

Student Name: _____

MATHEMATICAL METHODS (CAS)

Unit 1 – Written examination 2



2007 Trial Examination

Reading Time: 15 minutes

Writing Time: 2 hours

QUESTION AND ANSWER BOOK

Structure of book

<i>Section</i>	<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
1	26	26	26
2	4	4	54
			Total 80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, and rulers, a protractor, set-squares, aids for curve sketching, one bound reference, one approved CAS calculator (memory DOES NOT need to be cleared) and if desired, one scientific calculator. For approved computer based CAS, their full functionality may be used.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer book of 19 pages.

Instructions

- Print your name in the space provided on the top of this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

SECTION 1**Instructions for Section 1**

Choose the response that is **correct** for the question.

A correct answer scores 1, an incorrect answer scores 0.

Marks will **not** deducted for incorrect answers.

No marks will be given if more than one answer is completed for any question.

Question 1

What is the equation of the line passing through the origin and (5,10)

- A. $y = 2x$
- B. $y = 0.5x$
- C. $y = -.5x$
- D. $y = -2x$
- E. $y = x$

Question 2

Which of the following lines is parallel to $y = -3x + \frac{4}{3}$

- A. $y = \frac{1}{3}x + 3$
- B. $y = -\frac{1}{3}x - 2$
- C. $y = 3x + 1$
- D. $y = \frac{1}{3}x + \frac{4}{3}$
- E. $y = -3x - 5$

Question 3

The midpoint between the coordinates $A\left(\frac{1}{2}, 3\right)$ and $B\left(\frac{3}{2}, 8\right)$ is

- A. $\left(2, \frac{11}{2}\right)$
- B. $\left(1, \frac{11}{2}\right)$
- C. (2, 11)
- D. (1, 11)
- E. (1, 5)

SECTION 1- continued

Question 4

The angle of inclination for the positive x -axis for the point $(\sqrt{3}, 1)$ is

- A. 45°
- B. 60°
- C. 30°
- D. 90°
- E. 0°

Question 5

Which of the following is the expansion of $(x^{\frac{3}{2}} + 2)(x^{\frac{3}{2}} - 3)$?

- A. $x^{\frac{3}{2}} - 5x - 6$
- B. $x^3 - x^{\frac{3}{2}} + 6$
- C. $x^3 - x^{\frac{3}{2}} - 6$
- D. $x^{\frac{3}{2}} - x - 6$
- E. $x^3 - 5x^{\frac{3}{2}} + 6$

Question 6

Which of the following are the factors of $x^3 + 3x^2 - 3.25x - 5.25$?

- A. $(x - 3.5)(x + 1.5)(x - 1)$
- B. $(x - 3.25)(x - 5.25)(x + 3)$
- C. $(x + 3.5)(x - 1.5)(x + 1)$
- D. $(x + 3.25)(x + 5.25)(x - 3)$
- E. $(x + 3.5)(x - 1.5)(x - 1)$

Question 7

If $f(2) = 0$, then a factor of $f(x)$ is

- A. $x + 2$
- B. 2
- C. -2
- D. $x - 2$
- E. 0

SECTION 1- continued
TURN OVER

Question 8

The simplified form of $3 \leq 4x - 5 \leq 7$ is

- A. $\frac{3}{4} \leq x \leq \frac{7}{4}$
- B. $3 \leq x \leq 12$
- C. $8 \leq x \leq 12$
- D. $2 \leq x \leq 3$
- E. $2 \leq x \leq \frac{1}{2}$

Question 9

The solutions to $x^2 + 6x - 4 = 0$ are

- A. $-3 \pm \sqrt{13}$
- B. $-3 \pm \frac{\sqrt{13}}{2}$
- C. $3 \pm \sqrt{13}$
- D. $-6 \pm \sqrt{13}$
- E. $3 \pm \frac{\sqrt{13}}{2}$

Question 10

How many real solutions are there to $x^2 + x + 4 = 0$?

- A. One
- B. Four
- C. Two
- D. Three
- E. None

Question 11

The following are simultaneous equations

$$0.5x + 1.5y - 0.3z = 1.1$$

$$1.0x - 0.6y + 0.2z = 0.8$$

$$0.3x + 0.5y - 0.1z = 0.3$$

Solving the above simultaneous equations yields

- A. $x = -0.5, y = 5.5, z = 23$
- B. $x = 23, y = 5.5, z = -0.5$
- C. $x = 5.5, y = 23, z = -0.5$
- D. $x = -0.5, y = 23, z = 5.5$
- E. $x = 5.5, y = -0.5, z = 23$

Question 12

The following are simultaneous equations

$$y = x^2 - 4x - 3$$

$$y = x - 3$$

Solving the above simultaneous equations yields

- A. (3, 0) and (-2, 5)
- B. (0, -3) and (2, 5)
- C. (-3, 0) and (5, 2)
- D. (0, 3) and (5, 2)
- E. (0, -3) and (5, 2)

Question 13

Determine the domain and range of the following function: $f(x) = \frac{1}{x+4} - 1$

- A. Domain: $\mathbb{R} \setminus \{-4\}$ Range: $\mathbb{R} \setminus \{1\}$
- B. Domain: $\mathbb{R} \setminus \{4\}$ Range: $\mathbb{R} \setminus \{-1\}$
- C. Domain: $\mathbb{R}^+$ Range: $\mathbb{R}^+$
- D. Domain: $\mathbb{R}^-$ Range: $\mathbb{R}^-$
- E. Domain: $\mathbb{R} \setminus \{-4\}$ Range: $\mathbb{R} \setminus \{-1\}$

Question 14

$\mathbb{R}^- \cup [0, \pi)$ is equivalent to which of the following?

- A. $[-\infty, 0)$
- B. $(-\infty, 0]$
- C. $[-\infty, \pi)$
- D. $(-\infty, \pi)$
- E. $(-\infty, \pi]$

Question 15

Which of these relations is not a function?

- A. $3x - 2y = 5$
- B. $y = \sqrt{9 - x^2}$
- C. $y = 2$
- D. $x = 4$
- E. $y + x^2 = 4$

SECTION 1- continued
TURN OVER

Question 16

A function is given as $y = a(x - h)^2 + k$. Which of the following is **incorrect**?

- A. It has a turning point at (h, k)
- B. It is dilated a units in the horizontal direction
- C. It is a quadratic function
- D. It is shifted h units to the left/right
- E. It is shifted k units upwards/downwards

Question 17

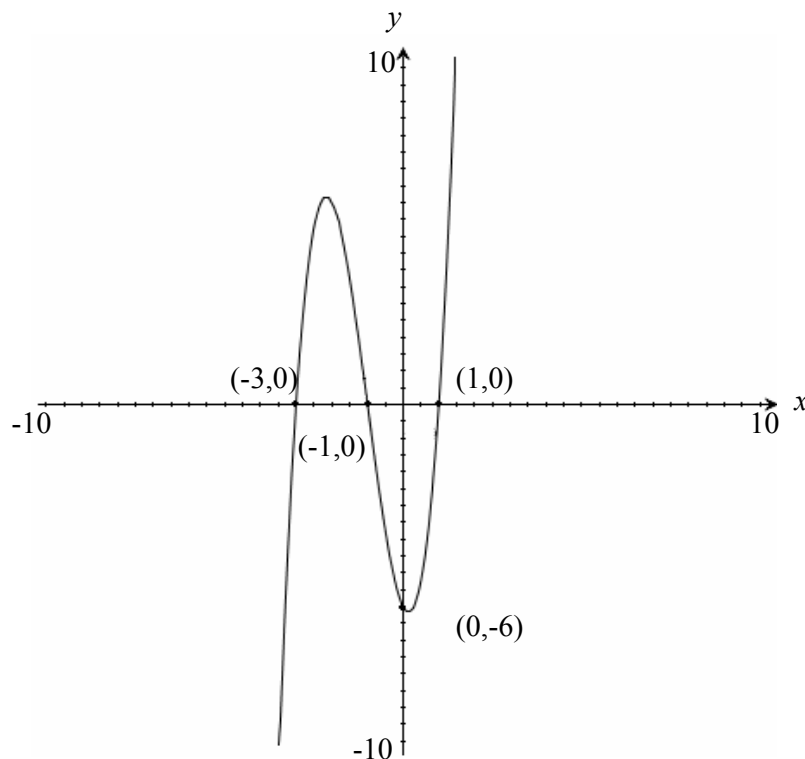
A polynomial of the form $y = ax^2 + bx + c$ passes through the points $(-2, -4)$, $(0.5, 2.25)$ and $(1, 8)$.

The actual equation of the polynomial is

- A. $y = 3x^2 - 7x - 2$
- B. $y = 3x^2 + 7x - 2$
- C. $y = 2x^2 + 7x - 3$
- D. $y = 2x^2 - 3x + 7$
- E. $y = x^2 + 7x - 6$

Question 18

This cubic is of the form $y = ax^3 + bx^2 + cx + d$.



The real equation of this cubic is

- A. $2x^3 + 6x^2 - 2x - 6$
- B. $x^3 - 3x^2 - x + 3$
- C. $x^3 + 3x^2 - x - 3$
- D. $2x^3 - 12x^2 - 8x + 48$
- E. $2x^3 - 10x^2 + 14x - 6$

SECTION 1- continued
TURN OVER

Use the following information to answer Questions 19 and 20.

The point $(-4,3)$ is transformed by a dilation of factor 2 from the y -axis, a reflection along $y = x$ and a translation of 3 units in the positive x direction.

Question 19

Which of the following sets of matrices accurately reflects such a transformation?

- A. $\begin{bmatrix} 3 \\ 1 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \times \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} -4 \\ 3 \end{bmatrix}$
- B. $\begin{bmatrix} -4 \\ 3 \end{bmatrix} \times \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \times \begin{bmatrix} 1 \\ 3 \end{bmatrix}$
- C. $\begin{bmatrix} 1 \\ 3 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \times \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} -4 \\ 3 \end{bmatrix}$
- D. $\begin{bmatrix} -4 \\ 3 \end{bmatrix} \times \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \times \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \times \begin{bmatrix} 3 \\ 1 \end{bmatrix}$
- E. $\begin{bmatrix} 3 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \times \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} -4 \\ 3 \end{bmatrix}$

Question 20

The coordinates of the new point are

- A. $(-24,3)$
- B. $(3,-24)$
- C. $(-8,9)$
- D. $(18,-4)$
- E. $(6,-8)$

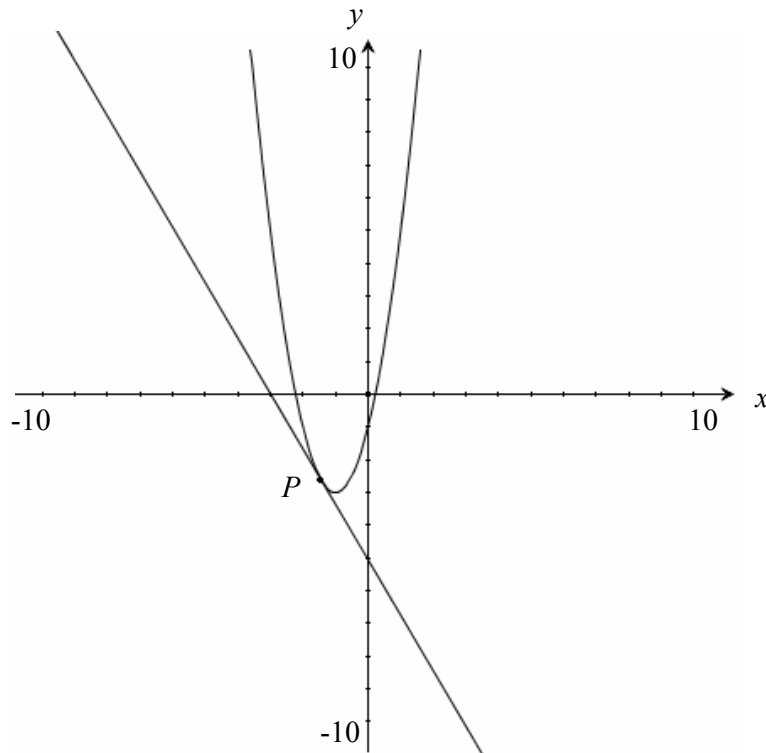
Question 21

The average rate of change for $\{(x,y): y = x^2 + 3x - 5, x \in [0,5]\}$ is

- A. 8
- B. 5
- C. -8
- D. 30
- E. -40

Question 22

A tangent has been drawn on this parabola to estimate the instantaneous rate of change at P .



This value is closest to

- A. 1.7
- B. 0.6
- C. -1.7
- D. -1.6
- E. -0.6

Question 23

If $\Pr(A \cup B) = 0.7$, $\Pr(A) = 0.3$ and $\Pr(A \cap B) = 0.1$, what is $\Pr(B)$?

- A. 0.3
- B. 0.5
- C. 0.4
- D. 0.7
- E. 0.6

Question 24

If $\Pr(A) = 0.8$, what is $\Pr(A')$?

- A. 0.5
- B. 0.4
- C. 0.1
- D. 0.2
- E. 0.8

SECTION 1- continued
TURN OVER

Question 25

If $\Pr(A \cap B) = 0.35$ and $\Pr(B) = 0.55$, what is $\Pr(A|B)$?

- A. $\frac{1}{5}$
- B. $\frac{11}{7}$
- C. $\frac{9}{13}$
- D. $\frac{7}{11}$
- E. 5

Question 26

What is the probability of rolling a 4 on a six-sided die and a 3 on a twenty-sided die?

- A. $\frac{1}{30}$
- B. $\frac{1}{26}$
- C. $\frac{1}{120}$
- D. $\frac{1}{14}$
- E. $\frac{1}{12}$

END OF SECTION 1

SECTION 2**Instructions for Section 2**

A decimal approximation will not be accepted if an **exact** answer is required to a question.
In questions worth more than one mark, appropriate working **must** be shown.
Marks are given as specified for each question.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

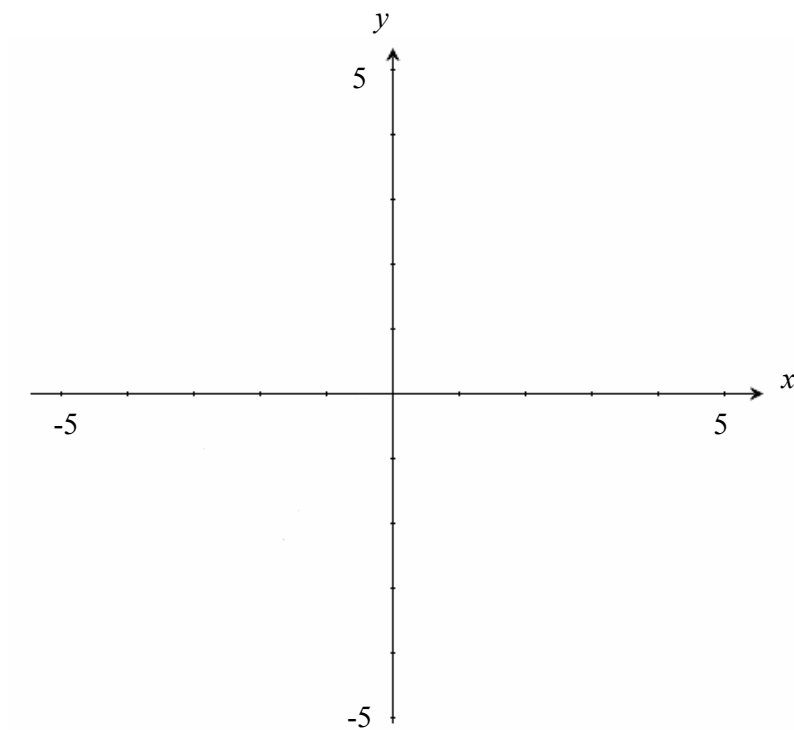
Consider the relation $\{(x, y) : x^2 - 4x + y^2 + y = 4.75\}$

- a. Showing your work, complete the square for both the x and the y components to express your answer in the form $(x - h)^2 + (y - k)^2 = r^2$.

4 marks

SECTION 2- Question 1- continued
TURN OVER

- b. Graph the relation on the axes provided, indicating any key points, including intercepts to the nearest tenth of a unit.



Working space

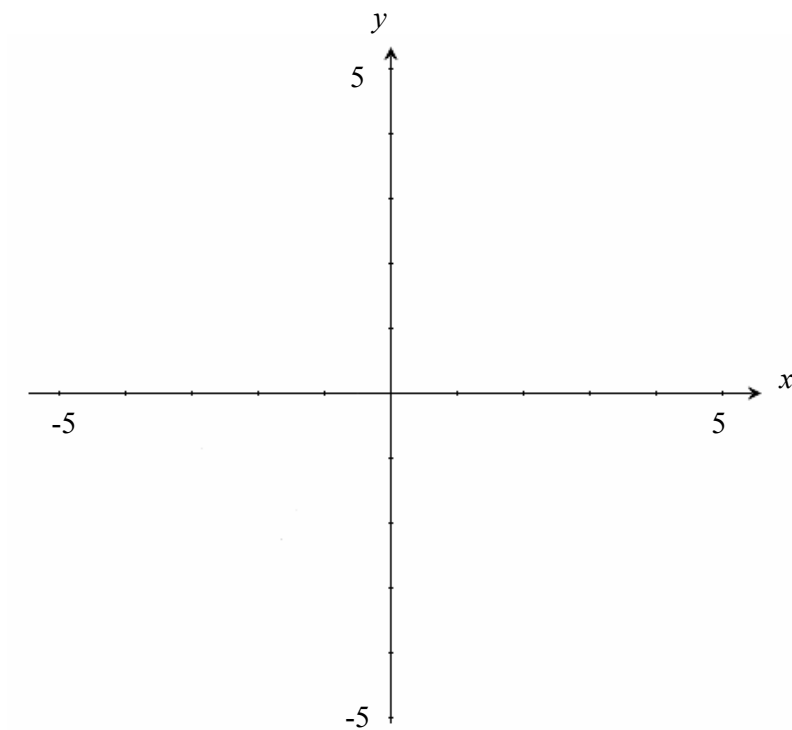
5 marks

- c. State the range and domain of the relation.

2 marks

SECTION 2- Question 1- continued

- d. Now graph the relation $\{(x, y) : (x-1)^2 + (y+1)^2 \leq 16\} \cap \{(x, y) : x^2 + y^2 \geq 1\}$



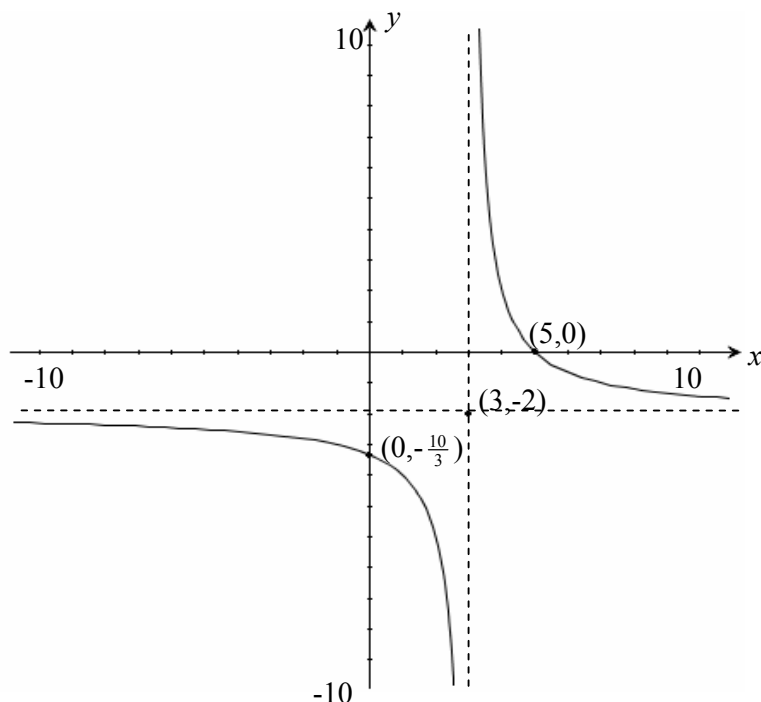
Working space

4 marks
Total 15 marks

SECTION 2- continued
TURN OVER

Question 2

The following function has the equation $y = a(x-h)^{-1} + k$



- a.** Determine the values of a , h , and k . Hence, find the equation of the function.

4 marks

- b.** Determine the inverse of the function found in **part a**.

2 marks

SECTION 2- Question 2- continued
TURN OVER

- c. On the original axes, graph the inverse function, identifying any asymptotes and intercepts (to the nearest unit).

5 marks

- d. Determine the equation of a hyperbola with vertical asymptote $x = -2$, horizontal asymptote $y = 4$ and contains the point $(1, 5)$.

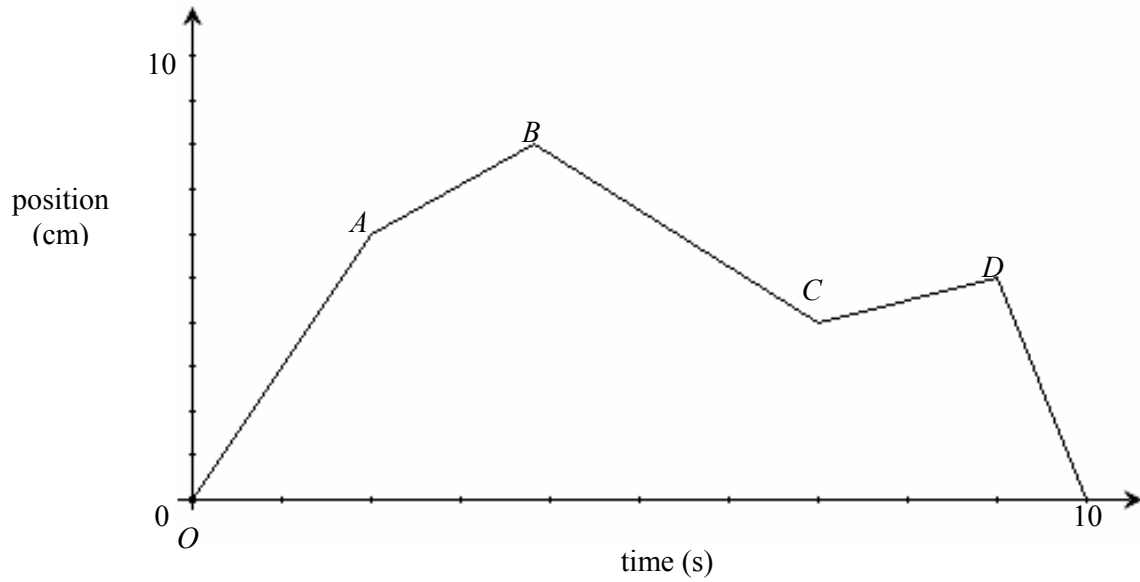
3 marks

Total 14 marks

SECTION 2- continued
TURN OVER

Question 3

The position of a remote-control car from its owner over 10 seconds is outlined below:



- a. Calculate the total distance travelled by the car during these 10 seconds.

2 marks

- b. Determine the average velocity for the following:

i. OA

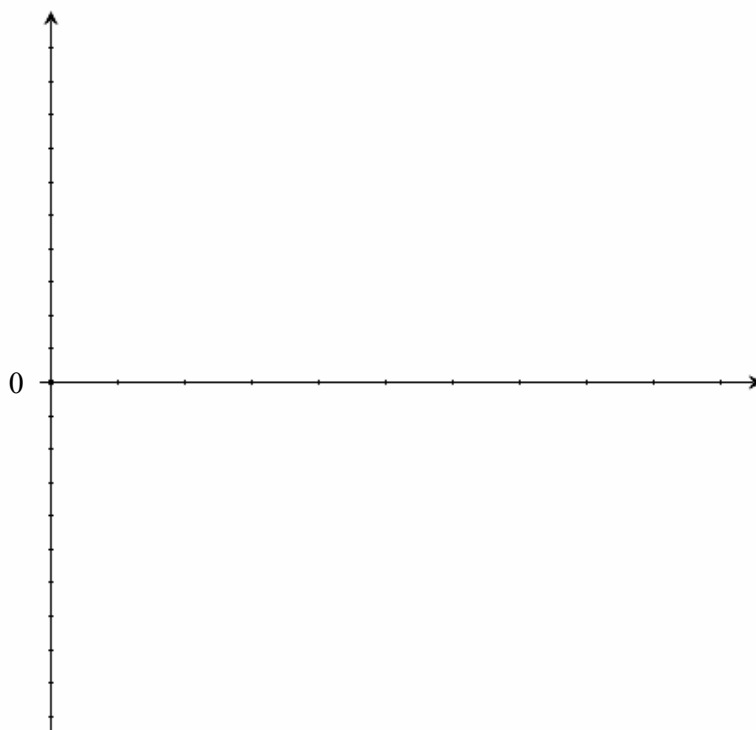
ii. AB

iii. BC

1 + 1 + 1 = 3 marks

SECTION 2- Question 3- continued

- c. Using the axes below, plot a velocity time graph from your information gathered in **part b**.



4 marks

- d. Showing your work, verify that your answer to **part a**. was accurate by calculating the total distance travelled using your graph from **part c**.

3 marks

Total 12 marks

SECTION 2- continued
TURN OVER

Question 4

Suppose that the outcome of The Ashes test series between Australia and England can be represented as follows:

$$\Pr(\text{England wins match 1}) = \frac{3}{8}$$

$$\Pr(\text{England wins the match if they won the preceding match}) = \frac{1}{3}$$

$$\Pr(\text{Australia wins match 1}) = \frac{5}{8}$$

$$\Pr(\text{Australia wins the match if they won the preceding match}) = \frac{1}{2}$$

- a. Construct a tree showing the possible outcomes (and probabilities) for the first three matches.

4 marks

- b. What is the probability that Australia wins the first three matches?

2 marks

SECTION 2- Question 4- continued

- c. What is the probability that England fails to win only the last two matches?

2 marks

- d. What is the probability that the series is tied at 1-1 after the first two matches?

2 marks

- e. What is the probability that England will lead 2-1 after the first three matches?

3 marks

Total 13 marks

END OF QUESTION AND ANSWER BOOK