

Since $\sin \theta = y$, it means that the triangle is on the unit circle, with hypotenuse of 1. Therefore, $\cos \theta = -x$, as it is in the 2nd quadrant.

Question 5

Answer: B

Explanation:

This means that $a = 4, k = -2$ and $b = \frac{2\pi}{\pi} = 2$, so the equation is $y = 4\sin(2x) - 2$

Question 6

Answer: E

Explanation:

$$4^{\frac{n}{2}} \times 3^n - 1 \times 6^{1-n} = 2^n \times 3^n - \frac{6}{6^n} = 6^n - \frac{6}{6^n} = 36^n - 6, \text{ therefore, } n = \frac{1}{2}$$

Question 7

Answer: A

Explanation:

Using CAS technology, the solution to $\frac{f(1+2h) - f(1)}{(1+2h) - 1} = 4(h+1)$

Question 8

Answer: C

Explanation:

$$\frac{(xz^2)^4}{2x^3} \div \frac{xy}{2z^2} = \frac{2x^4 z^{10}}{2x^4 y} = \frac{z^{10}}{y}$$

Question 9

Answer: A

Explanation:

They both have domains which include negative real numbers also.

Question 10

Answer: B

Explanation:

$$\log_{10} 1000 - (\log_{10} x + \log_{10} (y^2)) = 3 - \log_{10} (xy^2)$$

Question 11

Answer: D

Explanation:

$$2\log_5 125 \text{ becomes } 2(3) = 6$$

Question 12

Answer: E

Explanation:

They both have the point (1, 0) in common with each other.

Question 13

Answer: C

Explanation:

For the exponential growth equation $P(t) = Ab^{kt}$, $A = 4000$ and $b = 2$.

Solve $8000 = 4000 \times 2^{40k}$, to get $k = 0.025$. The equation is $P(t) = 4000 \times 2^{0.025t}$

Question 14

Answer: E

Explanation:

The limit is undefined at $x = 0$

Question 15

Answer: A

Explanation:

$$f'(x) = \sqrt{3}(3)x^2 - 7(2)x + \frac{1}{2}(0) - 0 = 3\sqrt{3}x^2 - 14x + \frac{1}{2}$$

Question 16

Answer: E

Explanation:

$$\frac{dV}{dr} = \pi r^2$$

Question 17

Answer: C

Explanation:

$$A'(r) = \frac{4}{3}\pi r, \text{ so } A'(5) = \frac{4}{3}\pi(5) = \frac{20}{3}\pi$$

Question 18

Answer: B

Explanation:

$$g'(x) = 9x^2 - 4x, \text{ so solve } 9x^2 - 4x = 0 \text{ to get } x = 0 \text{ and } x = \frac{4}{9}$$

Question 19

Answer: D

Explanation:

$x + y = 40$, so $y = 40 - x$. Now $P = xy$, which becomes $P = x(40 - x)$.
It follows that $P'(x) = 40 - 2x$, so there is a maximum at $x = 20$.
 $P(20) = 20(40 - 20) = 400$

Question 20

Answer: A

Explanation:

$$f(x) = \int (3x^2 - 2x + 7)dx = x^3 - x^2 + 7x + c$$

Solve $2 = 1 - 1 + 7 + c$, where $c = -5$, so that $f(x) = x^3 - x^2 + 7x - 5$

Question 21

Answer: D

Explanation:

$$3 \times \frac{5 \times 4 \times 3 \times 2 \times 1}{4 \times 3 \times 2 \times 1} - 3 \times 2 \times 1 = 3 \times 5 - 6 = 9$$

Question 22

Answer: B

Explanation:

$${}^8P_5 = 6720$$

Question 23

Answer: A

Explanation:

$${}^{10}C_5 = 252$$

Question 24

Answer: B

Explanation:

$$\frac{{}^8C_3 \times {}^5C_1}{{}^{13}C_4} = \frac{56 \times 5}{715} = \frac{280}{715} \approx 0.39$$

Question 25

Answer: E

Explanation:

$$\frac{1}{{}^6P_6} = \frac{1}{720} \approx 0.001$$

SECTION 2: Analysis questions

Question 1

a. Amplitude of $\frac{1}{2}$

M1

(negative as it is inverted), so $a = -0.5$.

M1

Shifted upwards 1.5 unit, so $c = 1.5$

M1

New period is used to find b : $\frac{2\pi}{3} = \frac{2\pi}{b}$, so $b = 3$

M1

Therefore, the equation is $y = -0.5\sin(3x) + 1.5$

A1

b.

i. Changes the amplitude (amplitude = a). In this case a reflection on the x -axis and a dilation of 0.5 from the x -axis.

A1

ii. Changes the period (new period = $\frac{2\pi}{b}$). In this case a dilation of $\frac{1}{3}$ from the y -axis.

A1

iii. Shifts the graph c units up (if positive) or down (if negative). In this case up 1.5 units in the y -direction.

A1

c. Remember that the maximum range of cosine is normally $[-1, 1]$, so, the maximum value is $y_{\max} = 2(1) + 5 = 7$ m

M1

The minimum value is: $y_{\min} = 2(-1) + 5 = 3$ m

A1

d. Solve $4 = 2\cos\left(\frac{\pi t}{12}\right) + 5$, to get $t = 8$ h (or 8AM)

A1

Total 11 marks

Question 2

a. $A = 2000$, so $P(t) = 2000 \times 10^{kt}$

A1

Solve: $75,000,000,000 = 2000 \times 10^{7k}$,

M1

to get $k \approx 1.082$

A1

b. Solve: $1,000,000 = 2000 \times 10^{1.082t}$,

M1

to get $t \approx 2.494$ days, which is 2 days and 12 hours (approximately)

A1

c. Solve: $P = 2000 \times 10^{1.082(10)}$,

M1

to get $P \approx 1.32 \times 10^{14}$ spores

A1

d. $A = 200$, so $P(t) = 200 \times b^{3.5t}$

Solve: $150,000 = 200 \times b^{3.5(7)}$,

M1

to get $b \approx 1.310$

A1

e. Solve: $10,000,000 = 200 \times 1.31^{3.5t}$,

M1

to get $t \approx 11.45$ days, which is 11 days and 11 hours (approximately)

A1

Total 11 marks

Question 3

a. Since $V = l \times w \times h$, and $l = 15 - 2x$, $w = 8 - 2x$ and $h = x$,

M1

it follows that $V = x(15 - 2x)(8 - 2x)$ or $V = 4x^3 - 46x^2 + 120x$

A1

b. $\frac{dV}{dx} = 12x^2 - 92x + 120$

M1

Solve for $12x^2 - 92x + 120 = 0$

M1

to get $x = \frac{5}{3}$ and $x = 6$ (we cannot use the second answer, it is outside of our domain for a positive volume)

M1

$$V\left(\frac{5}{3}\right) = 4\left(\frac{5}{3}\right)^3 - 46\left(\frac{5}{3}\right)^2 + 120\left(\frac{5}{3}\right) \\ \approx 90.7 \text{ m}^3$$

M1

A1

c. $V'(1) = 4 - 46 + 120 = 78$ and $V(2) = 48 - 184 + 120 = -16$

M1

Therefore, there is a local maximum at $(1.67, 90.3)$

A1

d. Length: $15 - 2\left(\frac{5}{3}\right) \approx 11.7 \text{ m}$

A1

Width: $8 - 2\left(\frac{5}{3}\right) \approx 4.7 \text{ m}$

A1

Height: $1.67 \text{ m} \approx 1.7 \text{ m}$

A1

Total 12 marks

Question 4

a. $h(t) = \int (20 - 9.8t)dt = 20t - 4.9t^2 + c$

M1

Since it was on the ground at rest: $c = 0$, therefore, the final equation is $h(t) = 20t - 4.9t^2$

A1

- b.** To find maximum height, find the time when the velocity equals zero.
 $20 - 9.8t = 0$

M1

Therefore, $t = 2.04$ seconds

M1

$$h(2.04) = 20(2.04) - 4.9(2.04)^2 \approx 20.4 \text{ metres}$$

A1

- c.** The time in the air will be twice that of the time taken to reach maximum

M1

height. Therefore, $t = 2(2.04) = 4.08$ seconds

A1

- d.** Hence, find $v(4.08) = 20 - 9.8(4.08)$

M1

$$= -20 \text{ m/s}$$

A1

Total 9 marks

Question 5

a. $T = \begin{bmatrix} 0.74 & 0.08 \\ 0.26 & 0.92 \end{bmatrix}$

M1, A1

b. $s_0 = \begin{bmatrix} 2450 \\ 7400 \end{bmatrix}$

A1

c. $s_1 = T \times s_0$

M1

$$= \begin{bmatrix} 0.74 & 0.08 \\ 0.26 & 0.92 \end{bmatrix} \times \begin{bmatrix} 2450 \\ 7400 \end{bmatrix}$$

M1

$$= \begin{bmatrix} 2405 \\ 7445 \end{bmatrix}$$

Therefore, 2405 Athenas **and** 7445 Aries will be leased as new models after the first year. A1

d. $s_2 = \begin{bmatrix} 0.74 & 0.08 \\ 0.26 & 0.92 \end{bmatrix} \times \begin{bmatrix} 2405 \\ 7445 \end{bmatrix} = \begin{bmatrix} 2375.3 \\ 7474.7 \end{bmatrix}$

M1

$$s_3 = \begin{bmatrix} 0.74 & 0.08 \\ 0.26 & 0.92 \end{bmatrix} \times \begin{bmatrix} 2375.3 \\ 7474.7 \end{bmatrix} = \begin{bmatrix} 2355.698 \\ 7494.302 \end{bmatrix}$$

M1

$$s_4 = \begin{bmatrix} 0.74 & 0.08 \\ 0.26 & 0.92 \end{bmatrix} \times \begin{bmatrix} 2355.698 \\ 7494.302 \end{bmatrix} = \begin{bmatrix} 2342.76068 \\ 7507.23932 \end{bmatrix}$$

M1

$$s_5 = \begin{bmatrix} 0.74 & 0.08 \\ 0.26 & 0.92 \end{bmatrix} \times \begin{bmatrix} 2342.76068 \\ 7507.23932 \end{bmatrix} = \begin{bmatrix} 2334.222049 \\ 7515.777951 \end{bmatrix}$$

M1

Therefore, after five years, they are leasing 2334 Athenas **and** 7516 Aries.

A2

Total 12 marks